Photons, Clocks, Gravity and the Concept of Mass

L.B.Okun\textsuperscript{a} *

\textsuperscript{a}ITEP, Moscow, 117218, Russia


\section{1. MASS IN SPECIAL RELATIVITY}

1. The term mass was introduced by Newton in \textit{Principia}, 1687: “Definition I: The quantity of matter is the measure of the same, arising from its density and bulk conjointly”.

2. The principle of relativity was formulated by Galileo in \textit{Dialogue}, 1632 (Galileo’s ship).

3. His predecessor: Nicolaus Cusanus in “De docta ignorata” (“On the scientific ignorance”), 1440.

4. Velocity of light $c$ was first measured by O.Roemer, 1676.

5. A. Michelson and E. Morley, 1887: $c$ is the maximal velocity of signals. $c = 1$.


7. Lorentz transformations of $t$, $\mathbf{r}$ and $E$, $\mathbf{p}$, where $\mathbf{r}$ and $\mathbf{p}$ are 3-vectors.

8. $m^2 = E^2 - \mathbf{p}^2$; $E$, $\mathbf{p}$ – 4-vector; $m$ – scalar.

9. Einstein 1905: rest energy $E_0 = m$.

10. $E = m\gamma$, $\mathbf{p} = m\gamma \mathbf{v} = E\mathbf{v}$, $\mathbf{v} = \frac{\mathbf{p}}{E}$

$\gamma = \frac{1}{\sqrt{1 - v^2}}$, $v \leq 1$.

11. Photon: $m = 0$, $E = p$, $v = 1$, no rest frame.

12. System of two bodies:

$E = E_1 + E_2$, $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$,

$m^2 = E^2 - \mathbf{p}^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = m_1^2 + m_2^2 + 2E_1 E_2 (1 - \mathbf{v}_1 \cdot \mathbf{v}_2)$.

Two photons: $m_{2\gamma}^2 = 2E_1 E_2 (1 - \cos \theta)$.

13. Mass of a system is conserved.

Example $e^+ e^- \rightarrow 2\gamma$ at rest.

$m_{e^+ e^-} = m_{2\gamma} = 2m_e$, though $m_{\gamma} = 0$.

14. Mass is not a measure of inertia.

This measure is derived from: $d\mathbf{p}/dt = \mathbf{F}$, $\mathbf{p} = E\mathbf{v}$.

15. Mass is not additive at $v \neq 0$.

$m = m_1 + m_2$ only at $v_1 = v_2 = 0$.

16. Mass is not a source and receptor of gravity at $v \neq 0$ (see item 20).

17. Mass is not a measure of an amount of matter. The concept of matter is much broader in relativity than in classical physics (examples: gas or flux of photons and/or neutrinos).

18. From mathematical point of view mass occupies a higher rank position in special relativity than in non-relativistic physics: it is a relativistic invariant.

*email: okun@heron.itep.ru
19. From practical point of view the role of mass of a particle is decreasing with increase of its energy.

2. ATOMS IN STATIC GRAVITY

20. According to General Relativity, the source and receptor of gravitational field is the density of energy-momentum tensor $T^{ik}$ which is coupled to gravitational field like density of electric current is coupled to electromagnetic field.

For a point-like particle

$$T^{ik} = m \delta(r) \frac{dx^i}{d\tau} \frac{dx^k}{dt}, \quad \tau = \sqrt{t^2 - r^2}.$$ 

For a particle at rest ($v = 0$) mass $m$ is the source and receptor of gravity: $T^{00} = m \delta(r)$.

Thus, according to GR, there is no such notion as gravitational mass $m_g$.

21. $\phi$ – gravitational potential

a) the potential of sun:

$$\phi = -\frac{GM_\odot}{rc^2} = -\frac{1}{2} \frac{r_g}{r}, \quad \phi(\infty) = 0$$

$G$ – Newton constant, $M_\odot$ – mass of the sun, gravitational radius $r_g = 2GM_\odot/c^2 = 3$ km.

b) the potential near the surface of earth at height $h$:

$$\phi = \frac{gh}{c^2}, \quad \phi(0) = 0, \quad g = 9.8 \text{ m s}^{-2}.$$ 

22. $E_{0g} = m + m\phi = m(1 + \phi), \quad E_{0g} \neq m$. 

$E_{0g}$ – rest energy in a static gravitational field. It increases with increasing distance from the source.

23. For an excited level with mass $m^*$

$$E_{0g}^* = m^*(1 + \phi).$$

The splitting between levels increases with $h$:

$$\Delta E_{0g} = E_{0g}^* - E_{0g} = (m^* - m)(1 + \phi).$$ 

24. Note that for electric potential the splitting is constant:

$$E_{0e} = m^* + e\phi_e.$$ 

Consider an excited ion He$^+$ in a capacitor:

$$\Delta E_{0e} = E_{0e}^* - E_{0e} = m^* - m.$$ 

25. The frequency $\omega$ of a photon emitted in the process of deexcitation

$$\omega = \Delta E_{0g}/h,$$

where $h = 2\pi\hbar$ is the Planck constant. 

