The heavy-to-light transitions in the light-cone QCD sum rules

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Abstract

We have analyzed the $B \rightarrow \pi$ and $B_s \rightarrow K$ semileptonic form factors and $B \rightarrow V\gamma (V = K^*, \rho, \omega)$ processes in the light-cone QCD sum rules. In order to enhance the predictivity and reliability of numerical results the chiral-current correlator is employed and the twist-3 light-cone wavefunction can be effectively eliminated from the sum rules.
$B$ decays into a light meson are an important ground to understand and test the standard model (SM), since they can provide the signal of CP-violation phenomena and perhaps, a window into new physics beyond the SM. Nevertheless, the definite interpretation for the relevant experimental data demand that we have the ability to precisely compute the physical amplitudes. One is forced to use some approximate methods, such as QCD factorization formula [1], heavy quark effective theory (HQET), chiral perturbative theory (CPT), Modified PQCD approach [2], QCD sum rules [3] and light cone QCD sum rules [4, 5]. Each of them has advantages and disadvantages. QCD factorization formula is viewed as a great progress in phenomenology of heavy flavors, however the underlying long distance effects included in a series of hadronic matrix elements confront still us, which are not being dealt rigorously with and thus would hinder us from doing such a desired calculation. CHPT and HQET, as two effective theories at low energy, can describe very well light-to-light and heavy-to-heavy transitions, respectively. They are not suitable for a study on heavy-to-light processes. It seems that the modified PQCD approach with a resummation of Sudakov logarithms is consistent with the physical picture of heavy-to-light transitions due to the hard gluon exchange. However, a detailed analysis shows that the reliable PQCD calculation depends on whether the singularities can be eliminated or suppressed. The singularities come from on shell gluon, on-shell light quark and on-shell heavy quark. QCD sum rule method is built on the basis of QCD theory, but the obtained form factors behave very badly in the heavy quark limit $M_Q \to \infty$ due to the fact that one omits the effects of the finite correlation length between the quarks and the gluons in the physical vacuum. In order to overcome the defect, QCD light cone sum rule (LCSR) approach has been developed in Ref. [4, 5] and has been regarded as an advanced tool to deal with heavy-to-light exclusive processes. In comparison with the case of the traditional QCD sum rules, in LCSR approach the operator product expansion (OPE) is carried out near the light cone $x^2 \approx 0$, instead of in small distance $x \approx 0$, and the non-perturbative dynamics is parametrized in term of so-called the light cone wavefunctions of light mesons, instead of the vacuum condensates. It is the striking advantage of the approach to describe heavy-to-light transitions in a way consistent with the universally accepted viewpoint that non-perturbative QCD dynamics occupies an dominant place and perturbative hard gluon exchanges contribute only a subleading effect in that case. Very recently, it was generalized to study the non-factorizable effects in $B \to \pi\pi$ [6] and to probe heavy-to-light form factors in the whole kinematically accessible ranges [7]. The technical details of LCSR can be found, for instance, in [8], while for a detailed comparison with the traditional sum rules, see [9]. However, a problem with the approach is that the relevant light cone wavefunctions are intrinsically non-perturbative and can only be determined via some phenomenological methods. This could considerably affect the reliability of sum rule predictions. Accordingly, it is essentially important to find out some certain way to control the pollution by the light cone wavefunctions. With this end in view, a pragmatic strategy has been suggested in Ref. [5,10-13], in which some specific chiral current operators act as the interpolating fields in the correlators used as a LCSR calculation on the form factors for heavy-to-light transitions, making some twist-3 components cancel out in the OPE. Taking it into account that these twist-3 distributions are equally important in most cases but poorly known to us compared with the corresponding twist-2 ones, we may think of this procedure as an effective way to narrow down the uncertainties in LCSR calculations. Here we intend to present a simple summary of the our previous works [10-13] on applications of the improved LCSR approach to some important heavy-to-light transition processes.

