1. INTRODUCTION

Exclusive (e,p) knockout reactions have been used since a long time to study the single particle properties of nuclear structure. Thermal analysis of the experimental cross sections was successfully carried out in the theoretical framework of the semi-relativistic distorted wave impulse approximation (DWIA) for $Q^2$ less than 0.4 (GeV/c)$^2$. In recent years, owing to the new data from JINR [8], also the comparison with the semi-relativistic DWIA [9] for $Q^2$ up to 2(GeV/c)$^2$ has been extended to the case of the semi-relativistic DWIA [10].

In this work we analyze the data for $Q^2$ up to 2(GeV/c)$^2$ and the semi-relativistic DWIA [10], and we discuss the implications for the semi-relativistic DWIA model.
the framework of the relativistic mean field theory. The effective Pauli reduction has been adopted for the outgoing nucleon wave function. The resulting Schrödinger-like equation is solved for each partial wave starting from relativistic optical potentials. The relativistic current is written following the most commonly used current conserving (cc) prescriptions for the \((e, e')p\) reaction introduced in Ref. [20]. The ambiguities connected with different choices of the electromagnetic current cannot generally be dismissed. In the \((e, e')p\) reaction the predictions of different prescriptions are generally in close agreement [21]. Large differences can however be found at high missing momenta [22, 23].

The formalism is outlined in Sec. II. Relativistic calculations of nuclear transparency are presented in Sec. III, where current ambiguities are also investigated. Some conclusions are drawn in Sec. IV.

II. FORMALISM

The nuclear transparency can be experimentally defined as the ratio of the measured cross section to the cross section in plane wave approximation, which is usually evaluated by means of a Monte Carlo simulation to take into account the kinematics of the experiment. Hence, we define nuclear transparency as

\[
T = \frac{\int V dE_m dP_m \sigma_{DW} (E_m, P_m, \mathbf{P'})}{\int V dE_m dP_m \sigma_{PW} (E_m, P_m)},
\]

where \(\sigma_{DW}\) is the distorted wave cross section and \(\sigma_{PW}\) is the plane wave one. Since the measured transparency depends upon the kinematics conditions and the spectrometer acceptance, we have to specify the space phase volume, \(V\), and use it for both the numerator and the denominator [24]. Because of final state interaction, the distorted cross section depends upon the momentum of the emitted nucleon \(\mathbf{p'}\), whereas the undistorted cross section only depends upon the missing energy \(E_m\) and the missing momentum \(P_m\).

In the one-photon exchange approximation the \((e, e')p\) cross section is given by the contraction between the lepton tensor and the hadron tensor. In the case of an unpolarized reaction it can be written as

\[
\sigma = \sigma_M f_{rec} E' |\mathbf{p'}|^2 \left[ \rho_{00} f_{00} + \rho_{11} f_{11} + \rho_{01} f_{01} \cos \left( \frac{1}{2} \theta_1 \right) + \rho_{1-1} f_{1-1} \cos (2\phi) \right],
\]

where \(\sigma_M\) is the Mott cross section, \(f_{rec}\) is the recoil factor [1, 2], \(E'\) and \(\mathbf{p'}\) are the energy and momentum of the emitted nucleon, and \(\theta_1\) is the out of plane angle between the electron scattering plane and the \((q, \mathbf{p'})\) plane. The coefficients \(\rho_{i\lambda}\) are obtained from the lepton tensor components and depend only upon the electron kinematics [1, 2]. The structure functions \(f_{i\lambda}\) are given by bilinear combinations of the components of the nuclear current as

\[
\begin{align*}
f_{00} &= \langle J^0 (J^0) \rangle, \\
f_{11} &= \langle J^z (J^z) \rangle + \langle J^y (J^y) \rangle, \\
f_{01} &= -2\sqrt{2} \text{Re} \left[ \langle J^y (J^y) \rangle \right], \\
f_{1-1} &= \langle J^y (J^y) \rangle - \langle J^x (J^x) \rangle,
\end{align*}
\]

where \(\langle \cdots \rangle\) means that average over the initial and sum over the final states is performed fulfilling energy conservation. In our frame of reference the \(z\) axis is along \(q\), and the \(y\) axis is parallel to \(q \times \mathbf{p'}\).

In RDWIA the matrix elements of the nuclear current operator, i.e.,

\[
J^\mu = \int d^3 r \mathbf{T_j}(r) \bar{\Psi}(r) e^{iq \cdot r} e_{\mu},
\]

are calculated using relativistic wave functions for initial and final states.

