Critical Tsallis exponent in heavy ion reaction

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The numerical solution of the nonlocal kinetic equation allows to simulate heavy ion reactions around Fermi energy. The expansion velocity and density profile show specific radial dependence which can be described with a Tsallis exponent of $q = 5/3$. This might be considered as an indication of a phase transition.

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The question whether multifragmentation in heavy ion collisions around the Fermi energy is associated with a nuclear matter phase transition has been investigated for decades. From the Van der Waals equation of state it is obvious that, in order to describe such possible phase transition, we must have a kinetic equation capable to describe the second virial coefficient, or, more specifically, the excluded volume and the correlated pressure in equilibrium. While the equilibrium virial correction including its quantum equivalent has been thoroughly investigated, it is astonishing that its nonequilibrium extension has not been considered. Therefore, we have developed a nonlocal quantum kinetic theory which leads to an excluded volume as well as to a correlated pressure [1]. Solving the resulting kinetic equation demands no more numerical effort than solving the standard local Boltzmann equation extended by Pauli-blocking effects. Some experimental features of correlations are described by this result [2].

Searching for signals of a possible phase transition during the reaction is hindered by the over-shading of various reaction channels and four orders of magnitude later observation. Therefore dynamical models provide the correct approach to search for appropriate signals. In [3] it was found that in central heavy ion reactions around the Fermi energy a non-Hubblean expansion profile appears. This was associated with a peculiar density profile and it vanishes towards standard Hubble expansion for higher energies. This has been
attributed to long range correlations which are typical for fluctuations near a phase transition.

Since the heavy ion reaction is a complicated evolution of a correlated finite size system we might search for a simpler effective description of main features of the numerical solution. A promising short cut to describe effectively a statistics including finite size effects is the nonextensive statistical mechanics, starting from nonextensive entropies, as suggested by Druyvenstein [4], Renyi [5], Sharma [6] or Tsallis [7]. For a discussion of the kinetics underlying generalized statistics see [8]. Here it will be shown that both the velocity as well as the density profile can be associated with an anomalous diffusion of fractional derivative Fokker-Planck-like equation [9] with a Tsallis exponent $q = 5/3$. Since the latter value represents the border between Gaussian and Lévy-like fluctuations [10], we consider it a hint of phase transition.

Within the (extended) quasi-particle and quasi-classical approximations [11] we keep the gradient terms in the scattering integral of the Kadanoff and Baym equation and obtain a quantum non-local kinetic equation [11],

\[
\frac{\partial f_1}{\partial t} + \frac{\partial \epsilon_1}{\partial k} \frac{\partial f_1}{\partial r} - \frac{\partial \epsilon_1}{\partial r} \frac{\partial f_1}{\partial k} = \int P^- \left[ f_3^- f_4^- (1 - f_1 - f_2^-) - (1 - f_3^- - f_4^-) f_1 f_2^- \right].
\]  

\( (1) \)

The arguments of the distribution functions and the corresponding ones of the quasiparticle energies \( \epsilon \) are nonlocal, \( f_1 \equiv f(k, r, t) \), \( f_2^- \equiv f(p, r - \Delta_2, t) \), \( f_3^- \equiv f(k - q - \Delta_K, r - \Delta_3, t - \Delta_t) \), and \( f_4^- \equiv f(p+q - \Delta_K, r - \Delta_4, t - \Delta_t) \). The scattering probability is the square of the T-matrix, \( P^- = \frac{4 \rho_{\text{th}}}{(2\pi)^5} \delta \left( \epsilon_1 + \epsilon_2^- - \epsilon_3^- - \epsilon_4^- - 2\Delta_E \right) \times \left| T \left( \epsilon_1 + \epsilon_2^- - \Delta_E, k - \frac{\Delta_K}{2}, p - \frac{\Delta_K}{2}, q, r - \Delta_r, t - \frac{\Delta_t}{2} \right) \right|^2 \). All non-local corrections are given by derivatives of the scattering phase shift \( \phi = \text{Im} \ln T(\Omega, k, p, q, r, t) \)

\[
\Delta_1 = \frac{\partial \phi}{\partial \Omega}, \quad \Delta_E = -\frac{1}{2} \frac{\partial \phi}{\partial t}, \quad \Delta_K = \frac{1}{2} \frac{\partial \phi}{\partial r},
\]

\[
\Delta_2 = \frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial q} \frac{\partial \phi}{\partial k} - \frac{\partial \phi}{\partial k}, \quad \Delta_3 = -\frac{\partial \phi}{\partial k} \frac{\partial \phi}{\partial r}, \quad \Delta_4 = -\frac{\partial \phi}{\partial k} - \frac{\partial \phi}{\partial q}.
\]  