2. Heavy-to-light Semileptonic Form Factors [10, 11]

Semileptonic $B$ decays included by the $b \to u$ transition are regarded as the most promising processes adequate to extract $|V_{ub}|$ from the relevant data. We will focus on a discussion on $B \to \pi$ and $B \to K$ semileptonic transition form factors within the framework of an improved LCSR.
\[
\langle \pi(p)|\pi\gamma_b|B(p+q)\rangle = 2f^{B-\pi}(q^2)p_\mu + \tilde{f}^{B-\pi}(q^2)q_\mu, \tag{1}
\]

with \( q \) being the momentum transfer. We choose to use a chiral current correlator

\[
\Pi_\mu(p,q) = i \int d^4xe^{ipx}\langle \pi(p)|T\{\pi(x)\gamma_\mu(1+\gamma_5)b(x),\bar{b}(0)i(1+\gamma_5)d(0)\}|0\rangle
= \Pi(q^2,(p+q)^2)p_\mu + \tilde{\Pi}(q^2,(p+q)^2)q_\mu, \tag{2}
\]

to calculate \( f^{B-\pi}(q^2) \) and \( \tilde{f}^{B-\pi}(q^2) \). Its hadronic expansion reads

\[
\Pi^H_\mu(p,q) = \Pi^H(q^2,(p+q)^2)p_\mu + \tilde{\Pi}^H(q^2,(p+q)^2)q_\mu
= \frac{\langle \pi|\pi\gamma_b|B\rangle\langle B|\bar{b}\gamma_5d|0\rangle}{m_B^2 - (p+q)^2}
+ \sum_H \frac{\langle \pi|\pi\gamma_\mu(1+\gamma_5)|B^H\rangle\langle B^H|\bar{b}(1+\gamma_5)d|0\rangle}{m_{B^H}^2 - (p+q)^2}. \tag{3}
\]

We argue that such a procedure is conceptually reasonable and technically feasible by observing the hadronic form of (2). Because of the special chiral constructs, the correlator would receive the contribution of the \( 0^+B \) mesons, in addition to the \( 0^-B \) meson, in its hadronic expression. However, this causes no difficulty in extracting the desired sum rule for the form factors. Since the lowest \( 0^+B \) meson is far from the ground state of \( 0^-B \) mesons and slightly below the first excited \( 0^-B \) mesons in mass so that we can safely isolate the pole contribution of the lowest \( 0^-B \) meson and parametrize them from the higher \( 0^- \) states as well as all the \( 0^+B \) mesons in a dispersion integral:

\[
\Pi^H(q^2,(p+q)^2) = \frac{2f(q^2)m_B^2f_B}{m_b(m_B^2 - (p+q)^2)} + \int_{s_0}^{\infty} \frac{\rho^H(s)}{s - (p+q)^2}ds + \text{subtractions}, \tag{4}
\]

\[
\tilde{\Pi}^H(q^2,(p+q)^2) = \frac{\tilde{f}(q^2)m_B^2f_B}{m_b(m_B^2 - (p+q)^2)} + \int_{s_0}^{\infty} \frac{\tilde{\rho}^H(s)}{s - (p+q)^2}ds + \text{subtractions}. \tag{5}
\]

Here the threshold parameter \( s_0 \) should be set near the squared mass of the lowest \( 0^+B \) meson.

QCD calculation of the underlying correlator may be allowed, on the other side, for the negative and large \( p^2 \) and \( (p+q)^2 \), in which case the OPE goes effectively in powers of the deviation from the light cone \( x^2 \approx 0 \). The chiral limit \( p^2 = m_\pi^2 = 0 \) will be taken throughout this discussion, for simplicity. Carrying out the OPE for the correlator, we have

\[
\Pi^{QCD}(q^2,(p+q)^2) = \Pi^{(\pi\pi)[q^2,(p+q)^2]} + \Pi^{(\pi\pi\pi)[q^2,(p+q)^2]}
= 2f_\pi m_b \left[ \frac{1}{u} \frac{du}{u^3} \varphi_\pi(u) \frac{1}{s - (p+q)^2} - 8m_b^2 \int_0^1 \frac{du}{u^2} \frac{G_1(u)}{(s - (p+q)^2)^3} \right]
+ 2 \int_0^1 \frac{du}{u^2} \frac{G_2(u)}{(s - (p+q)^2)^2} + 4 \int_0^1 \frac{du}{u^3} \frac{G_2(u)}{(s - (p+q)^2)^3}
+i g_\lambda m_b \int \frac{d^4kd^4xv}{(2\pi)^4(m_b^2 - k^2)} e^{i(q-k)\cdot x}(\langle \pi(p)|\bar{d}(x)\gamma_\mu G^{\alpha\beta}v\cdot x||\sigma_{\alpha\beta}u(0)|0\rangle)
+ (\pi(p)|\bar{d}(x)\gamma_\mu\gamma_5 G^{\alpha\beta}v\cdot x||\sigma_{\alpha\beta}u(0)|0\rangle). \tag{6}
\]
leading twist-2 wavefunction and the others have twist-4. They are defined as follows

