Searching for dynamical fermion effects in UKQCD simulations

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We present recent results from the UKQCD collaboration’s dynamical QCD simulations. This data has fixed lattice spacing but varying dynamical quark mass. We concentrate on searching for an unquenching signal in the mesonic mass spectrum where we do not find a significant effect at the quark masses considered.

1. Introduction

Computing resources available for simulations of lattice QCD are now powerful enough to investigate unquenching effects. UKQCD has embarked on a programme of studying these effects in the light hadron spectrum, static quark potential, glueball spectrum and topological sectors. The purpose of this paper is to investigate unquenching effects in the first of these areas.

It is well known that the lattice cut-off is a function of both the gauge coupling, $\beta$, and dynamical quark mass, $m^{sea}$ (see e.g. [1]). For this reason, the philosophy we have chosen is to simulate at points along the “matched” trajectory in the $(\beta, m^{sea})$ plane i.e. defined by fixed lattice spacing, $a$. This then disentangles lattice spacing artefacts with unquenching effects. Any variation of a physical quantity along this trajectory can sensibly be attributed to unquenching effects rather than lattice systematics. This work has been published in full in [2].

2. Simulation Details

A non-perturbatively improved clover action was used with lattice parameters displayed in Table 1 with a volume of $16^3 \times 32$. The last four rows contain the parameters for the matched ensembles, while the top simulation explores a lighter quark mass. Further details of the simulation appear in [2,3].

3. Mesonic Sector

We begin outlining our results by indicating our quark masses via $M^{unitary}_{PS}/M^{unitary}_V$, where $M_{V(PS)}$ is the pseudoscalar (vector) meson mass and the superscript “unitary” refers to the parameters $m^{sea} \equiv m^{val}$ in Table 1. As can be seen our quark masses are only modestly light (c.f. $M^{unitary}_{PS}/M^{unitary}_V \approx 0.18$ in nature).

The results for the hyperfine splitting are shown in Fig. 1 where the experimental points are plotted assuming $r_0 = (0.49 \pm 0)$ fm. Note that there is a tendency for the lattice data to flatten towards the experimental points as the dynamical quark mass decreases within the matched ensembles. However the “unmatched” simulation ($\kappa^{sea} = 0.1355$) shows an increased negative slope, presumably due to finite-volume effects.

![Figure 1. The hyperfine splitting.](image-url)
Table 1

Lattice parameters together with measurements of $a$ and quark masses.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\kappa_{\text{sea}}$</th>
<th>$c_{SW}$</th>
<th>#conf.</th>
<th>$a_{r_0}$ [Fm]</th>
<th>$a_J$ [Fm]</th>
<th>$M_{\text{unitary}}/M_{\text{PS}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.20</td>
<td>0.1355</td>
<td>2.0171</td>
<td>208</td>
<td>0.0972(8)</td>
<td>0.110+$^+3$</td>
<td>0.578+$^+13$</td>
</tr>
<tr>
<td>5.20</td>
<td>0.1355</td>
<td>2.0171</td>
<td>150</td>
<td>0.1031(10)</td>
<td>0.115+$^+3$</td>
<td>0.700+$^+12$</td>
</tr>
<tr>
<td>5.26</td>
<td>0.1345</td>
<td>1.9497</td>
<td>101</td>
<td>0.1041(12)</td>
<td>0.118+$^+2$</td>
<td>0.783+$^+5$</td>
</tr>
<tr>
<td>5.29</td>
<td>0.1340</td>
<td>1.9192</td>
<td>101</td>
<td>0.1018(10)</td>
<td>0.116+$^+3$</td>
<td>0.835+$^+7$</td>
</tr>
<tr>
<td>5.93</td>
<td>0.131 0.132 0.133 0.134 0.135 0.136 0.137 0.138 0.139 0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The $J$ parameter, defined as,

$$J = M_V \frac{dM_V}{dM_{PS}} |_{K,K^*},$$

was calculated using 3 approaches: (i) “Pseudo-Quenched” i.e. where $\kappa_{\text{sea}}$ is held fixed and the derivative in eq.(1) is w.r.t. variations in $\kappa_{\text{val}}$; (ii) “Unitary Trajectory” i.e. where $\kappa_{\text{sea}} \equiv \kappa_{\text{val}}$ and the derivative in eq.(1) is w.r.t. variations in both $\kappa_{\text{val}}$ and $\kappa_{\text{sea}}$ combined; and (iii) “Chiral Extrapolation of Pseudo-Quenched” i.e. taking the $J$ values from Approach (i) and performing the chiral extrapolation $m_{\text{sea}} \rightarrow 0$. The results of all three methods are shown in Fig. 2, together with the experimental point. There is evidence that the lattice value for $J$ approaches the experimental point as the sea quark mass decreases (see approaches 1 and 3). Note also that the “unmatched” simulation ($\kappa_{\text{sea}} = 0.1355$) has a smaller $J$ value. This fact is related to the comment in the previous paragraph.

