Searching for Failed Supernovae With Astrometric Binaries

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ABSTRACT

Stars in the mass range $8 M_\odot \lesssim M \lesssim 30 M_\odot$ are thought to end their lives as luminous supernovae that leave behind a neutron star. However, if a substantial fraction of these stars instead ended as black-hole remnants, without producing a supernova (a ‘failed’ supernova), how would one know? We show that, under plausible assumptions, the Hipparcos catalog should contain $\sim 30 f_{\text{fail}}$ astrometric binaries with black-hole companions, where $f_{\text{fail}}$ is the fraction of supernovae that fail. Since no black-hole astrometric binaries are found in Hipparcos, one might like to conclude that such failed supernovae are very rare. However, the most important assumption required for this argument, the initial companion mass function (ICMF) of G stars (the majority of Hipparcos stars) in the high-mass companion regime, is without any observational basis. We show how the ICMF of G stars can be measured using the Full-Sky Astrometric Explorer (FAME), thereby permitting an accurate measurement of the rate of supernovae that fail.

Subject headings: astrometry—binaries: general—stars: neutron—supernovae: general

1. Introduction

Massive stars ($M \gtrsim 8 M_\odot$) end their lives as supernovae (SNe), leaving behind black-hole (BH) or neutron-star (NS) remnants depending on whether they are more or less massive than a cutoff mass, $M_{\text{cut}} \sim 30 M_\odot$. So says the standard lore, but what is the observational evidence?

Certainly there exist NSs and BHs, and since pulsars are frequently found near the centers of SN remnants, there can be little doubt about their origin. But do the majority of massive stars $8 \lesssim M/M_\odot \lesssim 30$ really die gloriously in luminous SNe that give birth to a NS? Or do most of them fizzle out in failed SNe that collapse in on themselves, leaving behind a BH?
Failed SNe have been invoked by theorists for a number of reasons, mostly because of the difficulties of producing an explosion in analytical models and simulations (for a detailed review see Woosley & Weaver 1986). The failures in the earliest models in the 1960s were mostly ascribed to the models’ inability to transfer the energy of the core collapse into decoupling the mantle from the envelope. Improvements in the neutrino transport model and new estimates of neutrino cross sections still failed to produce explosions in the 1970s. Owing to improved nuclear rates and more sophisticated models, successful explosions were finally produced in the 1980s, however only for low-mass iron cores ($M \lesssim 1.3 M_\odot$), corresponding to progenitor masses of $M \lesssim 11 M_\odot$. This problem was somewhat alleviated by the introduction of a delayed neutrino energy transport, which supplies the energy to the lower parts of envelope and so helps the shock propagate outwards. The success of this mechanism remains inconclusive. One should keep in mind that the researchers were driven to make explosions happen and not to demonstrate that they do not happen. Mikaelian (1978) early on suggested that some SNe really ‘fail’, arguing that only stars that spin slowly can produce a SN, while the fast spinning stars just collapse into a BH. Others, however, have dismissed rotation as a significant factor. Failed SNe (of type Ib) were suggested by Woosley (1993) to represent the main mechanism for producing cosmological $\gamma$-ray bursts (GRBs). In this conjecture, for which the simulations have already produced substantial support, the collapsing core fails to produce an explosion, but instead an accretion disk forms around the core, which draws in mass from the mantle at the rate of $\sim 0.5 M_\odot$ s$^{-1}$, and then quickly collapses into a BH. Woosley (1993) suggests that in stars that have previously lost their H-envelope (Wolf-Rayet stars) the accretion could be accompanied by an energetic burst of gamma rays in the form of polar jets. This model is the basis for the currently favored “hypernova” scenario, first proposed by Paczynski (1998), which in addition requires the presence of very strong ($\sim 0.1$ T) magnetic fields.