$$\langle \pi(p)|T \bar{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle = -i p_\mu f_\pi \int_0^1 du e^{i p x} (\varphi_\pi(u) + x^2 g_1(u))$$

$$+ f_\pi(x_\mu - \frac{x^2 p_\mu}{p x}) \int_0^1 du e^{i p x} g_2(u), \quad (7)$$

$$\langle \pi(p)|\bar{d}(x)\gamma_\mu\gamma_5 g_{\alpha\beta}(vx) u(0)|0\rangle = f_\pi \left[ q_3(g_{\alpha\beta} - \frac{x_\alpha q_\mu}{q x}) - q_\alpha(g_{\beta\mu} - \frac{x_\beta q_\mu}{q x}) \right] \int D\alpha_i \varphi_{\perp}(\alpha_i) e^{i q x(\alpha_1 + \nu_3)}$$

$$+ f_\pi \left[ q_\mu x_\beta - q_\beta x_\alpha \right] \int D\alpha_i \varphi_{\parallel}(\alpha_i) e^{i q x(\alpha_1 + \nu_3)} \quad (8)$$

and

$$\langle \pi(p)|\bar{d}(x)\gamma_\mu g_{\alpha\beta}(vx) u(0)|0\rangle = if_\pi \left[ q_3(g_{\alpha\beta} - \frac{x_\alpha q_\mu}{q x}) - q_\alpha(g_{\beta\mu} - \frac{x_\beta q_\mu}{q x}) \right] \int D\alpha_i \tilde{\varphi}_{\perp}(\alpha_i) e^{i q x(\alpha_1 + \nu_3)}$$

$$+ if_\pi \left[ q_\mu x_\beta - q_\beta x_\alpha \right] \int D\alpha_i \tilde{\varphi}_{\parallel}(\alpha_i) e^{i q x(\alpha_1 + \nu_3)}. \quad (9)$$

At this point, the important observation is that difference from the standard LCSR calculations, in the present case the twist-3 components cancel precisely out in the OPE. It can improve greatly the precision of LCSR predictions.

Now the LCSR for $f(q^2)$ can be obtained using the standard procedure. The result is

$$f(q^2) = \frac{m_b^2 f_\pi}{m_B^2 f_B} \frac{m_b^2}{m_B^2} \left\{ \int d\Delta \frac{du}{u} e^{-\frac{m_b^2 - q^2(1-u)}{M^2}} \left( \varphi_{\pi}(u) - \frac{4 m_b^2}{u^2 M^2} g_1(u) + \frac{2}{u M^2} \right) \right\}$$

$$+ \int_0^1 dv \int D\alpha_i \frac{\theta(\alpha_1 + \nu \alpha_3 - \Delta)}{\alpha_1 + \nu \alpha_3} e^{\frac{m_b^2 - q^2(1-u)}{M^2}} \left[ 2 \varphi_{\perp}(\alpha_i) + 2 \varphi_{\parallel}(\alpha_i) - \varphi_{\parallel}(\alpha_i) - \tilde{\varphi}_{\parallel}(\alpha_i) \right]$$

$$- 4m_b^2 e^{\frac{-s_0}{2m_b^2}} \left( \frac{1}{(m_b^2 - q^2)^2} \right) \left( g_1(\Delta) - \frac{1}{(s_0 - q^2)(m_b^2 - q^2)} \right)$$

$$- 2e^{\frac{-s_0}{2m_b^2}} \left( \frac{m_b^2 + q^2}{s_0 - q^2} \right) \left( g_2(\Delta) - \frac{1}{(m_b^2 - q^2)^2} \right) \left( 1 + \frac{m_b^2 + q^2}{M^2} \int_0^\Delta g_2(v)dv \right)$$