We use two methods to extract the lattice spacing. The first is via the Sommer scale $r_0$, and the second is directly from the meson spectrum as outlined in [4]. The values obtained from both methods are displayed in Table 1. Note that $a_J$ is $\approx 10\%$ larger than $a_{r_0}$. This could be due to the experimental estimate of $r_0 = 0.49$ Fm being $\approx 10\%$ too small.

4. Unquenching Effects in Meson Spectrum

In order to investigate unquenching effects in the meson spectrum, we define the quantity

$$\delta_{i,j}(\beta, \kappa_{\text{sea}}) = 1 - \frac{a_i(\beta, \kappa_{\text{sea}})}{a_j(\beta, \kappa_{\text{sea}})},$$

where $a_i$ is the scale determined from the physical quantity $M_i$. When $\delta_{i,j} = 0$ then the lattice prediction of $M_i$ with scale taken from $M_j$ agrees with experiment. Thus $\delta$ is a good parameter to study unquenching effects. We expect that $\delta_{i,j}(\beta, m_{\text{sea}}) = \mathcal{O}(a^2)$ since we are using a non-perturbatively improved clover action.

In Fig. 3 we plot $\delta_{i,j}$ for the matched datasets where $j = r_0$, and $i$ is the scale determined from the string tension, $\sigma$, and the two mass pairs ($\rho, \pi$) and ($K, K^*$) following [4]. The $x$–axis in this plot is $(aM_{\text{PS}}^{\text{unitary}})^{-2} \sim 1/m_{\text{sea}}$, so the quenched data point lies on the $y$–axis.

We see disappointingly that, in the case of the scale determinations from the meson spectrum, there is no significant tendency of $\delta$ towards zero as $m_{\text{sea}} \rightarrow 0$, i.e. the quantity $\delta$ does not give us an indication of unquenching effects. However, in the case of $\sigma$, there is a statistically significant
Figure 3. $\delta_i$ as described in the text.

variation of $\delta \to 0$ as $m^{sea} \to 0$. This implies that we do see unquenching effects in the static quark potential.

5. Chiral Extrapolations

In [2] we used 3 approaches to perform the chiral extrapolations of hadron masses: “Pseudo-Quenched”; “Unitary Trajectory”; and a “Combined Chiral Fit”. In this publication we will refer only to the last approach. We take

$$M(k^{sea}, k^{val}) = A(k^{sea}) + B(k^{sea}) \tilde{M}_{PS}(k^{sea}, k^{val})^2$$

$$= A_0 + A_1 \tilde{M}_{PS}(k^{sea}, k^{sea})^{-2}$$

$$+ B_0 + B_1 \tilde{M}_{PS}(k^{sea}, k^{sea})^{-2} \tilde{M}_{PS}(k^{sea}, k^{val})^2,$$

using the nomenclature $\tilde{M} \equiv aM$ and the first argument of $M(k^{sea}; k^{val})$ refers to the sea quark and the second to the valence quark.

The results of these extrapolations are shown in Table 2. We stress that this functional form for the extrapolation is not motivated by theory, but is used as a numerical analysis technique in order to test for evidence of unquenching effects. As can be seen from Table 2, the parameters $A_1$ and $B_1$ are compatible with zero (to $2\sigma$) and therefore we conclude that there is no evidence of unquenching effects.

<table>
<thead>
<tr>
<th>hadron</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$B_0$</th>
<th>$B_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector</td>
<td>$0.492^{+9}_{-9}$</td>
<td>$-0.004^{+2}_{-3}$</td>
<td>$0.61^{+4}_{-4}$</td>
<td>$0.015^{+2}_{-2}$</td>
</tr>
<tr>
<td>Nucleon</td>
<td>$0.663^{+13}_{-13}$</td>
<td>$0.006^{+3}_{-3}$</td>
<td>$1.23^{+5}_{-5}$</td>
<td>$-0.001^{+1}_{-1}$</td>
</tr>
<tr>
<td>Delta</td>
<td>$0.84^{+2}_{-2}$</td>
<td>$-0.002^{+5}_{-5}$</td>
<td>$0.91^{+8}_{-8}$</td>
<td>$0.02^{+2}_{-2}$</td>
</tr>
</tbody>
</table>

Table 2

Fit parameters from the Chiral Extrapolations

6. Conclusions

This paper attempts to uncover unquenching effects in the dynamical lattice QCD simulations at a fixed (matched) lattice spacing (and volume) and various dynamical quark masses. This approach allows a more controlled study of unquenching effects without the possible entanglement of lattice and unquenching systematics.

However, we see no significant sign of unquenching effects in the meson spectrum. This is presumably since our dynamical quarks are relatively massive, and so the meson spectrum is dominated by the static quark potential. This potential is, by definition, matched amongst our ensembles at the hadronic length scale $r_0$, and so any variation of the meson spectrum within our matched ensemble must surely be a “higher” order unquenching effect which is beyond our present statistics.

We have however shown that unquenching effects exist in the static quark potential, and other work (using the same ensembles) has shown interesting unquenching effects in the glueball and topological sector [2].

REFERENCES

2. UKQCD, C.R.Allton et al., hep-lat/0107021.
3. D. Hepburn, these proceedings.