I. INTRODUCTION

denoted central estimates. Implications for neutrino mass and properties are
neutrino–nucleon cross-sections which are usually larger than some of the
neutrino–nucleon cross-sections. Table 1 shows the new range for the
and in some of the indirect searches. It is shown that the new range for
in turn, for detection rates for relic neutralinos in WIMP dark experimentals
signs term for the evaluation of the neutrino–nucleon cross-section, and,
We discuss the implications of a new determination of the pion–nucleon

ABSTRACT

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Size of the neutrino–nucleon cross–section in the light of a new
determination of the pion–nucleon sigma term.
contents in the nucleon; this point was also remarked in Refs. [3,4]. In [2] we quantified the range of these uncertainties and showed that, within their ranges, the couplings of neutralinos to nucleon may be sizeably stronger than some of those currently employed in the literature. Subsequently, an analysis of this problem was also undertaken in Refs. [5–8].

One of the crucial ingredients in the calculation of the hadronic matrix elements is the so-called pion–nucleon sigma term. Its numerical derivation from experimental data of pion–nucleon scattering is rather involved, and thus the origin of considerable uncertainties. Recent experimental results in pion–nucleon scattering have now prompted a new determination of the pion–nucleon sigma term [9]. In the present paper, we consider the implications of these new inputs for the neutralino–nucleon cross-section; in particular, we show that the new data reinforce our previous conclusions of Ref. [2]. As compared to Ref. [2], the present paper, apart from the use of the new evaluation for the sigma term, contains also some rather relevant updateings, mainly: i) radiatively corrected Higgs–quark couplings [10] and ii) recent experimental bounds on Higgs masses and supersymmetric parameters from LEP2 [11] and CDF [12].

II. NEUTRALINO–NUCLEON ELASTIC CROSS–SECTION

The neutralino–nucleon scalar cross-section may be written as

$$
\sigma_{\text{scalar}}^{(\text{nucleon})} = \frac{8G_F^2}{\pi} M_Z^2 m_{\text{red}}^2 \left[ \frac{F_h I_h}{m_h^2} + \frac{F_H I_H}{m_H^2} + \frac{M_Z^2}{2} \sum_{\tilde{q}} < N | \bar{q} q | N > \sum_i P_{\tilde{i}} (A_{\tilde{i}}^2 - B_{\tilde{i}}^2) \right] .
$$

(1)

The first two terms inside the brackets refer to the diagrams with exchanges of the two CP–even neutral Higgs bosons, $h$ and $H$, in the $t$–channel (the exchange diagram of the CP–odd one, $A$, is strongly kinematically suppressed and then omitted here) [13] and the third term refers to the graphs with squark–exchanges in the $s$– and $u$–channels [14]. The mass $m_{\text{red}}$ is the neutralino–nucleon reduced mass. Since, for simplicity, in the present paper we explicitly discuss only the Higgs–mediated terms, we do not report the expressions for the squark propagator $P_{\tilde{i}}$, and for the couplings $A_{\tilde{i}}, B_{\tilde{i}}$, which may be found in Ref. [15]. However, in the numerical results reported in this paper also the squark–exchange terms are included. The quark matrix elements $< N | \bar{q} q | N >$ are meant over the nucleonic state.

The quantities $F_{h,H}$ and $I_{h,H}$ are defined as follows

$$
F_h = (-a_1 \sin \theta_W + a_2 \cos \theta_W)(a_3 \sin \alpha + a_4 \cos \alpha)
F_H = (-a_1 \sin \theta_W + a_2 \cos \theta_W)(a_3 \cos \alpha - a_4 \sin \alpha)
I_{h,H} = \sum_{\tilde{q}} k_{\tilde{q},H}^2 m_{\tilde{q}} < N | \bar{q} q | N > ,
$$

(2)
where the $a_i$’s are the coefficients in the definition of the neutralino as the lowest-mass linear superposition of photino ($\tilde{\gamma}$), zino ($\tilde{Z}$) and the two higgsino states ($\tilde{H}_1^0$, $\tilde{H}_2^0$)

$$
\chi \equiv a_1 \tilde{\gamma} + a_2 \tilde{Z} + a_3 \tilde{H}_1^0 + a_4 \tilde{H}_2^0.
$$