To further proceed, we need to make a choice of input parameters. The parameters entering the sum rule are the $b$ quark mass $m_b$, $B$ meson mass $m_B$, decay constant $f_B$ and threshold parameter $s_0$. We take $m_b = 4.7 - 4.9$ GeV, $m_B = 5.279$ GeV. As for the decay constant $f_B$ and the threshold parameter $s_0$, the two-point correlator

$$K(q^2) = i \int d^4 x e^{i q x} \langle 0|\overline{q}(x)(1 + \gamma_5)b(x),\overline{q}(0)(1 - \gamma_5)q(0)|0\rangle$$

will be used to estimate it, for consistency. A standard manipulation yields three self-consistent sets of SR results: (1) $f_B = 165$ MeV for $m_b = 4.7$ GeV and $s_0 = 33$ GeV$^2$, (2) $f_B = 120$ MeV for $m_b = 4.8$ GeV and $s_0 = 32$ GeV$^2$, and (3) $f_B = 85$ MeV for $m_b = 4.9$ GeV and $s_0 = 30$ GeV$^2$. The parameters relevant to the $\pi$ meson contain the decay constant $f_\pi$ and the set of light cone wavefunctions. We use $f_\pi = 0.132$ GeV and the wavefunction models[14, 15] based on the conformal invariance of QCD:

$$\varphi_{\pi}(u, \mu) = 6u(1 - u)[1 + a_2(\mu)C_2^\frac{3}{2}(2u - 1) + a_4(\mu)C_4^\frac{3}{2}(2u - 1) + \cdots], \quad (11)$$
2.5 GeV, and the Gegenbar polynomials \( C_{2}^{2/3}(2u - 1) = \frac{3}{2} [5(2u - 1)^{2}] \) and \( C_{4}^{2/3}(2u - 1) = \frac{15}{8} [21(2u - 1)^{4} - 14(2u - 1)^{2} + 1] \), and

\[
g_{1}(u, u) = \frac{5}{2} z^{2} u^{2} \pi^{2} + \frac{1}{2} \varepsilon \delta^{2} [u \pi(2 + 13u \pi)] + 10u^{3} \ln u(2 - 3u + \frac{6}{5} u^{2}) + 10u^{3} \ln \pi(2 - 3\pi + \frac{6}{5} \pi^{2})],
\]

\[
g_{2}(u, u) = \frac{10}{3} \delta^{2} u \pi(u - \pi).
\]

\[
\varphi_{\perp}(\alpha_{i}) = 30\delta^{2}(\alpha_{1} - \alpha_{2})\alpha_{3}[\frac{1}{3} + 2\varepsilon(1 - 2\alpha_{3})],
\]

\[
\bar{\varphi}_{\perp}(\alpha_{i}) = 30\delta^{2}\alpha_{3}^{2}(1 - \alpha_{3})[\frac{1}{3} + 2\varepsilon(1 - 2\alpha_{3})],
\]

\[
\varphi_{\parallel}(\alpha_{i}) = 120\delta^{2}\varepsilon(\alpha_{1} - \alpha_{2})\alpha_{1}\alpha_{2}\alpha_{3},
\]

\[
\bar{\varphi}_{\parallel}(\alpha_{i}) = -120\delta^{2}\alpha_{1}\alpha_{2}\alpha_{3}[\frac{1}{3} + \varepsilon(1 - 3\alpha_{3})],
\]

with \( \delta^{2}(\mu_{b}) = 0.17 \text{ GeV}^{2} \) and \( \varepsilon(\mu_{b}) = 0.36 \).

Having fixed the input parameters, we can carry out the numerical analysis. The reasonable range of \( M^{2} \) is found to be \( 8 \text{ GeV}^{2} \leq M^{2} \leq 17 \text{ GeV}^{2} \), in which the variation of \( f(q^{2}) \) with \( M^{2} \) turns out to be negligibly small. For a specific \( M^{2} = 12 \text{ GeV}^{2} \), the sum rules for \( f^{B \rightarrow \pi}(q^{2} = 0) \) are predicted to \( f(0) = 0.27 \), 0.29 and 0.33 (corresponding to the set (3), set (2) and set (1), respectively), which are in basic agreement with an estimate from the standard LCSR[14].

As a matter of fact, numerical agreement between the two different approaches exists up to \( q^{2} = 10 \text{ GeV}^{2} \), the differences being within 20%; while for 10 GeV\(^2 \leq q^{2} \leq 18 \text{ GeV}^{2} \) there is a numerical disagreement of about 20 – 25%. For the region \( q^{2} \geq 18 \text{ GeV}^{2} \), application of the LCSR is questionable such that a comparison is meaningless between the two sum rule results.