Hence atomic clocks in gravitational field increase their rate with height, as predicted by Einstein in 1907.

26. This was confirmed by airplane experiments in 1970s.

Feynman: “atoms at the surface of the earth are a couple of days older than at its center”.

27. Thought experiment: Carry clock A from the first floor to the second. In a year carry clock B the same way. A will be ahead of B, $\Delta T/T = gh/c^2$. Absolute effect!

This thought experiment is used to get rid of disparity between a clock on an airplane and that on a desk. (Think of twin paradox in special relativity.)

28. Pendulum clocks are not standard clocks similar to wristwatches, or atomic clocks. $T \sim \sqrt{1/g}$. 

They are gravimeters, measuring the strength of gravitational field.

3. PHOTONS IN STATIC GRAVITY

29. In 1911 Einstein predicted redshift of photon frequency in static gravitational field: the gravitational redshift.

30. In 1960 Pound and Rebka discovered the shift $\Delta \omega/\omega = gh/c^2$ in $^{57}$Fe. Interpretation in terms of “the weight of the photon”.
31. For static field and static observers the frequency of photon does not depend on height. (Maxwell equations in static field). Hence the shift observed by Pound et al. was due to the larger splitting of $^{57}$Fe levels at the top of the Harvard tower. Redshift of photons relative to clocks.

32. Unlike frequency, the momentum of photon decreases with height. Schwarzshild metric in general relativity $g^{ij}$.

The masslessness of photon:

$$g^{ij} p_i p_j = 0, \ i, j = 0, 1, 2, 3, \ g^{00} p_0^2 - g^{rr} p_r^2 = 0$$

$$g^{00} = 1/(1 + 2\phi), \ g^{rr} = (1 + 2\phi)$$

$p_0 \equiv E$, hence,

$$p_r = \frac{E}{1 + 2\phi} = \frac{E}{1 - r_g/r}.$$  

33. Photon’s wave length $\lambda$ increases with height. In that sense photon is redshifted

$$\lambda = \frac{2\pi\hbar}{p_r} = \frac{2\pi\hbar}{E}(1 + 2\phi)$$

34. Coordinate velocity is smaller near the sun:

$$v = \frac{\lambda\omega}{2\pi} = (1 + 2\phi)$$

35. Another derivation: Consider interval

$$ds^2 = g_{00}dt^2 - g_{rr}dr^2 = 0$$

$$g_{00} = \frac{1}{g^{00}} = 1 + 2\phi, \ g_{rr} = \frac{1}{g^{rr}} = \frac{1}{1 + 2\phi}$$

$$v = \frac{dr}{dt} = \sqrt{g_{00}/g_{rr}} = 1 + 2\phi = 1 - \frac{r_g}{r} < 1$$

36. Before general relativity (1906, 1911) Einstein had $v = 1 + \phi$ (there was no $g_{rr}$ at that time). In 1915: $(1 + 2\phi)$.

37. Geometrical optics. Refraction index

$$n = \frac{1}{v} = \frac{1}{1 + 2\phi} \approx 1 + \frac{r_g}{r}; \ n - 1 = \frac{r_g}{r}.$$  

38. Deflection of star light by sun (1911, 1915, 1919)

$$\alpha = \frac{2r_g}{R_\odot} \sim 10^{-5}$$


40. Delay of radar echo from a planet in upper conjunction (Shapiro, 1964)

$$\Delta t = \frac{2}{c} \int \frac{dz}{v(z) - 1} = \frac{2}{c} \int \frac{dz}{z} \frac{r_g}{r} = 2 \frac{r_g}{c} \ln \frac{4r_p r_e}{R_\odot} \approx 240 \mu s$$

Earth: $r_e = 150 \cdot 10^6$ km.

Mercury: $r_p = 58 \cdot 10^6$ km,

$R_\odot = 0.7 \cdot 10^6$ km.

41. Deceleration of a particle approaching the sun is a relativistic effect, characteristic not only to photons, but to any fast enough particle. M.I. Vysotsky (private communication) has shown that "enough" means $v_\infty > c/\sqrt{3}$, where $v_\infty$ is velocity of the particle at $r = \infty$.

To prove this consider the expression for energy of a particle of mass $m$ falling on the sun along radius $r$ with coordinate velocity $v = dr/dt$ (use eq. (88.9) from "Field Theory" by L. Landau and L. Lifshitz):

$$E = \frac{mc^2 \sqrt{g_{00}}}{\sqrt{1 - \frac{2r_g}{g_{00}}v^2}}.$$  

Note that, according to Schwarzschild, $g_{00} = \frac{1}{g_r} = 1 - r_g/r$ and that energy does not depend on $r$. Hence

$$1 - v_\infty^2 = g_{00}^{-1} - g_{rr}^{-1} v^2.$$  

For $r_g/r < 1$ one gets

$$v^2 = v_\infty^2 + \frac{r_g}{r}(1 - 3v_\infty^2).$$

Thus, the particle accelerates like a non-relativistic stone only if $v_\infty < c/\sqrt{3}$. 