(3)

The coefficients $k_{\tilde{q},H}^b$ for up-type and down-type quarks are given in Table 1, in terms of the angle $\beta$, defined as $\tan \beta = <H_2^0^{(0)}>/ <H_1^0^{(0)}>$ and the angle $\alpha$, which rotates $H_1^{(0)}$ and $H_2^{(0)}$ into $h$ and $H$. In Table I also included are the coefficients $k_{\tilde{q},H}^{b,H}$ for the CP-odd Higgs boson $A$, which are important for the evaluation of the neutralino relic abundance. The entries include those radiative corrections which may be sizeable at large $\tan \beta$. These corrections affect the couplings to down-type quarks $k_{\tilde{d},\text{type}}$, and are parametrized in terms of the quantity $\epsilon \equiv 1/(1 + \Delta)$, where $\Delta$ enters in the relationship between the fermion running masses $m_d$ and the corresponding Yukawa couplings $h_d$ [10]:

$$
m_d = h_d <H_0^1>(1 + \Delta)
$$

(4)

These corrections take contributions mainly from gluino-squark, chargino-squark and neutralino-stau loops [10].

III. EVALUATION OF THE QUANTITIES $M_Q\langle N|\tilde{q}Q|N \rangle$

For the calculation of the quantities $m_q\langle N|\tilde{q}q|N \rangle$ it is useful to express them in terms of the pion-nucleon sigma term

$$
\sigma_{\pi N} = \frac{1}{2}(m_u + m_d) <N|\bar{u}u + \bar{d}d|N>,
$$

(5)

of the quantity $\sigma_0$, related to the size of the SU(3) symmetry breaking, and defined as

$$
\sigma_0 \equiv \frac{1}{2}(m_u + m_d) <N|\bar{u}u + \bar{d}d - 2\bar{s}s|N>,
$$

(6)

and of the ratio $r = 2m_s/(m_u + m_d)$.

Assuming isospin invariance for quarks $u$ and $d$, the quantities $m_q\langle N|\tilde{q}q|N \rangle$ for light quarks may be written as

$$
m_u <N|\bar{u}u|N> \simeq m_d <N|\bar{d}d|N> \simeq \frac{1}{2}\sigma_{\pi N}
$$

(7)

$$
m_s <N|\bar{s}s|N> \simeq \frac{1}{2}r(\sigma_{\pi N} - \sigma_0).
$$

(8)
For the heavy quarks $c, b, t$, use of the heavy quark expansion [16] provides

$$m_c < N|\bar{c}c|N| \simeq m_b < N|\bar{b}b|N| \simeq m_t < N|\bar{t}t|N| \simeq \frac{2}{2 T} \left[ m_N - \sigma_{\pi N} + \frac{1}{2} r(\sigma_{\pi N} - \sigma_0) \right] , \quad (9)$$

where $m_N$ is the nucleon mass. The quantities $I_{h,H}$ can then be rewritten as

$$I_{h,H} = k_{u-type}^h g_u + k_{d-type}^h g_d , \quad (10)$$

where

$$g_u \simeq m_l < N|\bar{u}u|N| + 2 m_h < N|\bar{t}t|N|$$

$$\simeq \frac{4}{2 T} (m_N + \frac{19}{8} \sigma_{\pi N} - \frac{1}{2} r(\sigma_{\pi N} - \sigma_0)) , \quad (11)$$

and

$$g_d \simeq m_l < N|\bar{d}d|N| + m_s < N|\bar{s}s|N| + m_h < N|\bar{t}t|N|$$

$$\simeq \frac{2}{2 T} (m_N + \frac{23}{4} \sigma_{\pi N} + \frac{25}{4} r(\sigma_{\pi N} - \sigma_0)) ; \quad (12)$$

$l$ stands for light quarks ($l = u, d$) and $h$ denotes the heavy ones ($h = c, b, t$).