The choice of the electromagnetic operator is a longstanding problem. Here we discuss the three
current conserving expressions [20, 25, 26]

\[ \tilde{J}_{\rho 1}^2 = G_M(Q^2)\gamma^\mu - \frac{\kappa}{2M} F_2(Q^2) \bar{p} p', \]

\[ \tilde{J}_{\rho 2}^2 = F_1(Q^2)\gamma^\mu + \frac{i}{2M} F_2(Q^2) \sigma^{\mu\nu} q_\nu, \]

\[ \tilde{J}_{\rho 3}^2 = F_1(Q^2)\frac{\bar{p} p'}{2M} + \frac{i}{2M} G_M(Q^2) \sigma^{\mu\nu} q_\nu, \]

where \( q^\mu = (\omega, q) \) is the four-momentum transfer, \( Q^2 = |q|^2 - \omega^2 \), \( \bar{p} p' = (E + E', p_m + p') \), \( \kappa \) is the anomalous part of the magnetic moment, \( F_1 \) and \( F_2 \) are the Dirac and Pauli nucleon form factors, \( G_M = F_1 + \kappa F_2 \) is the Sachs nucleon magnetic form factor, and \( \sigma^{\mu\nu} = (i/2) [\gamma^\mu, \gamma^\nu] \). These expressions are equivalent for on-shell particles thanks to Gordon identity. However, since nucleons in the nucleus are off-shell we expect that these formulas should give different results. Current conservation is restored by replacing the longitudinal current and the bound nucleon energy by [20]

\[ J_L = J_z = \frac{\omega}{|q|} J^0, \]

\[ E = \sqrt{|p_m|^2 + M^2} = \sqrt{|p' - q|^2 + M^2}. \]

The bound state wave function

\[ \Psi_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix}, \]

is given by the Dirac-Hartree solution of a relativistic Lagrangian containing scalar and vector potentials.

The ejectile wave function \( \Psi_f \) is written in terms of its positive energy component \( \Psi_{f+} \) following the direct Pauli reduction method [27]

\[ \Psi_f = \begin{pmatrix} \Psi_{f+} \\ \frac{M + E' + S - V}{M + E'} \Psi_{f+} \end{pmatrix}, \]

where \( S = S(r) \) and \( V = V(r) \) are the scalar and vector potentials for the nucleon with energy \( E' \).

The upper component \( \Psi_{f+} \) is related to a Schrödinger equivalent wave function \( \Phi_f \) by the Darwin factor \( D(r) \), i.e.,

\[ \Psi_{f+} = \sqrt{D(r)} \Phi_f, \]

\[ D(r) = \frac{M + E' + S - V}{M + E'}. \]

\( \Phi_f \) is a two-component wave function which is solution of a Schrödinger equation containing equivalent central and spin-orbit potentials obtained from the scalar and vector potentials. Hence, using the relativistic normalization, the emitted nucleon wave function is written as

\[ \overline{\Psi_f} = \Psi_f^\dagger \gamma^0 = \sqrt{\frac{M + E'}{2E'}} \left[ \left( \frac{1}{\sigma \cdot p'} \right) \sqrt{D} \Phi_f \right]^\dagger \gamma^0 = \sqrt{\frac{M + E'}{2E'}} \Phi_f^\dagger \left( \sqrt{D} \right)^\dagger \left( 1 ; \sigma \cdot p' \frac{1}{C C^\dagger} \right) \gamma^0, \]

where

\[ C = C(r) = M + E' + S(r) - V(r). \]

III. TRANSPARENCY AND THE \((e,e'p)\) REACTION

The \((e,e'p)\) reaction is a well-suited process to search for CT effects. The \( e-p \) cross section is accurately known from QED and the energy resolution guarantees the exclusivity of the reaction.
Several measurements of nuclear transparency to protons in quasi-free \((e, e'p)\) knockout have been carried out on several target nuclei and over a wide range of energies to look for CT onset.

Here, we calculated nuclear transparency for closed shell or subshell nuclei at kinematics conditions compatible with the experimental setups for which the measurements of nuclear transparency have been performed, and for which the RDWIA predictions are known to provide a good agreement with cross section data. The bound state wave functions and optical potentials are the same as in Refs. [18, 19], where the RDWIA results are in satisfactory agreement with \((e, e'p)\) and \((\gamma, p)\) data.

The relativistic bound-state wave functions have been obtained from the program ADFX of Ref. [28], where relativistic Hartree-Bogoliubov equations are solved in the mean field approximation to the description of ground state properties of several spherical nuclei. The model starts from a Lagrangian density containing sigma-meson, omega-meson, rho-meson and photon field, whose potentials are obtained by solving self-consistently Klein-Gordon equations. Moreover, finite range interactions are included to describe pairing correlations and the coupling to particle continuum states.

The outgoing nucleon wave function is calculated by means of the complex phenomenological optical potential EDAD1 of Ref. [29], which is obtained from fits to proton elastic scattering data on several nuclei in an energy range up to 10\(^4\) MeV.

Since no rigorous prescription exists for handling off-shell nucleons, we have studied the sensitivity to different \(cc\) choices of the nuclear current. The Dirac and Pauli form factors \(F_1\) and \(F_2\) are taken from Ref. [30].