There are remarkably few observational probes of failed SNe. The expected rate of core-collapse SNe derived from the LF of massive stars predicts many times fewer SNe than are historically observed in our Galaxy (van den Bergh & Tammann 1991), most likely indicating poor knowledge of the massive star LF. Therefore, it is not possible to infer from this whether failed SNe exist. Ultimately, with sufficiently sensitive neutrino detectors, one could directly detect many extragalactic SNe, and identify the failed ones among them from the lack of a (or very subluminous) optical counterpart. Microlensing observations with the Space Interferometry Mission (SIM) can directly measure the masses of isolated BHs and NSs in Galactic bulge fields (Gould & Salim 1999; Gould 2000). One could then compare the ratio of NS/BH detections to what would be expected based on, say, a Scalo (1986) mass function. The interpretation would be somewhat complicated by the fact that many NSs receive a large kick at birth, which could remove a large fraction of them from the
bulge. Nevertheless, given a sufficiently large sample, and with the information on the kick-velocity distribution gleaned from the NS transverse velocities (which come out of the same microlensing observations), it should be possible to reconstruct the remnant ratio. Finally, one could examine the ratio of NS/BH X-ray binaries, where these remnant objects are routinely found. However, since these are interacting systems with complex evolutionary histories, it is difficult to make inferences about the ratio of total populations, and therefore the production mechanisms, from these very special objects.

Here we propose another probe of failed SNe: BHs in astrometric binaries. In astrometric binaries the components are too close to be resolved, and only the motion of the photocenter is observed. If one component is invisible (BH or NS), only the motion of the visible component around the barycenter will be detected. This experiment would be sensitive to detecting failed SNe that did not undergo some process that would disrupt the system. MacFadyen & Woosley (1999) predict that the formation of a collapsar might always be accompanied by a hypernova explosion, however whether this necessarily precludes a BH from remaining in the binary system is uncertain.

In § 2, we discuss the sensitivity of astrometric surveys to dark companions, focusing specifically on Hipparcos. Note that only space-based surveys probe enough stars to allow an effective search for rare objects. We show that Hipparcos is sensitive to BH companions of the great majority of stars in its catalog. No BH binaries are found, which potentially places strong limits on the number of failed SNe. However, since the initial companion mass function (ICMF) of Hipparcos stars is unknown, there is a serious loophole in this argument: one does not know whether the absence of BH astrometric binaries reflects the absence of failed SNe or the absence of progenitors in binaries. In § 3, we show that astrometric binary searches using the Full-Sky Astrometric Explorer (FAME) can close this loophole.

### 2. Astrometric Detection of Dark Companions

Consider a binary whose components have masses and luminosities in the band of astrometric observations \((M, L)\) and \((m, l)\), respectively. From Kepler’s third law, the semi-major axis, \(a\), is related to the period, \(P\), by \([\frac{(m + M)}{M_\odot}](P/\text{yr})^2 = (a/\text{AU})^3\). If the motion of the photocenter is fit to a Keplerian orbit, the angular semimajor axis of the photocenter orbit, \(\alpha\) (measured in the orbital plane), will then be related to the other parameters by

\[
\frac{m^3}{M_\odot(m + M)^2} \left(\frac{L}{L + l}\right)^3 \left(\frac{P}{\text{yr}}\right)^{-2} \left(\frac{D\alpha}{\text{AU}}\right)^3,
\]

where \(D\) is the distance to the system. The quantities on the rhs of this equation can all be measured astrometrically. We will assume that \(M\), the mass of the more luminous
component, can be estimated photometrically or spectroscopically. And we will focus on the case in which the companion is known to be dark (or at least extremely dim compared to the primary), \( l \ll L \). Under these assumptions, it is straightforward to determine \( m \), the mass of the dark companion, from the astrometric observations.

In general, \( \alpha \) can be measured with approximately the same precision as the parallax, \( \pi \). Of course this does not hold exactly. Even for circular binary orbits, the inclination of the orbit will not match exactly the ecliptic latitude (i.e., the inclination of the parallactic circle), so there will be either more or less information about the binary orbit than about the reflex motion of the Earth’s orbit (parallax). Moreover, for certain binary orbits, notably edge-on highly eccentric orbits that “point” in our direction, the errors in \( \alpha \) will be much larger than the parallax errors because the binary will show almost no astrometric motion. Nevertheless, from the standpoint of making an estimate of the errors for a random ensemble of binaries, setting \( \sigma_\alpha \sim \sigma_\pi \) is a good approximation. This is confirmed by Figure 1, where we plot \( \sigma_\alpha/\sigma_\pi \) for astrometric binaries with orbital solutions (i.e., binaries of type ‘O’) in the Hipparcos catalog (ESA 1997, Vol. 10). While these fits made use of some auxiliary ground-based spectroscopic information (mainly to establish the period), or constrained orbits to be circular, this should not have a major impact on the errors in \( \alpha \) for periods \( P \lesssim 3.3 \) yr, the duration of the mission. While the figure shows some scatter, the two errors are roughly equal on average.