The total uncertainty in \( f^{B \rightarrow \pi}(q^{2}) \), which arises from the uncertainties in the \( b \) quark mass \( m_{b} \), the threshold parameter \( s_{0} \), the decay constant \( f_{B} \) and the pionic light cone wavefunctions, is estimated at the level of about 26%.

Following the same procedure, we may analyze the \( B_{s} \rightarrow K \) form factor \( f^{B_{s} \rightarrow K}(q^{2}) \). As compared with the case of \( B \rightarrow \pi \), however, the \( f^{B_{s} \rightarrow K}(q^{2}) \) is more difficult to calculate, for \( SU(3) \) breaking corrections to the twist-3 wavefunctions of \( K \) meson have not been investigated completely in the literature. Explicitly, this problem can be avoided in our approach.

The correlator used for the LCSR calculation on \( f^{B_{s} \rightarrow K}(q^{2}) \) may be obtained by an obvious replacement \( d \rightarrow s \) in Eq. (2). Carrying out the OPE and considering the non-negligible \( K \)-meson mass we have

\[
\Pi^{H}(q^{2}, (p + q)^{2}) = \frac{2f^{B_{s} \rightarrow \pi}(q^{2})m_{B_{s}}f_{B_{s}}}{(m_{b} + m_{s})(m_{B_{s}}^{2} - (p + q)^{2})} + \int_{s_{0}}^{\infty} \frac{\rho_{H}^{s}(s)}{s - (p + q)^{2}} ds \quad (12)
\]

where the definitions \( \beta = \alpha_{1} + \alpha \alpha_{3} \), \( \Delta = (m_{b}^{2} - q^{2})/(s_{0} - q^{2} - m_{K}^{2}) \) and \( D \alpha_{i} = d \alpha_{1} d \alpha_{2} d \alpha_{3} \delta(1 - \alpha_{1} - \alpha_{2} - \alpha_{3}) \) have been used, and the light cone wavefunctions of the \( K \) meson obey the same definitions as the corresponding cases of \( \pi \) meson.

In the following numerical analysis we adopt a model presented in [16],

\[
\varphi_{K}(u) = 6u(1 - u) \left\{ 1 + 1.8 \left[ (2u - 1)^{2} - 1/5 \right] - 0.5(2u - 1) \left[ 1 + 1.2[(2u - 1)^{2} - 3/7] \right] \right\} \quad (13)
\]

for the leading twist-2 wavefunctions of the \( K \) meson, and neglect the \( SU(3) \) breaking effects for all the twist-4 distributions. Numerically, we take \( f_{K} = 0.16 \text{ GeV} \) and \( m_{s} = 0.15 \text{ GeV} \).
Making the Borel improvements on the yielded theoretical expression and the resulting deviations from our predictions turn out to be between $-10\%$ and $-15\%$, depending on $q^2$, in the total kinematically accessible region.

Having in mind that all the above discussions are limited to the region of low and intermediate momentum transfer, for which the OPE is effective, one must find another way to estimate the form factors at the large momentum transfer. The heavy-to-light transition form factors in the whole kinematical accessible range, for example, $B_s \to K$ form factor $f_{B_s \to K}(q^2)$ can be precisely represented as

$$f_{B_s \to K}(q^2) = \frac{f_{B^+}g_{B^+B_K}}{2m_{B^+}(1-q^2/m_{B^+}^2)} + \int_{\sigma_0}^{\infty} \frac{\rho(\sigma)d\sigma}{1-q^2/\sigma} = F_G(q^2) + F_H(q^2),$$

with $g_{B^+B_K}$ being the strong coupling defined by $(B^*(q,e)K(p)|B_s(p+q)) = -g_{B^+B_K}(p\cdot e)$, and $\rho(\sigma)$ a spectral function with the threshold $\sigma_0$. Obviously, $F_G(q^2)$ stands for the contribution from the $B^*$ pole, which describes the principal behavior of the form factor around $q^2 = q_{\text{max}}^2$, and $F_H(q^2)$ parameterizes the higher state effects in the $B^*$ channel. The non-perturbative parameter $f_{B^+}g_{B^+B_K}$ is calculable with the correlator used for the LCSR estimate of $f_{B_s \to K}(q^2)$. Accordingly, modelling the higher state contributions by a certain assumption and then fitting Eq.(14) to its LCSR result $f_{B_s \to K}(q^2)$ in the region accessible to the light cone OPE, we might derive the form factor $f_{B_s \to K}(q^2)$ in the total kinematical range with a better accuracy.

