In order to analyse the role of the quasiparticle-phonon interaction in the origin of nuclear gap, we applied an approach which is similar to the Eliashberg theory for usual superconductors. We obtained that the averaged contribution of the quasiparticle-phonon mechanism to the observed value of the pairing gap for $^{120}$Sn is 26% and the BCS-type mechanism gives 74%. Thus, pairing is of a mixed nature at least in semi-magic nuclei – it is due to the quasiparticle-phonon and BCS mechanisms, the first one being mainly a surface mechanism and the second one mainly a volume mechanism. The calculations of the strength distribution for the odd-mass nuclei $^{119}$Sn and $^{121}$Sn have shown that the quasiparticle-phonon mechanism mainly improves the description of the observed spectroscopic factors in these nuclei. For the case of nuclei with pairing in both proton and neutron systems it is necessary to go beyond the Eliashberg-Migdal approximations and include the vertex correction graphs in addition to the rainbow ones. The estimations for spectroscopic factors performed within a three-level model have shown that the contribution of the vertex correction graphs was rather noticeable.

1 Introduction

As it is known in the microscopic theory of ordinary superconductors, the BCS model of superconductivity is the limit case $g^2 << 1$ of the Eliashberg theory in which the electron interaction is determined only by the quasiparticle-phonon coupling $g$. In the microscopic theory of finite nuclei, the BCS equation for the gap is used, as a rule, which contains a phenomenologically chosen pp interaction $^2,^3$. It would be justified if $g^2 << 1$ for real nuclei where $g$ is the correspondingly dimensionless phonon creation amplitude. However, we have only $g^2 < 1$ in magic $^4$ and in semi-magic $^5$ nuclei and the case of strong
coupling $g^2 > 1$ for nuclei with pairing in both nucleon systems. Therefore, one can expect a contribution of the quasiparticle-phonon interaction (QPI) to the observed nuclear gap.

This question has another important aspect. As it is known, the most collective low-lying phonons, which make the largest contribution to the QPI effects in nuclei, are mainly surface excitations. Thus the explicit singling out of the quasiparticle-phonon mechanism of nuclear pairing will make it possible to answer the old question whether nuclear pairing is a volume or surface effect.

And last but not the least, a consistent study of the QPI contribution to pairing should improve the description of nuclear excitations. At present, this is especially interesting in connection with the quick development and using the qualitatively new gamma-ray arrays like EUROBALL and GAMMASPHERE.

The problem of the origin of nuclear pairing has been long discussed on a phenomenological level within the theory of finite Fermi systems using its idea of the internal (volume) and external (surface) pp interaction (see 6). Microscopically, the question about a contribution of the QPI to the gap has been discussed in 7 and recently in 8,9,10. In 9 the authors solved a BCS equation with the pp interaction caused by an exchange of a collective phonon between two quasiparticles. In 7,8 the authors also considered the "insertion" graphs which correct single-particle moving. However, for the nuclear case of $g^2 > 1$ it is necessary to go beyond the Eliashberg-Migdal approximation which complicates the problem strongly.

In this article we calculate the contribution of the QPI to the observed gap within the approximation of $g^2 < 1$ for the nucleus $^{120}$Sn taking into account both types of graphs and also consider the case of strong coupling $g^2 > 1$ using a simple 3-level model.

2 General relations

The initial equations of our approach are very general equations 11 for the one-particle Green functions in a Fermi system with pairing $G, G^h, F^{(1)}$ and $F^{(2)}$ which contain general mass operators $\Sigma, \Sigma^h, \Sigma^{(1)}$ and $\Sigma^{(2)}$. In order to single out the well-known components, i.e. the mean field and pairing, we represent each of the mass operators as a sum of two terms the first one being energy-independent and the second one energy-dependent:

$$\begin{align*}
\Sigma(\varepsilon) &= \tilde{\Sigma} + M(\varepsilon), \\
\Sigma^{(h)}(\varepsilon) &= \tilde{\Sigma}^{(h)} + M^{(h)}(\varepsilon), \\
\Sigma^{(1,2)}(\varepsilon) &= \tilde{\Sigma}^{(1,2)} + M^{(1,2)}(\varepsilon),
\end{align*}$$

$$\begin{align*}
\Sigma^{(1,2)}(\varepsilon) &= \tilde{\Sigma}^{(1,2)} + M^{(1,2)}(\varepsilon),
\end{align*}$$

2
where the quantities $\tilde{\Sigma}, \tilde{\Sigma}^{(h)}$ correspond to a mean field and $\tilde{\Sigma}^{(1)}, \tilde{\Sigma}^{(2)}$ describe a pairing of the BCS type. The quantities $M, M^{(1)}, M^{(2)}, M^{(h)}$ contain the QPI and do not fix so far.

