Thermal instability of an optically thin dusty plasma

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Abstract

We investigate the role of thermal instability, arising from radiative cooling of an optically thin, dusty plasma, by linear stability analysis. The corresponding isobaric stability condition for condensation mode is found to be modified significantly by the degree of ionization and concentration of finite sized, relatively heavy, and negatively charged dust particles. It has been shown that though the fundamental wave mode is similar in nature to the one in absence of dust particles, a new dust-wave mode can propagate in such a plasma. It is conjectured that the presence of negatively charged dust particles may considerably affect the stability of various astrophysical structures against thermal instability, which can not be explained with the help of gravitational instability.

Key words: thermal instability, dusty plasma

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The formation and existence of a large number of astrophysical structures such as interstellar clouds, solar prominence, localized structures in planetary nebulae etc. can not be explained by means of conventional gravitational instability [1]. However, it has been argued that thermal instability may be a reasonably good candidate, with can accelerate condensation, giving rise to localized structures which grow in density by loosing heat, mainly through radiation [1,2]. The first comprehensive analysis of thermal instability in a diffuse interstellar gas is first given by Field [2]. Subsequently, many authors considered the process of thermal instability in different circumstances e.g. in hydrogen plasmas [3], to explain formation of clumpy gas clouds in two phase medium [4], and thermal instabilities in photoioized interstellar gas [5]. Recently, Birk [7] has considered thermal condensations in a weakly ionized hydrogen plasma.

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In recent years, however, it has been realized that heavier dust particles constitutes an ubiquitous and important component of many astrophysical plasmas. [7–9] including interstellar clouds, stellar and planetary atmosphere, planetary nebulae etc. It is, thus of particular interest to see the effect of dust particles on the role of thermal instability in astrophysical situations. One of the obvious facts of introducing dust as a component of plasma is the propagation of very low frequency compressional modes [10]. We, in this paper, investigate thermal instabilities in a multicomponent plasma with electrons, ions, and dust particles, assuming that $a \ll d \ll \lambda_D$, where $a$ is the average dust size, $d$ is the average inter-dust distance, and $\lambda_D$ is the Debye length. We consider the dust particles to be negatively charged and cold, whereas the ions can loose energy through radiation by de-excitation. We further assume that electrons are isothermal. We introduce a heat-loss function [2] exclusive of thermal conduction, which takes care of the radiative loss. For simplicity, we do not take into account the dust-charge fluctuation, which usually has a stabilizing effect [6]. The resultant equations are two-fluid magnetohydrodynamics (MHD) equations with Boltzmann distribution for the electrons. The dust acoustic (DA) and dust-ion acoustic (DIA) modes [10] are ruled out by virtue of ‘cold dust’ assumption. We show that the isobaric condition for thermal condensation mode is significantly modified by the inclusion of dust dynamics. We also find the existence of a new wave mode, solely because of the presence of the dust particles.

We begin by writing out the equations for a unbounded, weakly collisional dusty plasma with a heat-loss function $\mathcal{L}(\rho, T)$, which represents the rate of net loss of energy per unit mass, through radiation. We consider the dust particles to be negatively charged and cold and the electrons obey Boltzmann relation. We assume here that that ion-neutral collisional cross-section is of the same order as that of the electron-neutral collisions, so that the electrons can be treated as Boltzmann distribution if $(T_i/T_e)(v_{Te}/v_{Ti}) \gg 1$, and in practice this condition is satisfied in the cases we are going to consider in this work. The equations are,

\[
\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0, \quad (1)
\]

\[
m_d \frac{d\mathbf{v}_d}{dt} = -eZ_d \mathbf{E}, \quad (2)
\]

\[
n_e = n_{e0} \exp \left( \frac{e\phi}{T_e} \right), \quad (3)
\]

