Detecting Outer Planets in Edge-On Orbits: Combining Radial Velocity and Astrometric Techniques

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ABSTRACT

The astrometric and radial velocity techniques of extra-solar planet detection attempt to detect the periodic reflex motion of the parent star by extracting this periodic signal from a time-sampled set of observations. The extraction is generally accomplished using periodogram analysis or the functionally equivalent technique of Least Squares fitting of sinusoids. In this paper, we use a Frequentist approach to examine the sensitivity of Least Squares technique when applied to a combination of radial velocity and astrometric observations. We derive a semi-analytical expression for the sensitivity and show that the combined approach yields significantly better sensitivity than either technique on its own. We discuss the ramifications of this result to upcoming astrometric surveys with FAME, the Keck Interferometer, and SIM.

1. Introduction

Radial velocity (RV) surveys of nearby stars have been employed in the search for extra-solar planets for nearly two decades (see Marcy, Cochran & Mayor 2000). As these efforts continue into the next decade, they will be supplemented by precision astrometric searches, e.g., by FAME, Keck Interferometer, and SIM (Horner et al. 2000; van Belle et al. 1998; Danner et al. 1999). In previous papers, we examined the RV and astrometric techniques in detail, paying particular attention to the regime where the time-baseline of the observations is shorter than the orbital period of the extra-solar companion (Eisner & Kulkarni 2001a,b; hereafter EK2001a,b). This regime is interesting because one expects giant planets to form in the colder regions of the proto-planetary nebula, and thus one expects such objects to possess periods of many years to centuries (Boss 1995). In EK2001a,b we demonstrated that one can achieve a significant improvement in sensitivity (over current techniques) if the orbital amplitude and phase are included in the analysis.

Here, we examine the benefits of combining simultaneous astrometric and RV observations. Specifically, we examine the sensitivity of a combined astrometric and RV detection
technique applied to an edge-on orbit, where the full RV signature and one dimension of the astrometric signature can be observed.

The plan for the paper is straightforward. First, we simulate large numbers of hypothetical data sets containing (1) noise only, and (2) signal and noise, and determine the Frequentist Type I and II errors. As in EK2001a,b we acknowledge that a Frequentist approach is not as rigorous as a full Bayesian analysis. However, this approach is simple enough that it is amenable to deriving (semi-)analytical estimates of the sensitivity – a principal goal of the paper. We conclude by discussing the parameter space opened up by combining FAME, Keck Interferometer, or SIM astrometric surveys with ongoing precision RV studies.

2. Basic Equations

We will assume edge-on circular orbits throughout this discussion. The astrometric signature of an edge-on circular orbit is given by

$$\theta(t) = A \sin \left( \frac{2\pi t}{\tau} + \phi \right) + \lambda t + \mu,$$

where $\lambda$ and $\mu$ are the proper motion and parallax of the planetary system, respectively, and

$$A = \frac{M_p D}{D^2} \left( \frac{G \tau^2}{4\pi^2 M_*^2} \right)^{\frac{1}{3}}.$$

Here, $D$ is the distance to the system, $M_*$ is the mass of the star, and $M_p$ is the mass of the planet. We ignore the annual parallax. However, annual parallax should be included in modeling of planets with periods around one year.

The RV signature of this orbit is given by the derivative of the orbital position along the line of sight:

$$v(t) = V \sin \left( \frac{2\pi t}{\tau} + \phi \right) + \gamma.$$

Here $\gamma$ is the radial velocity of the planetary system, and

$$V = M_p \left( \frac{2\pi G \tau^2}{M_*^2} \right)^{\frac{1}{3}} = \frac{2\pi D}{\tau} \times A.$$