With taking into account Eqs. (1) the general system for the Green functions can be transformed into the following equations (see 5,10 for derivation):

\begin{align*}
G &= \tilde{G} + \tilde{G}MG - \tilde{F}^{(1)}M^{(h)}F^{(2)} - \tilde{G}M^{(1)}F^{(2)} - \tilde{F}^{(1)}M^{(2)}G \\
F^{(2)} &= \tilde{F}^{(2)} + \tilde{F}^{(2)}MG + \tilde{G}^{(h)}M^{(h)}F^{(2)} - \tilde{F}^{(2)}M^{(1)}F^{(2)} + \tilde{G}^{(h)}M^{(2)}G,
\end{align*}

(and the same for $G^{(h)}$ and $F^{(1)}$). The bare Green functions $\tilde{G}, \tilde{G}^{(h)}$ and $\tilde{F}^{(1)}, \tilde{F}^{(2)}$ are the well-known Green functions of Gorkov which contain single-particle energies $\tilde{\varepsilon}_\lambda$ and gap $\tilde{\Delta}_\lambda$.

In what follows the input parameters of our problem are phenomenological (observed) single-particle energies $\varepsilon_\lambda$ and gap values $\Delta_\lambda$. The latter are usually taken from the experiment or obtained from solving the BSC equation with a phenomenologically chosen pp interaction. Because in the phenomenological $\varepsilon_\lambda, \Delta_\lambda$ there is a contribution of the corresponding $M^{(i)}$ terms, the quantities $\varepsilon_\lambda, \Delta_\lambda$ should be "refined" from these contributions to avoid a double counting of the $M^{(i)}$ terms or, in our case, of the QPI. In other words, a refined basis $\tilde{\varepsilon}_\lambda, \tilde{\Delta}_\lambda$ should correspond to the mass operators $\tilde{\Sigma}^{(i)}$ and should be used in the calculations. The procedure to obtain a connection between $\varepsilon_\lambda, \Delta_\lambda$ and $\tilde{\varepsilon}_\lambda, \tilde{\Delta}_\lambda$ has been described in 5,10. The final formulae are the following

\begin{align*}
\varepsilon_\lambda &= \frac{\tilde{\varepsilon}_\lambda + M^{\text{even}}_\lambda(E_\lambda)}{1 + q_\lambda(E_\lambda)} \\
\Delta_\lambda \equiv \Delta^{(1,2)}_\lambda &= \frac{\tilde{\Delta}_\lambda + M^{(1,2)}_\lambda(E_\lambda)}{1 + q_\lambda(E_\lambda)},
\end{align*}

where $E_\lambda = \sqrt{\tilde{\varepsilon}_\lambda^2 + \Delta_\lambda^2}$, $q_\lambda = -M^{\text{odd}}_\lambda(E_\lambda)/E_\lambda$ and $M^{\text{even}}, M^{\text{odd}}$ are the even and odd terms of the mass operator $M$. In equations (3) the energies $\tilde{\varepsilon}_\lambda$ and $\varepsilon_\lambda$ are reckoned from the corresponding chemical potential $\tilde{\mu}$ and $\mu$. These results were obtained by using the approximation which is diagonal in the single-particle index $\lambda$.

Thus, in order to calculate physical characteristics it is necessary to solve Eqs. (3) to obtain the new basis $\tilde{\varepsilon}_\lambda, \tilde{\Delta}_\lambda$. However, from the point of view of the nature of nuclear pairing the question arises about a possible contribution of the QPI to the quantity $\tilde{\Delta}_\lambda$ because the difference between $\Delta_\lambda$ and $\tilde{\Delta}_\lambda$ is only due to the explicit singling out of the QPI contained in the $M^{(i)}$ terms. Because $\tilde{\Delta}_\lambda$ is energy-independent one can write a general equation for it (see
also \(^{8,10}\)

\[ \hat{\Delta}_\lambda = \sum_{\lambda'} W_{\lambda\lambda'} \hat{F}^{(2)}_{\lambda}\lambda' \]  

(4)

where \(W\) is a new energy-independent pp interaction and \(\hat{F}^{(2)}\) satisfies Eq.(2) in which the quantities \(M^{(j)}\) may be taken in any approximation we need. Further we will consider our \(g^2\) approximation \((g^2 < 1)\) and a model case of the strong coupling \(g^2 > 1\).