We work in the large space-like momentum regions ($q^2 \ll 0$ and $(p+q)^2 \ll 0$) for the correlator in question. Making the Borel improvements on the yielded theoretical expression $F^{QCD}(q^2, (p+q)^2) \to F^{QCD}(M_1^2, M_2^2)$, and then matching it onto the corresponding Borel improved hadronic form via the use of quark-hadronic duality ansatz, the final sum rule for $f_{B^+}g_{B^+B_K}$ reads,

$$f_{B^+}g_{B^+B_K} = \frac{2m_b(m_b + m_s)f_K}{m_{B_s}^2 m_{B^+}} e^{-\frac{m_{B_s}^2 + m_{B^+}^2}{2M^2}} \left\{ \frac{M^4}{M^2} \left[ e^{-\frac{m_{B_s}^2 + m_{B^+}^2}{M^2}} - e^{-\frac{\sigma_0}{M^2}} \right] \varphi_K(1/2) + e^{-\frac{m_{B_s}^2 + m_{B^+}^2}{M^2}} \left[ g_2(1/2) - \frac{4m_b^2}{M^2} g_1(1/2) - \int_0^{1/2} g_2(v)dv \right] \right\} \left[ 2\varphi_{\perp}(\alpha_i) + 2\tilde{\varphi}_{\perp}(\alpha_i) - \varphi_{\parallel}(\alpha_i) - \tilde{\varphi}_{\parallel}(\alpha_i) \right] \right\}.$$  

The resulting sum rule for $f_{B^+}g_{B^+B_K}$ is numerically $f_{B^+}g_{B^+B_K} = 3.88$ GeV, with the uncertainty of about $18\%$. Using its central value, we obtain a $B^*$ pole approximation for $f_{B_s \to K}(q^2)$, which is shown by the dotted line in Fig. 1. It is explicitly demonstrate that a perfect match between the direct LCSR and $B^*$ pole predictions appears at $q^2 \approx 15 - 20$ GeV$^2$.

It is important and interesting to make a comparison of our sum rule results and those from the standard LCSR based on the correlator vector and pseudoscalar currents, which are easy to obtain using the twist-3 wavefunction suggested in Ref. [17], leaving the twist-4 distribution amplitudes unchanged and making a corresponding replacement of the other relevant input parameters in Eq.(79) and (44) of Ref. [14]. We observe that the standard approach gives the same matching range as in our case and the resulting deviations from our predictions turn out to be between $-10\%$ and $-15\%$, depending on $q^2$, in the total kinematically accessible region.

Assuming the higher state contribution in Eq.(14) to obey $F_H(q^2) = a/ \left(1 - bq^2/m_{B^+}^2 - cq^4/m_{B^+}^4\right)$, we could give a model for the form factor

$$f_{B_s \to K}(q^2) = \frac{f_{B^+}g_{B^+B_K}}{2m_{B^+}(1-q^2/m_{B^+}^2)} + \frac{a}{1-bq^2/m_{B^+}^2 - cq^4/m_{B^+}^4}.$$  

\text{(16)}
f_B g_{B^* B,K}$. In the region $q^2 = 0 - 18 \text{ GeV}^2$, the best fit of Eq.(16) to $f_{LC}(q^2)$ yields $b = 1.11$ and $c = -8.33$. It turns out that the fitting results (the dashed line in Fig.1) reproduce precisely the LCSR prediction up to $q^2 = 18 \text{ GeV}^2$ and support considerably the $B^*$ pole description of the $B_s \to K$ form factor in large $q^2$ region.

Finally, we look roughly into SU(3) breaking effects in heavy-to-light decays by considering the ratio of the derived $f_{B_s \to K}(q^2)$ over $f_{B_s \to \pi}(q^2)$, which can be modelled using all the same method as in the $B_s \to K$ case. For the common kinematical region the resulting ratios, a comparable result $1.05 - 1.15$ with that from the standard approach, favor a small SU(3) breaking effect.