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = -\nu_R n_i + \nu_I n_e, \quad (4)
\]

\[
m_i n_i \frac{d\mathbf{v}_i}{dt} = eZ_i n_i \mathbf{E} - \nabla p_i, \quad (5)
\]
\[
\frac{3}{2} n_i \frac{dT_i}{dt} + p_i (\nabla \cdot \mathbf{v}_i) = \nabla \cdot (\chi_i \nabla T_i) - m_i n_i \mathcal{L}(n_i, T_i),
\]

where \(\chi_i\) is the ion thermal conductivity, \(\nu_{I,R}\) are ionization and recombination frequencies, and the other symbols have their usual meanings. The subscripts \(e, i, d\) respectively refer to the electron, ion, and dust populations. Equations (1) through (6) are the continuity and momentum equations for dusts, Boltzmann relation for the electrons, and continuity, momentum, and energy equations for the ions. The set of equations is closed by the quasineutrality condition,

\[
Z_i n_i = n_e + Z_d n_d,
\]

where \(Z_{i,d}\) represents the ion and dust charge numbers, respectively. Note that we have taken into account the possibility of multiply ionized ions. The ion pressure is given by \(p_i = n_i T_i\) (\(T_j\) are expressed in energy units) and we have assumed that \(\Gamma\), the ratio of specific heats for the ions to be 5/3. The recombination loss term in the ion-continuity equation represents the electron-ion recombination on the surface of the dust particles [11] and \(\nu_R \propto \pi r_d^2 n_d n_i c_s^2\), where \(r_d\) is the dust radius and \(c_s = (T/e_i m_i)^{1/2}\) is the sound speed.

We consider now a small perturbation of the form \(\exp(-i\omega t + ik \cdot \mathbf{r})\). The linearized expression for electron, dust, and ion densities are given by

\[
\tilde{n}_e = \tilde{\phi},
\]

\[
\tilde{n}_d = \frac{\omega_d^2}{\omega^2} \tilde{\phi},
\]

\[
\tilde{n}_i = \frac{1}{Z_i} \left( \delta_e - \frac{\omega_d^2}{\omega^2} \delta_d Z_d \right) \tilde{\phi},
\]

where we have used the linearized form of the Boltzmann relation for the electrons Eq.(3), dust-continuity equation Eq.(1), and the quasineutrality condition Eq.(7). We have used \(\omega_d = kc_d\) and \(c_d^2 = Z_d T_{e0}/n_{d0}\). The quantities with subscript ‘0’ and ‘1’ are equilibrium and perturbed quantities, respectively and ‘~’ represents normalized perturbed quantities. We have normalized the perturbed densities of the species by their respective equilibrium values and perturbed electrostatic potential energy \(e\tilde{\phi}_1\) by equilibrium electron thermal energy, \(\tilde{\phi} = e\tilde{\phi}_1/T_{e0}\). The quantities \(\delta_j = n_{j0}/n_{i0}\) are the ratios of equilibrium electron and dust densities to that of the ions for \(j = e, d\).

The linearized ion-continuity and momentum equations are given by

\[
-\omega \tilde{n}_i + i(k \cdot v_{i1}) = -\nu_R (\tilde{n}_d + \tilde{n}_i) + \nu_I \tilde{n}_e \delta_e,
\]

\[
\omega (k \cdot v_{i1}) = \omega_i^2 (\tau \tilde{\phi} + \tilde{p}_i),
\]
where \( \omega_i = k c_i \) is the ion-sound frequency, \( c_i = (T_{i0}/m_i)^{1/2} \) is the ion-thermal speed, \( \tau = Z_i T_{e0}/T_{i0} \), and \( \tilde{p}_i = p_{i1}/p_{i0} \) is the normalized perturbed ion pressure. Finally we write down the linearized energy equation for the ions as

\[
-\frac{3}{2} i \omega \tilde{T}_i + i \tilde{\varphi} \left[ \frac{1}{Z_i} \left( \delta_e - \delta_d Z_d \frac{\omega_d^2}{\omega^2} \right) (\omega + i \nu_R) - i \left( \nu_I \delta_e + \nu_R \frac{\omega_d^2}{\omega^2} \right) \right] = 0 \tag{13}
\]

\[
+ \omega_p \frac{1}{Z_i} \left( \delta_e - \delta_d Z_d \frac{\omega_d^2}{\omega^2} \right) \tilde{\varphi} + \omega_T \tilde{T}_i,
\]

where we have defined \( \tilde{T}_i = T_{i1}/T_{e0} \), \( \omega_p = k_p c_i \), and \( \omega_T = (k_T + k^3/k_K)c_i \). Following Field [2], we define the wave numbers \( k_{\rho,T,K} \) as

\[
k_{\rho} = \mathcal{L}_\rho \frac{m_i n_{i0}}{c_i^3}, \quad k_T = \mathcal{L}_T \frac{T_{i0}}{c_i^3}, \quad \text{and} \quad k_K = \frac{c_i n_{i0}}{\chi_i}. \tag{14}
\]