3 \(g^2\) approximation \((g^2 < 1)\)

In this approximation we should take

\[ \hat{M} = \quad \circ \quad \circ \]  

(5)

where the circle is the phonon creation amplitude \(g\) and the single line means the Green function in its matrix form which includes \(\hat{G}, \hat{G}^h, \hat{F}^{(1)}, \hat{F}^{(2)}\). Here, the difference between \(\Delta_\lambda\) and \(\hat{\Delta}_\lambda\) is due to the exchange of one phonon between two quasiparticles (see Eq.(3)) but, in accordance with the Eliashberg theory \(^1\), this difference is corrected by the \((1 + g_\lambda)^{-1}\) factor.

The system (3) has been solved for the semi-magic nucleus \(^{120}\)Sn. At first phenomenological \(\varepsilon_\lambda\) and \(\Delta_\lambda\) were obtained from the existing experimental data for the neighbouring \(^{119}\)Sn and \(^{121}\)Sn nuclei. The BCS equation for \(\Delta_\lambda\) was solved with the phenomenological volume pp interaction taken from \(^{14}\). The phonons have been calculated within the theory of finite Fermi systems \(^2\). We used 21 of the most collective \(2^+, 3^-, 4^+, 5^-\) and \(6^+\) phonons with the energy not exceeding the neutron binding energy. See \(^5\), where values of \(\Delta_\lambda\) and \(\hat{\Delta}_\lambda\) are given, for detailed discussions. Here, for simplicity we use a simple averaging procedure, for example for \(\Delta_\lambda\):

\[ \Delta_{av} = \frac{\sum_j \Delta_\lambda (2j+1)}{\sum_j (2j+1)} \]  

(6)

For the quantity \((\Delta_{av} - \hat{\Delta}_{av})/\Delta_{av}\), which gives the QPI contribution caused only by the phonon exchange between two quasiparticles (or by the retarded pp interaction), we obtained the value of 31%.

Further, we have performed the calculations in the \(g^2\) approximation for \(\hat{\Delta}_\lambda\) according to Eq.(4), i.e. we have solved this equation with the Green function \(\hat{F}^{(2)}\) obtained from Eq.(2) in our \(g^2\) approximation \(^{10}\). The very first
term of Eq.(2) is \(\tilde{F}(2)\) so that the equation \(\Delta(BCS) = W\tilde{F}(2)\) is a BCS-like equation which determines a "pure" BCS part of the phenomenological gap value if the new pp interaction \(W\) is known. For simplicity the interaction \(W\) was taken in the same functional form as in \(^{14}\), but the parameter \(c_p\) was determined from the condition that the average value \(\tilde{\Delta}_{av}\) found by the solving of the equation obtained and of the system (3) are the same. The results of these calculations are as follows. We obtained that the contribution of \(\Delta_{av}(BCS) = 74\%\) of the average phenomenological gap, which is 1.42 MeV. The contribution of the terms containing the QPI terms to \(\tilde{\Delta}_{av}\) is equal to (-5)\% of the average phenomenological gap. This result is similar to that for the ph channel in the sense that the contribution of terms corresponding to phonon exchange diagram and to the self-energy diagrams are opposite in sign, but in our case the contribution of the self-energy diagrams in the pp channel is small.

Thus, we obtained that the total contribution of the QPI to the averaged phenomenological gap for \(^{120}\)Sn is (31-5)=26\% and the BCS-like part is 74\%. One should emphasize however, that these are just averaged figures and our method of determining the new pp interaction is the simplest one. But in any case we see that pairing in semi-magic nuclei is of a mixed nature - it is due to the BCS-like mechanism and the quasiparticle-phonon one.

As it was mentioned above, for the calculations of nuclear physical characteristics with taking into account the QPI it is enough to know the new basis \(\tilde{\varepsilon}_\lambda, \tilde{\Delta}_\lambda\). The question arises where one can see the effect of this new basis. The simplest reply is to compare the calculations with new and old basis. We have made this comparison (in the framework of our \(g^2\) approximation) for the spectroscopic factors in \(^{119}\)Sn and \(^{121}\)Sn. The method of calculations has been described in \(^{5,12}\). The results are given in Table 1.

One can see easily that using our new (refined) basis improves mainly the description of the experiment. Of course, it would be desirable to find a better confirmation, i.e. a clearer manifestation of the effect. Probably it can be seen in the calculations of other characteristics of low-lying levels of odd-mass nuclei. The experiments using the gamma-spectrometers of the EUROBALL-type can clarify this question.