Thus, we can express the sensitivity (defined as the minimum-mass planet that can be detected) of the RV and astrometric techniques in terms of $A$:

$$M_p = D A \left( \frac{4\pi^2 M_*^2}{G \tau^2} \right)^{\frac{1}{3}}.$$
However, it is more difficult to identify planets with long periods than Equation 5 might suggest. In the so-called “long-period regime”, defined as $T_0 << \tau$ where $T_0$ is the duration of the survey, we observe a fraction of the orbit. As a result, in this regime, the sensitivity is expected to depend critically on the orbital phase. The reflex velocity is covariant with $\gamma$ and thus the RV technique is most sensitive when $2\pi t/\tau + \phi = n\pi$ (EK2001a). In contrast, the astrometric signal of an edge-on orbit is covariant with $\lambda t$ and $\mu$, and thus the astrometric technique is sensitive when $2\pi t/\tau + \phi = (n+1/2)\pi$ (EK2001b). Thus the RV and astrometric techniques achieve their maximal sensitivities for different orbital phases, and we expect, on general grounds, that combining the two techniques should yield a substantial benefit in the long-period regime.

3. Monte Carlo Analysis

The signal analysis for the astrometric and RV techniques consists of fitting the observations to the models specified in Equations 1 and 3. As noted by several authors (e.g.) EK2001a, SCARGLE82, NA98 the most optimal fitting is obtained by using the technique of Least Squares. First, we convert the physical model specified by Equations 1 and 3 to equations linear in the unknowns:

$$\theta(t) = A_c \cos(\omega t) + A_s \sin(\omega t) + \lambda t + \mu,$$

$$v(t) = V_c \cos(\omega t) + V_s \sin(\omega t) + \gamma.$$  

Here, $A_c = A \sin \phi$, $A_s = A \cos \phi$, $V_c = V \sin \phi$, $V_s = V \cos \phi$, and $\omega = 2\pi/\tau$. In EK2001a,b we discuss the importance of the $\gamma$, $\lambda$ and $\mu$ terms. These three variables are not directly relevant in detecting or characterizing a companion planet but they are unknown and in the long-period regime are covariant with some of the orbital parameters (Black & Scargle 1982). Thus the three variables must be solved for in order to correctly model the observations.

Using Equations 6 and 7 as our physical model, we perform the following analysis. First, we simulate a large number of data-sets containing only Gaussian noise (i.e. no signal). For each of these data sets, we perform a Least Squares fit to three models: a model using only astrometric measurements (Equation 6), a model using only RV measurements (Equation 7), and a model that utilizes both astrometric and RV measurements. In each case, for each simulated data set we fit for amplitude and phase. We note here that for the RV+astrometry model, since the two measurements have different variances we minimize the $\chi^2$ (where $\chi$ is the difference between the model and the rms-weighted measurements).

Specifically, we simulate $N = 1000$ data-sets, sampled at one month intervals for $T_0 = 10$ years (with no loss of generality, we take the time interval to go from $-T_0/2$ to $T_0/2$), and
we explore periods from 5 to 100 years. We assume that the measurement noise in both the RV and astrometric surveys is characterized by Gaussian noise with rms of $\sigma_{\text{RV}}$ and $\sigma_{\text{ast}}$ respectively. The best achieved $\sigma_{\text{RV}} = 3 \text{ m s}^{-1}$ (Butler et al. 1996). The anticipated astrometric precision of FAME is between 50 and 100 $\mu$as (Horner et al. 2000), that of the Keck Interferometer (narrow angle) between 30 and 50 $\mu$as (van Belle et al. 1998), and that of SIM between 1 and 10 $\mu$as (Danner et al. 1999). We note that $\sigma_{\text{ast}} = 100 \mu$as yields approximately equivalent sensitivity to RV technique with $3 \text{ m s}^{-1}$ rms for a planet orbiting a star located at distance $D = 10 \text{ pc}$ with $\tau \sim 2 T_{0} \sim 20$ years (Equation 4).

Next, for each of three models, we determine the ellipse (in $A-\phi$ space) within which 99% of the fitted amplitudes and phases lie. This ellipse, denoted by $\epsilon_{1}$, describes the “Type I” errors of the detection technique. Thus the inferred $A$ and $\phi$ have a 1% chance of being outside the $\epsilon_{1}$ ellipse (in the absence of a signal).