We recall that from the values of $\sigma_{\pi N}$ and $\sigma_0$ one can derive the fractional strange–quark content of the nucleon $y$

$$y = 2 \frac{< N|\bar{s} s|N|}{< N|\bar{u} u + \bar{d} d|N|} , \quad (13)$$

using the expression

$$y = 1 - \frac{\sigma_0}{\sigma_{\pi N}} . \quad (14)$$

**A. Values for $\sigma_{\pi N}$, $\sigma_0$ and $r$**

The range of $\sigma_{\pi N}$ we used previously in Ref. [2] is

$$41 \text{ MeV} \lesssim \sigma_{\pi N} \lesssim 57 \text{ MeV} . \quad (15)$$
This was derived from the pion–nucleon scattering amplitude, calculated at the so-called
Cheng–Dashen point by Koch [17], and the evolution of the nucleon scalar form factor, as a
function of the momentum transfer from $t = 2m^2$ to $t = 0$, evaluated in Ref. [18].

We now consider the new determination of $\sigma_{\pi N}$ presented in Ref. [9]. The George
Washington University/TRIUMF group, using up–dated pion–nucleon scattering data [20] and a
new partial–wave and dispersion relation analysis program, has derived a range for $\sigma_{\pi N}$ [9]

$$55 \text{ MeV} \lesssim \sigma_{\pi N} \lesssim 73 \text{ MeV},$$

which turns out to be sizeably larger than the one of Eq. (15). Values of the nucleon scalar
form factor at the Cheng–Dashen point higher than those of Ref. [17] were also reported in
Ref. [21].

In the present paper we consider the effect of employing the new range for $\sigma_{\pi N}$, given in
Eq. (16), in the evaluation of the quantities $m_q\langle N|\bar{q}q|N\rangle$‘s.

Here, $\sigma_0$ is taken in the range [19]

$$\sigma_0 = 30 \div 40 \text{ MeV}.$$  \hspace{1cm} (17)

Also the mass ratio $r = 2m_\pi/(m_u + m_d)$ may be affected by significant uncertainties [2].
However, here, for consistency with some of the previous determinations, we use the default
value $r = 25$.

It has to be noted that combining together values of $\sigma_{\pi N}$ and $\sigma_0$ within their ranges
in Eqs. (16) and (17) leads to rather large values for the fractional strange–quark content
of the nucleon, as given by Eq. (14). This is a puzzle that urges further investigation in
hadron physics. In what follows, in the variations of the quantities $\sigma_{\pi N}$ and $\sigma_0$ we impose
the constraint that, anyway, $y \leq 0.5$.

**B. Values for $m_q\langle N|\bar{q}q|N\rangle$, $g_u$ and $g_d$**

Inserting the numerical values of $\sigma_{\pi N}$, $\sigma_0$ and $r$ in the expressions given at the beginning
of this section, we finally obtain estimates for the quantities $m_q\langle N|\bar{q}q|N\rangle$, $g_u$ and $g_d$. Since
$m_s < N[\bar{s}s]\ N >$ is the most important term among the $m_q < N[\bar{q}q]\ N >$’s [22], unless tan $\beta$
is very small, the extremes of the range of the neutralino–nucleon cross-section are provided by
the extremes of the range for $m_s < N[\bar{s}s]\ N >$. These, in turn, are given by: $(m_s < N[\bar{s}s]\ N >)_{\text{min}} \approx \frac{1}{4} r(\sigma_{\pi N,\text{min}} - \sigma_0^{\text{max}})$ and $(m_s < N[\bar{s}s]\ N >)_{\text{max}} \approx \frac{1}{4} r\sigma_{\pi N,\text{max}}$ (to satisfy the
constraint $y \leq 0.5$). We call Set $b$ the one with $m_s < N[\bar{s}s]\ N >= (m_s < N[\bar{s}s]\ N >)^{\text{min}}$ and
Set c the one with $m_s < N \langle \bar{s}s \rangle_N \Rightarrow (m_s < N \langle \bar{s}s \rangle_N )^{max}$; we denote by $Set a$ the reference set of Ref. [4]. The values of the quantities $m_q (N \langle \bar{q}q \rangle_N)$, $g_u$ and $g_d$ are given in Table II.