4 Strong coupling \((g^2 > 1)\). Model calculations.

Unfortunately, in this case it is necessary, generally speaking, to go beyond the Eliashberg-Migdal approximations, i.e. to add the vertex corrections to the rainbow ones. The reason is that, because our phonons are made up of the same quasiparticles, there is no such a small parameter as in the Eliashberg
Table 1: Spectroscopic factors in $^{119}$Sn (first line) and $^{121}$Sn (second line) calculated with (third column) and without (fourth column) taking into account the effect of the new (refined) basis.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$S_\lambda$</th>
<th>Exp. $^{13}$</th>
<th>R+</th>
<th>R-</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1g7/2$</td>
<td></td>
<td>0.75;0.6</td>
<td>0.66(0.0-3.0)</td>
<td>0.54</td>
</tr>
<tr>
<td>$2d3/2$</td>
<td>0.4;0.45</td>
<td>0.4</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.44;0.65</td>
<td>0.35</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>$3s1/2$</td>
<td>0.26;0.32</td>
<td>0.36</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.43</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>$1h11/2$</td>
<td>0.29</td>
<td>0.26</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.49</td>
<td>0.43</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

theory due to the existence of which these corrections are negligible. Actually, there can be some specific reasons for that in nuclei (see below) but, in any case, in finite nuclei we do not have such a small parameter $g^2$ as in the theory of ordinary superconductors. Thus, instead of the mass operators (5), we should take into account at least the following mass operators

$$\hat{M} = \begin{array}{c}
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{array}$$

where the double line means the full Green function in its matrix form which includes $G$, $G^h$, $F^{(1)}$, $F^{(2)}$.

It is necessary now to solve Eqs. 2 with these mass operators. For the qualitative understanding and analysis of the case with strong coupling, here we consider a schematic 3-level model. We will take three levels with energies $\varepsilon= -1, 0, 1$, each of them has the quantum number $j = 11/2$, the first one being occupied completely, parity of the low- and high-lying levels being positive and that of intermediate level negative. The occupation number of the intermediate level will be changed from 0 to $2j + 1 = 12$. In this model, to obtain $\Delta \neq 0$, the parameter of pp interaction should be $G_{pp} = 0.035$. The ph interaction was taken in our case to obtain the Bohr parameter $\alpha = g^2/(2j+1)\omega_s$ ($\omega_s$ is the energy of phonon) equal to 2 for the most collective $2^+$ phonon.

To estimate the contribution of the vertex correction we used the well-known results by Belyaev and Zelevinsky according to which every vertex contains the factor $w$ with a 6j-symbol $\omega = -(2j + 1) \begin{Bmatrix}{j \ 2 \ j} \end{Bmatrix}$ so that the $n$-iteration contains a factor $w^n$ which decreases the contribution of the vertex.
Figure 1: Distribution of the single-particle strength calculated in the three-level model for the case with pairing, $j=11/2$, $\alpha=2$ (see the text for details) The dispersion was taken as a smearing parameter for each of the calculations.

correction as compared to the rainbow contribution. In our case $w=0.76$, so in what follows we restricted ourselves to $w$ only. That means that at least the $g^4$ terms are taken into account reasonably enough in our nonlinear model with the mass operators (7). For the beginning and simplification we have calculated the spectroscopic factors in our non-linear model with the mass operators (7), the above-mentioned parameter $\alpha$ being taken $\alpha=2$, which corresponds to a realistic nuclear case. To obtain the distribution of the single-particle strength the iteration procedure was organized for the solution of the Dyson equation Eq. (2) with our mass operators. In our case this procedure must include also the refinement procedure for $\tilde{\varepsilon}_i$ and $\tilde{\Delta}_i$.

The results are given in Fig.1. It turned out that it was necessary to do four iterations to obtain the convergency. This corresponds to the approximation $1\text{quasiparticle} \otimes 4$ phonons. We can see that the inclusion of the vertex
correction is noticeable in the strength distribution.

In particular, for the dominant level we obtained the decrease of the spectroscopic factor from 0.49 to 0.40 due to the inclusion of the vertex corrections. The calculations have shown also that the role of the mass operators $M^{(1)}$ and $M^{(2)}$ for the levels far off the Fermi surface was small.

5 Conclusion

We obtained that, if the simple procedures proposed are used to determine the new particle-particle interaction and to estimate the average effect according to Eq.(6), then the QPI contribution was 26% and the BCS-like mechanism gave 74% of the gap observed for $^{120}$Sn. This means that at least for semimagic nuclei pairing is of a mixed nature—it is due to the quasiparticle-phonon and the BCS-like mechanisms, the first being mainly a surface one and the second mainly a volume mechanism. The effect of the QPI in the gap value can be observed probably in the experiments using the gamma-spectrometers of the EUROBALL-type to measure the characteristics of low-lying levels.

For the case of nuclei with unfilled shells in both proton and neutron systems (strong coupling) our estimation has shown that the contribution of the vertex correction to spectroscopic factors is rather noticeable. One can think that this contribution is even more noticeable in transition probabilities for low-lying levels.

S.K. thanks Prof. G.M. Eliashberg for useful discussions of the results.

References