The first two are the wave numbers of isothermal and isochoric perturbations, respectively and the third one is the reciprocal of mean free path of the heat-conducting particles. It has been assumed throughout the derivation that the perturbed ion thermal conductivity \( \tilde{\chi}_i = 0 \) and the equilibrium heat-loss function \( \mathcal{L}(\rho_0 = m_i n_{i0}, T_{i0}) = 0 \), where \( \mathcal{L}_{\rho,T} \) represents \( (\partial \mathcal{L}/\partial T)_\rho \) and \( (\partial \mathcal{L}/\partial \rho)_T \), evaluated for the equilibrium state.

Using Eqs.(8)-(13), the dispersion relation for this radiating dusty plasma can now be written as

\[
\left( 1 + i \frac{2 \omega T}{3 \omega} \right) \left[ \omega^2 \frac{1}{\omega^2} \left( \delta_e - \delta_d Z_d \frac{\omega_d^2}{\omega^2} \right) \left( 1 + i \frac{\nu R}{\omega} \right) \right] = 0 \tag{15}
\]

\[
- \tau - \frac{1}{Z_i} \left( \delta_e - \delta_d Z_d \frac{\omega_d^2}{\omega^2} \right) - i \frac{\omega}{\omega_i} \left( \nu_I \delta_e + \nu_R \frac{\omega_d^2}{\omega^2} \right) \]

\[
+ \frac{2}{3} \left[ \frac{1}{Z_i} \left( \delta_e - \delta_d Z_d \frac{\omega_d^2}{\omega^2} \right) \left\{ i \frac{\omega}{\omega} (\omega_p - \nu_R) - 1 \right\} + i \frac{\nu_I \delta_e + \nu_R \omega_d^2}{\omega^2} \right] ,
\]

where we have used the relation \( \tilde{\rho}_i = \tilde{n}_i + \tilde{T}_i \).

Before numerically solving the dispersion relation Eq.(15), however, it is instructive to analyze Eq.(15) under certain limits. In the limit of collisionless electron-ion plasma with no dust particles and thermal radiation and zero heat-conductivity, Eq.(15) reduces to

\[
\omega^2 = \omega_i^2 \left( \frac{5}{3} + Z_i \frac{T_{e0}}{T_{i0}} \right), \tag{16}
\]

which is just the ion-sound wave modified by the ion-charge number. We now
look at the expansion of Eq.(15) [2], in the limit of weak collisions ($\nu_{I,R} \ll \omega$) and vanishing thermal perturbation i.e. $\omega_{\rho,T} \to 0$, which, for the condensation mode, is given by

\begin{align}
\nonumber n_{\text{cond}} &\simeq -\frac{2\delta_e}{(5\delta_e + 3Z_i \tau)} \left[ \omega_{\rho} - \left( 1 + \frac{Z_i}{\delta_e \tau} \right) \omega_T - (\nu_R - Z_i \nu_I) \right] \\
&\quad + \frac{8\delta_e^4}{\omega_T^2 (5\delta_e + 3Z_i \tau)^4} (3\omega_{\rho} + 2\omega_T) \left[ \omega_{\rho} - \omega_T \left( 1 + \frac{Z_i}{\delta_e \tau} \right) \right]^2 \\
&\quad + O(\omega_{\rho,T}^5),
\end{align}

where we have substituted $\omega = i\omega$ and neglected $\omega_d$. The characteristic velocity of the modes, we are interested, is $c_i$ and in a plasma with cold dusts ($T_d \ll T_{i,e}$), as this one, $c_i/c_d \gg 1$. For example, in interstellar clouds, $n_i \sim 10^{-3}$, $n_d \sim 10^{-7}$, $T_e \approx T_i \sim 10^2$K and with $Z_d \sim 10^2$ and $m_d \sim 10^8 m_i$, $c_i/c_d \sim 10^3$, so that we can safely assume $\omega_d \ll \omega$. To the leading order, we have the radiation condensation mode if,

$$\omega_{\rho} + Z_i \nu_I > \nu_R + \left( 1 + \frac{Z_i}{\delta_e \tau} \right) \omega_T,$$