We note that, with the new values of $\sigma_{\chi N}$, the coefficient $g_d$, which usually dominates in the neutralino–nucleon cross-section, turns out to fall in the range

$$266 \text{ MeV} \lesssim g_d \lesssim 523 \text{ MeV},$$

(18)

to be compared with the reference value $g_d = 241 \text{ MeV}$ of $Set a$. Thus, for most supersymmetric configurations, one expects $(\sigma_{\text{nucleon}})_{Set b} / (\sigma_{\text{scalar}})_{Set a} \simeq 1.5$ and $(\sigma_{\text{scalar}})_{Set c} / (\sigma_{\text{scalar}})_{Set a} \simeq 6$.

IV. RESULTS AND CONCLUSIONS

The supersymmetric model employed in the present paper for the calculation of the neutralino–nucleon cross-section and for the neutralino relic abundance is the one defined in Ref. [23, 24] and denoted there as effMSSM. We refer to [23, 24] for details on the theoretical aspects and on the updated experimental bounds.

Inserting the values of Table II into Eq.(1), one obtains the results displayed in Fig. 1. The two ratios in the cross-sections are plotted as a function of $\xi \sigma_{\text{scalar}}^{(\text{nucleon})}$, where $\xi$ is a rescaling factor for the neutralino local density. $\xi$ is taken to be $\xi = \min \{1, \Omega \chi h^2 / (\Omega_m h^2)_{\text{min}} \}$, in order to have rescaling, when $\Omega \chi h^2$ turns out to be less than $(\Omega_m h^2)_{\text{min}}$ (here $(\Omega_m h^2)_{\text{min}}$ is set to the value 0.05).

The use of $\xi \sigma_{\text{scalar}}^{(\text{nucleon})}$ instead of simply $\sigma_{\text{scalar}}^{(\text{nucleon})}$ allows one to better identify the range of sensitivity in current WIMP direct experiments [25], in Fig. 1. Taking into account astrophysical uncertainties [26], this range may established to be

$$4 \cdot 10^{-10} \text{ nbarn} \lesssim \xi \sigma_{\text{scalar}}^{(\text{nucleon})} \lesssim 2 \cdot 10^{-8} \text{ nbarn},$$

(19)

for WIMP masses in the interval $40 \text{ GeV} \lesssim m_W \lesssim 200 \text{ GeV}$. In Fig. 1 we notice that, in the sensitivity range of Eq.(19), the cross-section ratios are actually of the sizes obtained by our previous estimate based on dominance of the term $g_d$, i.e. $(\sigma_{\text{scalar}}^{(\text{nucleon})})_{Set b} / (\sigma_{\text{scalar}}^{(\text{nucleon})})_{Set a} \simeq 1.5$ and $(\sigma_{\text{scalar}}^{(\text{nucleon})})_{Set c} / (\sigma_{\text{scalar}}^{(\text{nucleon})})_{Set a} \simeq 6$.

We turn now to the implications that our present analysis has for the cosmological properties of relic neutralinos, as explored by present WIMP direct detection experiments. Fig. 2 provides the essential information. The detection of relic neutralinos would be of great
relevance even if these particles constitute only a subdominant dark matter population [24]; however, it is obvious that the most attractive case is when the neutralino relic abundance falls into the interval of cosmological interest: $0.05 \lesssim \Omega_\chi h^2 \lesssim 0.3$. Fig. 2 shows to what extent the most interesting region: $0.05 \lesssim \Omega_\chi h^2 \lesssim 0.3$ and $4 \cdot 10^{-10}$ nbarn $\lesssim \sigma_{\text{scalar}}^{(\text{nucleon})} \lesssim 2 \cdot 10^{-8}$ nbarn, is covered by neutralino configurations, depending on the values of the input parameters of Table II. The upper-right frontier of the scatter plots moves progressively upward, as we move from Set a of inputs to Set b and to Set c.