Figure 2 shows the sensitivity (5\( \sigma \) detection) of Hipparcos to dark companions as a function of stellar type, i.e., the number of Hipparcos stars that can be probed for companions of a given mass. These types were assigned based on position in the color-magnitude diagram when the parallaxes were sufficiently accurate, and on position in the reduced proper-motion diagram otherwise. In the latter case, distances were assigned based on stellar type and color and magnitude. The figure shows that white dwarf (WD), NS, and BH companions of mass 0.6, 1.4 and 7 \( M_\odot \), are respectively detectable among 39\%, 68\%, and 89\% of all Hipparcos stars \( (N_{Hip} = 118,000) \). For periods of \( P = 1.5 \) yr, these fractions fall to 21\%, 47\%, 52\%. At \( P \sim 1 \) yr, sensitivity is seriously compromised by parallax aliasing and at shorter periods the sensitivity falls off rapidly. On the other hand, for \( P \gtrsim 3.3 \) yr, orbital solutions become rapidly unstable. Hence, the sensitivities peak fairly sharply at \( P \sim 3.3 \) yr.

The overwhelming majority of these Hipparcos stars are F and G dwarfs, or giant stars whose progenitors are overwhelmingly F and G dwarfs. The frequency of companions per log period for \( P \sim 3.3 \) yr among such stars is \( df_b/d \log P \sim 7\% \) (Duquennoy & Mayor 1991). From the previous paragraph, Hipparcos is sensitive to companions over about half a dex in period, \( \Delta \log P \sim 0.5 \). The total number of Hipparcos stars that were born with NS/BH
progenitor companions in this period range is then,

$$N_{\text{progen}} = N_{\text{Hip}} \frac{df_b}{d \log P} \Delta \log P \ f_{\text{progen}} = 40 \ f_{\text{progen}} \left( \frac{\%}{1} \right),$$

where we have normalized the fraction of companions that are NS/BH progenitors to $f_{\text{progen}} = 1\%$, in accord with an estimate by (Gould 2000) for their relative frequency among all stars (both binary and single). If a fraction $f_{\text{fail}}$ of these progenitors ended their lives as failed SNe, then there should be $\sim 40 f_{\text{fail}}$ BHs in orbits within the period range covered by Hipparcos of which Hipparcos should be sensitive to $\sim 80\%$ of them (from the previous paragraph), giving a total of $\sim 30 f_{\text{fail}}$ BHs. In fact, none of the 235 Hipparcos astrometric binaries with orbital solution (188 have $P < 3.3$ yr) contain a clear BH candidate component. In all cases we find the mass of the companion (if we assume it to be invisible) either well below the BH range, or of order or smaller than that of the luminous star, which implies that the companion is a main sequence star that is fainter than the primary.

One would like to use this result to argue that less than 10\% of massive stars end as failed SNe. That is, if more than 10\% failed, we would expect more than 3 BH companions. Since we find none, the hypothesis would be ruled out at the 95\% confidence level.

There are, however, two objections to this line of reasoning. First, we do not actually know that at formation the fraction of companions that are NS/BH progenitors is $f_{\text{progen}} = 1\%$. Indeed, there are no observational constraints on this parameter, and no theoretical reason to believe (or not to believe) that the fraction of NS/BH progenitors is the same for G star companions as it is for stars in the field. We address this problem in § 3.