As discussed earlier ($\S$2) we expect RV and astrometric models to show orthogonal sensitivity. Indeed, as can be seen from Figure 1, the $\epsilon_{1,\text{RV}}$ and $\epsilon_{1,\text{ast}}$ are 90° out of phase in the long period regime. On a basic level, we can understand the benefit of combining RV and astrometric observations by noting that the intersection of $\epsilon_{1,\text{rv}}$ and $\epsilon_{1,\text{ast}}$ is much smaller than either of the individual ellipses, and thus it is easier to detect signals over the level of the noise. In fact, this is verified by the combined analysis: the ellipse for the combined analysis, $\epsilon_{1,C}$, lies entirely within both $\epsilon_{1,\text{RV}}$ and $\epsilon_{1,\text{ast}}$ (Figure 1).

Analytic expressions for the Type I errors for RV or astrometric techniques are given in EK2001a and EK2001b, respectively. Given these expressions, it is not difficult to infer an analytic expression for the Type I errors in the case of combined astrometric+RV technique. As noted earlier for $\sigma_{\text{RV}} = 3 \text{ m s}^{-1}$ and $\sigma_{\text{ast}} = 100 \mu$as, the semi-minor axes for $\epsilon_{1,\text{rv}}$ and $\epsilon_{1,\text{ast}}$ are approximately equal ($D = 10 \text{ pc}$), and thus $\epsilon_{1}$ for the combined analysis will be a circle whose radius is given by

$$A_{c1} = \left\{ \begin{array}{ll}
2^{-1/2} \min(A_{1s}, \frac{T}{2\pi D} v_{1s}) & \text{for } \tau < T_{0} \\
\frac{2A_{1s}}{1-\cos(\pi T_{0}/\tau)} & \text{for } \tau > T_{0} \end{array} \right.. \quad (8)$$

Here, $A_{1s} = 3.69\sigma_{\text{ast}}n_{r}^{-1/2}$, $v_{1s} = 3.69\sigma_{\text{rv}}n_{r}^{-1/2}$, and the factor of $2^{-1/2}$ reflects the fact that in the short-period regime, there are essentially twice as many measurements. As illustrated in Figure 2, this analytic function provides an excellent fit to the data.

Next, we evaluate “Type II” errors for the three models. Type II errors describe the probability of failing to detect a genuine signal due to contamination by noise. To understand the type II statistics, we simulate a large number of data sets consisting of a simulated signal and noise (see EK2001a,b for further details). The signal is a sinusoidal wave with an amplitude $A_{0}$ (astrometry), and the corresponding velocity amplitude is $2\pi D A_{0}/\tau$ (we set
For each model, we increment the amplitude[s] until 99% of the fitted orbital parameters lie outside of the appropriate $\epsilon_1$ ellipse; this amplitude is denoted by $A_{99}$ (for each method).

4. Results and Discussion

The benefit of combining RV and astrometric analysis accrues mainly from the fact that the error ellipses for the two techniques in $A-\phi$ parameter space are perpendicular to each other (Figure 1). RV+astrometry analysis will be most useful in cases where the error ellipses for the two techniques are roughly the same size (otherwise, one error ellipse might lie entirely within the other, and no additional benefit would arise from combining the two techniques). As mentioned above, the current precision of RV techniques is $\sim 3 \text{ m s}^{-1}$ (Butler et al. 1996), which means that for a 10 year survey, we must use astrometric measurements with $\sim 100 \mu\text{as}$ precision (for a system at $D = 10$ pc) to reap the maximal benefit from RV+astrometry technique. This is approximately the sensitivity that will be obtained by future instruments like Keck Interferometer and FAME (van Belle et al. 1998; Horner et al. 2000).

As illustrated by Figure 3a, RV+astrometry analysis (with comparable RV and astrometric measurement accuracies) applied to edge-on orbits attains approximately the same sensitivity as an astrometric analysis applied to face-on orbits. This similarity stems from the fact that in both cases, the highly elliptical 1-D $\epsilon_1$ is circularized through the addition of a second dimension. Another way of thinking about this is that no matter what part of the orbit, when we observe both dimensions we can always see the full orbital curvature. Thus, combining astrometric and RV techniques in a large survey ensures good sensitivity for all orbital inclination angles.