The main results of the present paper may be summarized as follows:

- The new range of the pion–nucleon sigma term favours values of the neutralino–nucleon cross-section which are sizeably larger (a factor of 1.5 to 6) than some of the current estimates. However, a word of caution has to be said here: the new derivation of the pion–nucleon sigma term implies a rather high value for the fractional strange–quark content of the nucleon; this point requires further investigation in the framework of hadron physics. Thus, for instance, the consistency between the present indication for higher values of the pion–nucleon sigma term and the determination of the range of $\sigma_0$ has to be understood.

- Uncertainties implied by the hadronic quantities for the neutralino–nucleon cross-section are still quite sizeable; these have to be taken in due consideration in evaluations of the neutralino event rates.

- The previous remark applies not only to event rates for WIMP direct detection, but also to the evaluation of the neutrino fluxes expected at neutrino telescopes, as a consequence of possible neutralino–neutralino annihilations inside the Earth and the Sun. In fact, these fluxes depend on the capture rate of neutralinos by the macroscopic bodies; in turn, this rate depends on the neutralino–nucleon cross-section.

- A larger size of the neutralino–nucleon cross-section, as indicated by the new hadronic data, implies that present WIMP direct experiments (and some indirect measurements) explore larger sectors of the supersymmetric space where the neutralino may be of cosmological interest.
REFERENCES


TABLES

TABLE I. Values of the coefficients \( k_q^H \) in Eq.(2).

| \( k_u \)-type | \( \frac{\cos \alpha}{\sin \beta} \) | \( \frac{\sin \alpha}{\sin \beta} \) | \( \frac{1}{\tan \beta} \) |
|---|---|---|

| \( k_d \)-type | \( -\sin \alpha \cos \beta - \epsilon \cos(\alpha - \beta) \tan \beta \) | \( \cos \alpha / \cos \beta - \epsilon \sin(\alpha - \beta) \tan \beta \) | \( \tan \beta(1 + \epsilon) \) |

TABLE II. Values of the quantities \( m_q < |N|\bar{q}q|N> \), \( g_u \) and \( g_d \). Set a is the reference set given in Ref. [4]. Set b and Set c are the sets corresponding to the minimal and maximal values of \( m_s < |N|\bar{s}s|N> \), respectively. All quantities are in units of MeV.

| \( m_l < |N|\bar{q}q|N> \) | \( m_s < |N|\bar{s}s|N> \) | \( m_h < |N|\bar{h}h|N> \) | \( g_u \) | \( g_d \) |
|---|---|---|---|---|
| Set a [4] | 27 | 131 | 56 | 139 | 214 |
| Set b | 28 | 186 | 52 | 132 | 266 |
| Set c | 37 | 456 | 30 | 97 | 523 |
FIG. 1. Ratios of the neutralino–nucleon cross-sections, for values of the quantities $m_{3/2}|\bar{q}q|N)$, $g_u$ and $g_d$, as given in Table II.
FIG. 2. Scatter plot of $\sigma_{\text{scalar}}$ versus $\Omega_\chi h^2$, when Set a is employed for the quantities given in Table II. The curve denoted by Set a visualizes the up—right frontier of the scatter plot. The curves denoted by Set b and by Set c give the locations of the frontiers of the scatters plots (not shown in this figure) obtained by using values of Set b and Set c, respectively. $m_\chi$ is taken in the range $40 \text{ GeV} \leq m_\chi \leq 200 \text{ GeV}$. The two horizontal lines bracket the sensitivity region defined by Eq. (19), when the rescaling factor $\xi$ is set equal to one. The two vertical lines denote the range $0.05 \leq \Omega_\chi h^2 \leq 0.3$. Dots denote gauginos ($P > 0.9$), circles denote higgsinos ($P < 0.1$) and crosses denote mixed ($0.1 \leq P \leq 0.9$) configurations ($P$ being defined as $P \equiv a_1^2 + a_2^2$).