Second, even if this fraction is the same at formation, it could be that the very process of the failed SNe disrupts the binary. Certainly, binaries of this sort will very often be disrupted by an ordinary SNe. For example, consider a binary composed of an $M = 1 M_\odot$ and an $m' = 8 M_\odot$ star, the latter of which “instantaneously” ejects 83\% of its mass to become an $m = 1.4 M_\odot$ NS. Even if the NS receives no kick, the system will become unbound unless it is near apocenter in a fairly eccentric orbit, $e > 1 - 2(m + M)/(m' + M) = 0.47$. This eccentricity constraint becomes more severe for larger progenitor masses. Also, many NSs are known to receive a significant kick, often several hundred km s$^{-1}$, which would certainly disrupt the binary. However, in a failed SN, a large fraction of the progenitor would fall back on the BH, so the ratio $(m + M)/(m' + M)$ would be much larger. Hence, at least for the relatively less massive progenitors, the binary would not be disrupted. It remains possible that the BH remnant would also receive a strong kick in a failed SN, but since the mechanism behind the kick is not well understood, this must remain a matter of speculation.

Finally we note that for the specific case of the Hipparcos sample, there is some question as to its real sensitivity. Vol. 3]hip does not quote a specific threshold of detection, i.e.,
the required goodness of the fit, but from Figure 3, which shows the distribution of $\alpha/\sigma_\alpha$ as a function of period, we judge this threshold to be $\sim 5 \sigma$, i.e., the same value we used in estimating the total number of BH companions that should have been detected. However, Quist & Lindegren (2000) simulated the number of detections of $V < 7$ MS binaries that *Hipparcos* should have made (based on a Galactic model and the Duquennoy & Mayor 1991 binary distribution model), and find that the *Hipparcos* catalog should contain between 35% and 200% more binaries with $P < 3.3$ yr orbital solutions (type ‘O’) than it actually does. They suggest that many binaries for which ESA (1997) finds no orbital solution, and thus classifies as type ‘X’, or ‘G’, could have produced an orbit if, for example, the period were known from spectroscopic observations. Careful reanalysis of *Hipparcos* transit data might lead to new orbital solutions.

3. The Binary Mass Function

While there is some uncertainty as to the fate of binaries containing failed SNe, the main problem with deriving robust conclusions from equation (2) is that the observational constraints on $f_{\text{progen}}$, the fraction of binaries born with a massive companion, a NS/BH progenitor, are weak. The main difficulty here is that the “primaries” of these systems (mostly F and G dwarfs and their giant-star descendants – see Fig. 2) are all about $1 M_\odot$, whereas the “secondaries” of these systems are substantially more massive. They are therefore both more luminous (making the F-G star difficult to detect) and shorter-lived than the “primaries” (meaning that they are long gone in a field sample of F-G stars). Here we present two complementary astrometric methods to overcome this difficulty, and show that these can be implemented using FAME.

We wish to determine the ICMF of G stars as a function of companion mass, $m$, with some period $P$. For definiteness, let us consider a period range $3 \text{yr} < P < 5 \text{yr}$, and restrict ourselves to one point of the ICMF: B2 stars corresponding to a mass range $11 < m/M_\odot < 15$ and magnitude range $-2.8 < M_V < -2.0$. What we seek is ratio of formation rate of B2-G binaries (in this period range) to the formation rate of all G stars. From this formulation of the problem, it would seem that one should just survey OB associations and count the number of B2-G binaries and the total number of G stars. However, since the IMF of OB associations may be significantly different from the disk-averaged IMF, this procedure would produce a biased result. One must somehow compare formation rates in the disk as a whole.

Assume for the moment that the star formation rate has been uniform over the lifetime
of the disk. The ratio we seek, $F^P_{B2|G}$, is then given by

$$F^P_{B2|G} = \frac{\Sigma_{B2}}{\Sigma_G} \frac{\tau_G}{\tau_{B2}} f^P_{G|B2}. \quad (3)$$