which reduces, in the collisionless limit to

$$\omega_{\rho} > \left( 1 + \frac{Z_i}{\delta_e \tau} \right) \omega_T.$$ 

Note that for zero thermal-conductivity, $\omega_T = k_T c_i$ and the above conditions are independent of $k$. The conditions given by inequalities (18) and (19) are similar to the isobaric instability condition given by Field [2], modified by collisions and dust particles. We observe that the presence of a significant dust population (smaller $\delta_e$) considerably modifies the instability criteria for the condensation mode and has a stabilizing effect. It is interesting to note that the degree of ionization has also a similar effect on the stability of the condensation mode which becomes important in certain situations. For example, in condensations observed in planetary nebulae, the thermal instability is strongly associated with collisions of electrons with $N^+$ and $N^{++}$ ions [2]. It should, however, be noted that the conditions (18) and (19) do not reduce to one given by Field even in absence of dust particles, which is because our consideration of two-fluid approximation.

The inequality (18) also implies that the growth rate of the condensation mode is independent of the wave number in the short wavelength limit, when the thermal-conductivity is zero in opposite to a thermally conducting plasma (finite $k_K$), where there is no condensation for sufficiently larger $k$. Plots of the normalized growth rate ($\hat{\omega} = \omega/\omega_{\rho}$) versus normalized wave number ($\hat{k} = k/k_i$).
Fig. 1. Normalized growth rate of the collisionless condensation mode vs. wave number. The solid curves are with dust particles whereas the dashed ones are without any dust. The topmost curves are for $\hat{k}^{-1} = k_p/k_K = 0$ and the others for $\hat{k}^{-1} = 0.01$ and 0.1, respectively.

$k/k_p$ for the condensation mode are shown in Fig.1, in absence of collisions. The growth rate of the mode is definitely smaller in presence of dust particles, as we have guessed, but note the lengthening of the cut-off wave number. In fact, the "knees" of the lower curves with finite $k_K$ prove to a bifurcation point in a collisional plasma, as we shall see shortly. The parameters for Fig.1 are $\hat{k}_T = k_T/k_p = 0.2$, $Z_i = 1$, $Z_d = 100$, $T_e = T_i = 10^9$K, $m_d/m_i = 10^8$, $n_i = 10^{-3}$cm$^{-3}$, and $n_d = 10^{-6}$cm$^{-3}$. These values are typical for interstellar clouds [12].

An approximation similar to Eq.(17) for collisionless wave modes is given by

$$n_{\text{wave}} = -\frac{\delta_e}{(5\delta_e + 3Z_i\tau)} \left(\omega_p + \frac{2}{3}\omega_T\right) \pm i\omega_i \left(\frac{5\delta_e + 3Z_i\tau}{3\delta_e}\right)^{1/2} + O(\omega_{p,T}^2).$$

The instability criteria for a growing wave mode is thus $(\omega_p + 2/3\omega_T) < 0$, which is just the isentropic instability criteria, given by Field [2]. We can thus argue that the principal wave mode is unaffected by the presence of dust particles, but for a decrease in magnitude, dictated by the term $(5+3Z_i\tau/\delta_e)^{-1}$, which proportional to $\delta_e/Z_i^2$. However, there is one more wave mode which is not apparent from the above analysis. We can detect the latter only by appropriately solving Eq.(15), numerically. In Fig.2, we show the the growing-wave modes in a collisionless plasma with considerable dust presence as in case of a planetary nebula [13]. There is a purely dust-wave mode (second figure in Fig.2) which is otherwise absent in a pure electron-ion plasma. The growth rate of the mode is, however, much smaller. Notice the cut-offs in all these curves. These figures are with $\hat{k}_T = -2$, $\hat{k}^{-1}_K = 0.01$, $Z_i = 2$, $Z_d = 100$ and $n_d = 10^{-5}$cm$^{-3}$, all other parameters being same as in Fig.1. The value of $\hat{k}_K$ is typical in planetary nebulae. We have chosen a large negative value for $\hat{k}_T$ so as to drive the wave mode unstable (see Eq.(20)). Note that a condensation mode will always accompany these wave modes (not shown) as the condition
Fig. 2. Normalized growth rates vs. wave number for the condensation modes in absence of collisions. The first figure shows the usual wave mode present irrespective of the dust particles. However, the growth rate in presence of dust particles is considerably smaller (solid curve). The second figure shows the dust-wave mode which is absent in a pure electron-ion plasma.