4. MISLEADING TERMINOLOGY

42. If, instead of \( p = E v / c^2 \), the non-relativistic formula \( p = m v \) is kept, then \( m = E / c^2 \). This “relativistic mass”, often denoted as \( m_r \), is another notation for energy.

43. When applied to photon “\( m_\gamma = E_\gamma / c^2 \)”, Poincare, Lorentz, Born, Pauli, partly Einstein.

44. Einstein coined the expression “energy - mass equivalence”, which sometimes he used in the sense “whenever there is energy, there is mass”. (Recall massless photon.)

45. A redundant term “rest mass” \( m_0 \) is abundantly used.

46. Gravitational redshift is interpreted as “loss of photon’s energy as it climbs out of gravitational potential”.

47. From the point of view of relativity no \( m_r, m_0, m_g, m_i, m_l, m_t \). Only \( m! \)

5. UNSOLVED PROBLEMS OF \( m \)

48. Is the Higgs mechanism correct? If yes, we have to discover higgses and to study the pattern of their couplings.

49. Can the hierarchy problem \( (m_{Pl} \approx 10^{19} \text{ GeV} \ vs \ m_Z \approx 100 \text{ GeV}) \) be solved by SUSY? If yes, we have to discover numerous superparticles and to ascertain the pattern of their masses and couplings.

50. SUSY, if confirmed, would lead us into the quantum dimensions of space and time, into superspace.

51. Supergravity might lead to superunification of all forces.

52. The scale and mechanism of SUSY breaking is of paramount importance.

53. What makes the critical energy density of vacuum \( \varepsilon_c \approx 10^{-47} \text{ GeV}^4 \) so small compared with \( \eta^4 \), the density of vacuum expectation value (VEV) of the Higgs field: \( \eta = (246 \text{ GeV})? \)

54. If the recent data on the universe expansion are correct, how to explain the pattern of cosmological energy densities of baryons of dark matter, and of vacuum:

\[
\Omega_B \equiv \varepsilon_B / \varepsilon_c \approx 0.03, \quad \Omega_{DM} \equiv \varepsilon_{DM} / \varepsilon_c \approx 0.3 , \quad \Omega_\Lambda \equiv \varepsilon_\Lambda / \varepsilon_c \approx 0.7 .
\]

55. Compared with the above enigmas, the problem of neutrino masses and mixings may seem to be not so lofty. But it could reveal important clues to the concept of mass.

56. A serious twist to the concept of mass was given by quarks and gluons. The basic definition \( m^2 = E^2 - p^2 \) is not applicable to them, as they never exist as free particles with definite value of momentum. This color particles are confined within colorless hadrons.

57. That reminds us that when speaking about mass of a particle we have to specify the distance (momentum transfer \( q \)) at which it is measured. Masses as well as charges are running functions of \( q^2 \). It is not enough to write down the fundamental Lagrangian. One has to indicate the scale to which its couplings and masses refer.

58. At short distances the masses of light \( u \)-quarks and \( d \)-quarks, the building blocks of nucleons, are smaller than 10 MeV, while masses of nucleons are hundred times larger. These short-distance quarks are called "current quarks".

59. The same quarks with their “gluon coats” are called "constituent quarks", their masses being about 300 MeV. There should exist "constituent gluons". Hadrons without "valent" quarks are called "glueballs", hadrons containing both "valent" quarks and gluons are referred to as "hybrids".
60. The quark-antiquark and gluonic vacuum condensates: \( \langle q \bar{q} \rangle, \langle GG \rangle \), etc. play an important role. The masses of hadrons appear due to “burning out” of these (negative energy density) vacuum condensates inside hadrons by the intense color fields of the valent constituents. Though this picture has not been rigorously proved, it seems evident that the origin of mass is in vacuum.

61. The existing terminology is obsolete and non-adequate. Fermions (quarks and leptons) are usually referred to as “matter”, though neutrinos kinematically behave like photons. Even very heavy bosons are often referred to as “radiation”. Actually all fields and their quantum excitations (including photons) are matter.

Massive vacuum; is it also matter?

62. The literature on the subjects discussed in this lecture is very vast and controversial. The interested reader might look for further references in article [1–12]

ACKNOWLEDGEMENTS

I am grateful to Paul Langacker, Alfred Mann and their colleagues at the Physics Department of University of Pennsylvania and also to Alessandro Bettini and Venya Berezinsky at LNGS for the kind invitation to give this lecture and for their warm hospitality. The work was partially supported by A. von Humbold Award and by RFBR grant No. 00-15-96562.

REFERENCES

2. The concept of mass. Physics Today, June 1989, pp. 31-36