### 3. The rare decays $B \to (K^*, \rho, \omega) + \gamma$[12, 13]

The rare decays $B \to (K^*, \rho, \omega) + \gamma$ induced by the flavor-changing neutral currents (FCNC’s) provide us with a good opportunity to search for new physics. These decay modes are dominated by the electromagnetic penguin operators responsible for $b \to (s, d) + \gamma$. The relevant form factors $F_{B \to (K^*, \rho, \omega)}(0)$ have been studied widely by using the various tricks. With the standard LCSR, ones found $F_{B \to K^*}(0) = 0.32 + 0.05$ [18] and $F_{B \to \rho}(0) = 0.285 \pm 15\%$[19].

Here we reanalyze them using the improved LCSR approach, to eliminate the pollution by some nonleading distributions. First, we focus on the case of $B \to K^* \gamma$. The relevant decay amplitude reads

$$A(B \to K^* \gamma) = C m_B \epsilon^\mu < K^*(p, \eta)|\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)q^\nu|B(p + q) >,$$  

where $\epsilon$ and $q$ are the emitted photon polarization vector and momentum, respectively. The constant $C$ depends on the CKM matrix elements $V_{cb}^* V_{ub}$ and its apparent form can be found in Ref.[20].

The hadronic matrix element in [7] may be parametrized in terms of the form factor $F_{B \to K^*}(0)$,

$$< K^*(p, \eta)|\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)q^\nu|B(p + q) > = (2i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta + 2\epsilon \cdot q \eta^\alpha - 2q \cdot \eta^\nu p_\mu)F_{B \to K^*}(0).$$

A correlator, which is suitable for our purpose, is

$$F_{\mu}(p, q) = i \int d^4x e^{ipx} < K^*(p, \eta)|T(s(x)\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b(x), \bar{b}(0)i(1 + \gamma_5)u(0)|0 > = -2 [\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta - \epsilon \cdot q \eta^\alpha + q \cdot \eta^\nu p_\mu] F[(p + q)^2].$$

The hadronic expression for the invariant function $F[(p + q)^2]$ is of the following form

$$F^H[(p + q)^2] = \frac{m_B f_B F_{B \to K^*}(0)}{m_B^2 - (p + q)^2} + \int_{s_0}^{\infty} \frac{\rho^H(s)}{s - (p + q)^2} ds.$$  

Here the dispersion integral stands for the contribution of both higher pseudoscalar states $B^h_p$ and scalar resonance states $B^h_s$.

On the other hand, for the region of large spacelike momenta $(p + q)^2 \ll 0$ the resulting QCD form of $F[(p + q)^2]$, to the twist-2 accuracy, reads,

$$F^{QCD}[(p + q)^2] = \frac{m_B^2}{m_B^2} \int_0^\infty \frac{\varphi_\perp(u, u_h^2)}{s - (p + q)^2} \frac{u}{u^2 m_{K^*}^2 + m_h^2} ds.$$  

The variable $u$, the fraction of the $K^*$ meson momentum carried by the $s$ quark, is related to $s$ by $s = m_B^2/u - (1 - u)m_{K^*}^2$. The leading twist-2 wavefunction $\varphi_\perp(u, u_h^2)$ parametrizes the nonlocal matrix element $< K^*(p, \eta)|\bar{s}(x)\sigma_{\mu\nu}q^\nu(1 + \gamma_5)u(0)|0 >$ as the following,

$$< K^*(p, \eta)|\bar{s}(x)\sigma_{\mu\nu}q^\nu(1 + \gamma_5)u(0)|0 > = i [(q \cdot \eta^\nu p_\mu - \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta] f_{K^*}^\perp \times \int_0^1 du e^{ip \cdot x} \varphi_\perp(u, u_h^2),$$  

\[22\]
The final LCSR for the form factor $f^{K^*}(0)$ is

$$
\frac{m_B^2}{m_b} f_B F^{B \to K^*}(0) e^{-\frac{(m_B^2 - m^2)}{2}} = \int_{u(s_0)}^{1} \frac{m_b f_\perp}{u} \varphi_\perp(u, \mu^2) e^{\frac{u}{\mu^2}(\frac{m_B^2}{2} + m_{K^*}^2)},
$$

(23)

which, in comparison with that in Ref.[18], does not receive the contribution of distribution amplitudes $\varphi_\parallel^0$, $g_\parallel^1$ and $g_\perp^1$, and thus is more reliable.