Fig. 3. Growth rates in presence of collisions with $\hat{k}_T = 0.2, \hat{k}_K^{-1} = 0.01, \nu_R/\omega_\rho = 0.2, \nu_I/\omega_\rho = 0$, and $n_{d0} = 10^{-6} \text{ cm}^{-3}$. Other parameters are same as in Figs. 1 and 2.

for condensation instability is readily satisfied in this case ($\hat{k}_T < 0$).

In Fig. 3, we show the growth rates of modes with collisional effects. The solid curves represent condensation modes and the dashed one represents the wave mode. We see a merger of the condensation modes to a growing wave mode at a bifurcation point near $\hat{k} \simeq 4.0$.

We should emphasize that we do not have an explicit expression for the interstellar heat-loss function $\mathcal{L}(\rho, T)$ in general, because of an wide variety of compositions of material in different interstellar regions. Besides, several heating and cooling mechanisms may be operative simultaneously and their dependence on local density and temperature may not be known exactly. Dalgarno and McCray [14], in their classic review, have mentioned about different cooling processes in HI regions. A relatively concise expression for the heat-loss function in photo-ionized interstellar gases with high metallicity is given by Corbelli and Ferrara [5]. In the simplest case, for radiative cooling, the heat-loss function can be assumed to be proportional to $\rho^n \rho^m$, where $n = 5/2, m = -1/2$ for free-bound transitions and $n = 3/2, m = +1/2$ for free-free transitions [3,14,15]. However, the exact form the heat-loss function is not important at this point. As a prototype example, we assume the heat-
Fig. 4. Range of instability in terms of temperature with $\delta_e$. The value of $\bar{T} = 4.0, 4.5,$ and $5.0$, respectively for the upper, middle, and lower curves.

loss function used by Field [2] for thermal condensations in planetary nebulae, which is represented by

$$
\frac{T_0 \mathcal{L}_T}{\rho_0 \mathcal{L}_\rho} = \frac{\frac{3}{2} k_K^{-1} T - (\frac{1}{2} \bar{T} + \hat{k}_K^{-1}) + \bar{T} / T}{\bar{T} - \hat{k}_K^{-1} T}
$$

(21)

where $\bar{T}$ is the mean excess ionization temperature of N$^+$ ions. The temperatures are given in the units of $T_{N^+} = 21800$ °K, the mean excitation temperature of the N$^+$ level. Applying the instability condition (19) in the collisionless case, we have, thermal condensation if,

$$
T_-(\bar{T}, \zeta) < T < T_+(\bar{T}, \zeta),
$$

(22)

where $\zeta = (1 + 1/\delta_e)$ and

$$
T_\pm(\bar{T}, \zeta) = [2\zeta + \hat{k}_K \bar{T}(2 + \zeta) \pm \{[2\zeta + \hat{k}_K \bar{T}(2 + \zeta)]^2 - 8\hat{k}_K \bar{T} \zeta(2 + 3\zeta)\}^{1/2}] (4 + 6\zeta)^{-1}.
$$

(23)

For simplicity, we have assumed $T_{e0} = T_{d0}$ and $Z_i = 1$. The above expression of $T_\pm$ reduces to usual one when $\zeta = 1$ (single-fluid approximation). For $\hat{k}_K = 3/2$, $Z_d = 100$, $n_{i0} = 10^{-3}$ cm$^{-3}$, $n_{d0} = 10^{-7}$ cm$^{-3}$, and $\bar{T} = 4.5$, we have $T_\pm = 1.3293 - 2.542$, which in absence of dust becomes $0.7248 - 3.7254$ [2]. So, the presence of dust provides a stabilizing effect against radiative condensations. However, the range of instability shrinks to zero at $\bar{T} = 3.902$, much higher than 1.974, which is for the dust-less case. This implies that for a planetary nebula with a considerable amount of dust particles, to be stable against the radiative condensation mode, it has to be at a much higher temperature. In Fig.4, we show the range of unstable temperature with $\delta_e$. Note that a lower value of $\delta_e$ corresponds to higher dust concentration.

To conclude, we have found that the instability criterion for the thermal condensation mode is significantly affected by the presence of dust particles, which
can severely alter the stability regimes of many astrophysical condensations where a significant number of dust particles are present. The fundamental wave mode is similar in both cases with and without dust particles, though we now have another wave mode, which is due to the presence of dust particles. Finally, we have shown that the conventional stability regimes of many astrophysical condensations may be considerably changed.

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References