It is also worth noting that RV+astrometry yields valuable gains in the short-period regime ($\tau < T_0$). When $\sigma_{\text{RV}}$ is comparable to $\sigma_{\text{ast}}$, the sensitivity of RV+astrometry is better by $2^{-1/2}$ over RV or astrometry alone. Furthermore, noting that the sensitivity of astrometry to face-on orbits is $2^{-1/2}\sigma_{\text{ast}}$ (EK2001b), we see that the short-period sensitivity of RV+astrometry is approximately independent of orbital inclination.

We have also examined the sensitivity when astrometric data is combined with RV data of a longer time-baseline, specifically for several upcoming missions (recall that RV surveys will have been underway for 15–20 years by the time astrometric surveys commence). We investigate FAME ($T_{0,\text{RV}} = 20$ years, $T_{0,\text{ast}} = 5$ years), Keck Interferometer ($T_{0,\text{RV}} = 20$ years, $T_{0,\text{ast}} = 10$ years), and SIM ($T_{0,\text{RV}} = 30$ years, $T_{0,\text{ast}} = 10$ years). We find that in the cases of FAME and Keck Interferometer, the addition of longer time-baseline RV measurements
has a significant impact (Figures 3b–3c). In fact, RV+astrometry analysis can easily detect Saturns when astrometric analysis alone doesn’t come close (Figure 3b). In the case of SIM, the main benefit of RV+astrometry is that one can achieve optimal sensitivity over a wider range of inclination angles (Figure 3d).

4.1. Shorter Surveys

We have also examined the sensitivities of the RV, astrometric, and combined techniques applied to shorter duration surveys, in order to compare the various sensitivities for short-period companions. Specifically, we investigate the prospect of finding companions around M dwarfs with a 2-year survey combining RV measurements with astrometric measurements from AO systems on large telescopes like Keck or the Palomar 200''. Previous authors have successfully searched for companions around M-dwarfs using RV measurements and astrometric measurements on AO systems (\cite{Delfosse+99} although they have not used the combined RV+astrometry analysis described here (i.e., they analysed the RV data and the astrometric data separately).

As illustrated by Figure 4, the RV technique gains significantly over astrometry for short periods (because $V \propto A/\tau$; Equation 4). If astrometry is to contribute meaningfully then $\sigma_{\text{ast}} \lesssim 1$ mas. There is some expectation that such a precision can be obtained for binary stars (Dekany et al. 1994). If so, combined RV+Astrometry surveys of nearby M dwarfs can measure masses of Jupiter and Saturn-mass companions.

REFERENCES


Fig. 1.— A plot of the 1% error ellipses for the astrometric and RV techniques for a 90 year orbit ($\tau \sim 9 T_0$). The error ellipse for combined RV+astrometry technique is also shown. These are the ellipses, $\epsilon_1$, for which 99% of Least Squares fits to simulated Gaussian noise produce fitted amplitudes and phases that lie within $\epsilon_1$ (§3). We have scaled the ellipses to units of companion mass via Equation 5.
Fig. 2.— A plot of $A_{c1}$ versus orbital period (solid line). The analytic expression given by Equation 8 is also plotted (dashed line). $A_{c1}$ is the value of $|A_c|$ that is exceeded in 1% of least-squares fits to Gaussian noise, and describes the radius of the Type I error ellipse (actually a circle in this case).
Fig. 3.— Plots of $\log(M_{99})$, in units of Jupiter masses, versus $\log(\tau/T_0)$. $M_{99}$ is $A_{99}$ expressed in units of companion mass, assuming $M_* = M_\odot$ (Equation 5). The positions of Jupiter, Saturn and Uranus in this parameter space are also indicated. In (a), both the RV and astrometric surveys have a duration of $T_0 = 10$ years. (b) Shows the simulated sensitivity for FAME+RV, (c) shows Keck Interferometer+RV, and (d) shows SIM+RV.
Fig. 4.— Plots of \( \log(M_{99}) \), in units of Jupiter masses, versus \( \log(\tau/T_0) \). \( M_{99} \) is \( A_{99} \) expressed in units of companion mass, assuming \( M_* = 0.25M_\odot \) (Equation 5).