Using as inputs the Chernyak-Zhitnitsky(CZ) model for $\varphi_\perp^{K^*}(u, \mu^2)$,

$$
\varphi_\perp^{K^*}(u, \mu^2) = 6u(1 - u) \left[1 + 0.57\omega - 1.35(\omega^2 - \frac{1}{3}) + 0.46(\frac{7}{3}\omega^3 - \omega)\right].
$$

(24)

with $\omega = 2u - 1$, and taking into account all the possible uncertainties in the numerical analysis, we have $F^{B \to K^*}(0) = 0.34 \pm 0.05$, slightly greater than that in Ref.[18] and $B(B \to K^*\gamma) = (5.1 \pm 1.7) \times 10^{-5}$, a result comparable with the experimental observation[21] $B_\gamma(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$.

The corresponding relations in $B \to (\rho, \omega) + \gamma$ cases can be obtained by making replacements $s \to d$, $\varphi_\perp^{K^*}(u, \mu^2) \to \varphi_\perp^{(\rho, \omega)}(u, \mu^2)$, $m_{K^*} \to m_{(\rho, \omega)}$ and $f_\perp^{K^*} \to f_\perp^{(\rho, \omega)}$. The isospin symmetry allows us to adopt the same wavefunction model for $\rho^+$, $\rho^-$, $\rho^0$ and $\omega$. We choose the two different models

$$
\varphi_\perp(u, \mu^2) = 6u(1 - u) \left[1 + 0.138 \times 75(\omega^2 - 0.2)\right]
$$

(25)

and

$$
\varphi_\perp(u, \mu^2) = 6u(1 - u) \left[1 + 0.077 \times 7.5(\omega^2 - 0.2) - 0.077 \times \left(39.375\omega^4 - 26.25\omega^2 + 1.875\right)\right].
$$

(26)

which are respectively introduced by Ball and Braun in Ref.[22] and Bakulev and Mikhailo in Ref.[23], and called the model I and the model II. In the two cases, the resulting sum rule predictions are summarized as follows:

$$
F^{B \to \rho^\pm}(0) = 0.335 \pm 0.050, \quad Br(B \to \rho^\pm\gamma) = (2.71 \pm 1.00) \times 10^{-6}
$$

(27)

and

$$
F^{B \to \rho^0/\omega}(0) = 0.237 \pm 0.035, \quad Br(B \to (\rho^0, \omega) + \gamma) = (1.36 \pm 0.50) \times 10^{-6},
$$

(28)

for the model I, and

$$
F^{B \to \rho^\pm}(0) = 0.272 \pm 0.029, \quad Br(B \to \rho^\pm\gamma) = (1.79 \pm 0.61) \times 10^{-6}
$$

(29)

and

$$
F^{B \to \rho^0/\omega}(0) = 0.192 \pm 0.021, \quad Br(B \to (\rho^0, \omega) + \gamma) = (0.90 \pm 0.31) \times 10^{-6},
$$

(30)

for the model II. The results with the model I are slightly greater than those in Ref. [19], while the ones with the model II accord with those in Ref.[19]. Compared with the experimental observation Ref.[24] $Br(B^0 \to \rho^0\gamma) \leq 3.9 \times 10^{-5}$, $Br(B^0 \to \omega\gamma) \leq 1.3 \times 10^{-5}$ and $Br(B^- \to \rho^-\gamma) \leq 1.1 \times 10^{-5}$, our predictions are below the experimental upper limits.

4. Conclusion

Light-cone QCD sum rules is a good framework for calculating heavy-to-light form factors. We have analyzed the $B \to \pi$ and $B_s \to K$ semileptonic form factors, also $B \to K^*\gamma$ and $B \to (\rho, \omega)\gamma$ processes. In order to enhance the predictivity and reliability of numerical results it is worthwhile to study how to reduce the various uncertainties in the light-cone QCD sum rules. Here we employ the chiral-current correlator. It is explicitly shown that the twist-3 light-cone wavefunction, which have not been understood well, can be effectively eliminated from the sum rules.
Figure 1: The $B_s \rightarrow K$ form factor $f(q^2)$ in the total kinematical range. The solid line denotes the LCSR result $f_{LC}(q^2)$, which is reliable for $0 \leq q^2 \leq 17 \text{ GeV}^2$. The dotted line expresses the $B^*$ pole prediction suitable for large $q^2$. The best fit of Eq. (11) to $f_{LC}(q^2)$ is illustrated by the dashed line. It should be understood that the plotted curves correspond to the central values of all the relevant parameters.