In Fig. 1 our RDWIA results for nuclear transparency, calculated with the \(cc2\) prescription for the nuclear current are shown. The \(Q^2\) of the exchanged photon is taken between 0.3 (GeV/\(c\))^2 and 1.8 (GeV/\(c\))^2 in constant \((q, \omega)\) kinematics. Calculations have been performed for selected closed shell or subshell nuclei \((^{12}C, ^{16}O, ^{28}Si, ^{40}Ca, ^{90}Zr, \text{and } ^{208}Pb)\) for which the relativistic mean field code easily converges. The agreement with the data is rather satisfactory. At \(Q^2 = 0.3\) (GeV/\(c\))^2 our results lie below the data and are comparable with those presented in Ref. [14], where it was shown that the EDAD1 optical potential led to a smaller transparency, while better agreement was found using an empirical effective interaction which fits both proton elastic and inelastic scattering data. However, we have to note that the DWIA model of Ref. [14] uses a different approach to obtain single particle bound state wave functions. The calculations at \(Q^2 = 0.6, 1.3, \text{and } 1.8\) (GeV/\(c\))^2 are closer to the data and fall down only for higher mass numbers.

In Fig. 2 the energy dependence of nuclear transparency is shown. The calculations have been performed for the same nuclei and at the same kinematics as in Fig. 1. The calculated transparency is approximately constant for each nucleus and decreases for increasing mass number.

In Refs. [10, 12] it is reported that the transparency data can be fitted with an exponential law of the form \(T = A^{-\alpha}\), with \(\alpha \approx 0.24\). Since our model is based on a single particle picture of nuclear structure, we expect our results to be sensible to the discontinuities of the shell structure. These clearly appear in the changes in shape of the \(A\)-dependent curves.

In Fig. 3 the sensitivity of transparency calculations for \(^{12}C\) and \(^{40}Ca\) to different choices for the electromagnetic current is shown. The results with the \(cc1\) current are larger than those obtained with the \(cc2\) current, whereas \(cc3\) results are smaller than the \(cc2\) ones. A similar behavior was already found out in Ref. [19] for \((\gamma, N)\) differential cross section. Here it is mainly due to the fact that, when using the \(cc1\) current, the distorted cross section, \(\sigma_{DW}\) in Eq. 1, is enhanced with respect to the calculations with the \(cc2\) or the \(cc3\) current, whereas the plane wave cross sections, \(\sigma_{PW}\), are almost independent of the operator form.

**IV. SUMMARY AND CONCLUSIONS**

In this paper we have presented relativistic DWIA calculations for nuclear transparency of \((e, e'p)\) reaction in a momentum transfer range between 0.3 and 1.8 (GeV/\(c\))^2.

The transition matrix element of the nuclear current operator in RDWIA is calculated using the bound state wave functions obtained in the framework of the relativistic mean field theory, and the direct Pauli reduction method with scalar and vector potentials for the scattering state. In order to analyze the ambiguities in the choice of the electromagnetic vertex due to the off-shell character of the initial nucleon, we have used three current conserving expressions in our calculations.
We have performed calculations for selected closed shell or subshell nuclei. The dependence of nuclear transparency upon the mass number and the energy has been discussed. Low $Q^2$ results underestimate the data, thus indicating the presence of too strong an absorptive term in the optical potential. In contrast, results at higher $Q^2$ are closer to the data. We find little evidence of energy dependence or momentum transfer of the transparency for each nucleus.

The sensitivity to different choices of the nuclear current has been investigated for $^{12}$C and $^{40}$Ca. The results with the $e1$ current are larger than the $e2$ results, whereas those obtained with the $e3$ current are more similar to the $e2$ ones. This effect is due to the enhancement of the $e1$ distorted cross section with respect to the $e2$ and $e3$ cross sections.

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J. M. Udías, P. Sarriguren, E. May de Guerra, E. Garrido, and J. A. Caballero, Phys. Rev. C 48, 2731 (1993);
FIG. 1: The nuclear transparency for the quasi-free $A(e,e'p)$ reaction as a function of the mass number for $Q^2$ ranging from 0.3 to 1.8 (GeV/c)^2. Calculations have been performed for selected closed-shell or subshell nuclei with mass numbers indicated by open circles. The data at $Q^2 = 0.3$ (GeV/c)^2 are from Ref. [9]. The data at $Q^2 = 0.6, 1.3$, and $1.8$ (GeV/c)^2 are from Ref. [11].
FIG. 2: The energy dependence of nuclear transparency for $^{12}$C (open circles), $^{16}$O (open stars), $^{40}$Ca (open squares), $^{90}$Zr (open triangles), $^{208}$Pb (open diamonds), at the same kinematics as in Fig. 1. Calculations were performed for $Q^2$ values marked by symbols. The $^{12}$C data are from Refs. [9, 10, 11].
FIG. 3: The electromagnetic current dependence of nuclear transparency for $^{12}$C and $^{40}$Ca, at the same kinematics as in Fig. 1. Calculations were performed for $Q^2$ values marked by symbols.