\( (2) \)

The collision is of finite duration \( \Delta_t \). During this time particles can gain momentum and energy \( \Delta_{K,E} \) due to the medium effect on the collision. Three displacements \( \Delta_{2,3,4} \) correspond to the initial and final positions of two colliding particles/holes. The numerical values of the shifts calculated with realistic potentials are available [12] and lead to simple off-set in the the algorithm which simulates the collision [2].

We plot in Fig. 1 the time dependence of the power of radial velocity \( v \propto r^\alpha \) for different bombarding energies and the corresponding power of radial density dependence in Fig. 1. There are two distinct behaviors dependent
Fig. 1. The time dependence of the power law velocity profile (left) with respect to the radius \( v \propto r^\alpha \) for different lab energies [3]. The dotted lines show the surface matter behavior and the solid lines depict the bulk matter behavior which was distinguished at the radius \( R \sim 10\text{fm} \). The time dependence of the power law fit of the particle density \( n \propto r^{-\beta} \) in the surface region \((R > 10 \text{ fm})\) is seen on the right hand side plot.

on the bombarding energy. For higher energies, 60MeV and 90MeV, we see that the velocity exponent approaches \( \alpha \sim 1 \), in agreement with the Hubble expansion. This is associated with a vanishing density exponent. The slightly negative values in Fig. 1 can be understood as ring-like expansions of matter (pancakes). The interesting observation is that for energies below or around Fermi energy the density exponent is \( \beta \sim 3 \) and the surface velocity exponent \( \alpha \sim 2 \). This remarkable feature is present in very different simulations we made with different reactions. It is more clearly pronounced in the nonlocal scenario when compared to the local scenario. Therefore, we propose that it is connected with correlations.

Both exponents can be understood from a critical Tsallis exponent \( q = 5/3 \). To see this, we first interpret the radial and time dependent density as a probability distribution in the sense of a Fokker-Planck equation with fractional derivatives, the solution of which reads [9]

\[
\frac{\partial}{\partial t} P_\gamma(x,t) = D \nabla^\gamma P_\gamma(x,t)^\nu, \quad P_\gamma(x,t) \propto \frac{1}{(t)^{\frac{\gamma+1}{\gamma+1} - \frac{1}{\gamma+1}} \left( 1 + bz \right)^{1-\gamma}},
\]

with \( z = x(|k_1|t)^{-\frac{\gamma+1}{\gamma+1}} \). The order of fractional derivative \( \gamma \) is linked to the Tsallis exponent by \( q = \frac{\gamma+3}{\gamma+1} \approx \frac{5}{3} \). For a critical value of \( \gamma = 2 \) or \( q = 5/3 \), one obtains indeed the above observed cubic density profile \( P_2(x,t,\gamma) \propto x^{-3} \). The associated velocity profile can be explained also by observing that for \( \gamma = 2 \) from (3) follows
\[
\frac{\partial}{\partial t} P_2(x, t) \propto \frac{1}{t^2} \frac{1}{z^2 (1 + bz)^2} \quad z = x(|k_1|t)^{-1},
\] (4)

and the velocity profile can be estimated to

\[
v(r) \approx \langle x \dot{P}_2(x, t) \rangle \propto \frac{1}{b^2} \left( \ln(1 + bz) + \frac{1}{1 + bz} \right) \approx \frac{1}{b^2} + \frac{z^2}{2} + o(bz^3). \quad (5)
\]

This explains the radial quadratic exponent of the observed velocity profile.

In conclusion, we saw that for bombarding energies below the Fermi energy there appears a peculiar quadratic velocity profile with respect to the radius and an associated cubic power law of the density. Both features can be fitted by a fractional derivative Fokker-Planck equation with the critical Tsallis exponent \( q = 5/3 \). Since this exponent marks exactly the transition between Gauss- and Lévy-like diffusion we suggest that the observed radial dependence might be associated with a phase transition during the reaction stage.

References