Here $\Sigma_{B2}$ is the column density (number per square parsec) of B2 stars (averaged over spiral-arm and inter-arm regions), $\Sigma_G$ is the column density of G stars, $\tau_G$ and $\tau_{B2}$ are the lifetimes of G and B2 stars respectively (capped by the age of the disk in the case of late G stars), and $f^P_{G|B2}$ is the fraction of B2 stars with G companions in the appropriate period range $P$. The first two ratios in equation (3) are reasonably well known. The last factor $f^P_{G|B2}$ is very poorly known but can be measured using FAME, which will probe a volume $\sim 10,000$ larger than Hipparcos did. Then using equation (3), one can calculate from the current fraction of B stars with a G dwarf companion, the fraction of G dwarfs that were born with B companions. Figure 4 shows the number of stars to which FAME is sensitive to dark (or dim, since a G dwarf is much fainter than a B star) companions with a given mass in 5-year orbits, for various spectral types. There are $2 \times 10^6$ stars with $1.2 M_\odot \lesssim M \lesssim 8 M_\odot$ (WD progenitors) for which FAME will be sensitive to companions of $1 M_\odot$ (G dwarfs), as well as $2 \times 10^4$ heavier stars ($8 M_\odot \lesssim M \lesssim 20 M_\odot$) (NS or BH progenitors). If the companion rate in this period range is 7%/dex, and 5% of companions are G stars, then FAME will detect $\sim 2000$ G star companions of WD progenitors within an octave of period and $\sim 20$ G star companions of heavier stars. Thus, the WD-progenitor ICMF will be mapped out in great detail, and can then be extended with reasonably good confidence into the higher-mass range of the progenitors of SNe, luminous or failed. There will also be a direct measurement the ICMF in this high mass regime, albeit somewhat crude.

Of course, the star formation rate has not been uniform over the lifetime of the disk, but it is straightforward to take account of this variation by modifying equation (3).

This method of determining the ICMF assumes implicitly that $f^P_{G|B2}$ has not changed from today’s value over the age of the disk. While this assumption is plausible, it can also be partially checked using a different application of FAME astrometry, namely by detecting the remnant object companions themselves.

In contrast to NS/BH progenitors, the evolution of WD-progenitor binaries is deterministic provided that the pair is not close enough to interact during the asymptotic giant branch (AGB) phase. This is because the mass loss of the progenitor proceeds on timescales that are long compared to the period, so that the evolution is adiabatic. One finds that the final semimajor axis is $a = a'(M + m')/(M + m)$, where $a'$ is the initial semimajor axis, and $m'$ is the initial mass. Hence the initial period $P'$ is related to the final period $P$ by

$$P' = P \frac{(M + m)^2}{(M + m')^2}. \quad (4)$$
This means that, while it is not possible to directly probe the time-averaged ICMF in the NS/BH progenitor regime (because the binaries could have been disrupted by the SNe), it might be possible to probe it in the WD-progenitor regime.

A significant problem in determining the ICMF for G stars is that when a 0.6 \( M_\odot \) companion to a G dwarf is discovered, one does not immediately know whether this companion is a WD or an M dwarf (with \( M_V \sim 8 \)). Neither has much luminosity compared to the G dwarf and so in neither case would the mass determination be significantly affected (see eq. [1]). By the same token, however, there would be no obvious signatures that the companion was one type or the other. It is possible that with precision photometry one could detect the IR excess due to the M dwarf. High signal-to-noise ratio spectroscopy could certainly detect the M dwarf if it were there. The scope of the spectroscopy project would be significantly reduced if one surveyed K dwarfs rather than G dwarfs because the K/M magnitude difference is substantially smaller than the G/M difference. Figure 4 shows that \textit{FAME} will be sensitive to WD companions of \( 10^6 \) K dwarfs and \( 2 \times 10^6 \) G dwarfs, which is certainly enough to obtain a large sample of WD/dG-dK binaries.

The major limitation of this method comes from the fact that during its 5-yr mission, \textit{FAME} can obtain accurate mass measurements only for \( P < 5 \) yr. According to equation (4), the periods \( P' \) of the progenitor systems were substantially shorter than the periods \( P \) of their present-day descendants. Specifically, for G dwarf primaries, we have \( P'(M + m')^2 < 12.8 M_\odot^2 \text{yr} \). If the periods were too short, then during the AGB phase, the binary would have suffered mass transfer and its evolution would have taken a complex course. If we assume that no mass transfer occurs for \( a' > 1.5 \text{AU} \) (the exact value is model dependent), then this condition implies \( (M + m')(a'/\text{AU}) < 12.8^{2/3} M_\odot \), or \( m' < 2.6 M_\odot \). Hence only the lower-mass WD-progenitor population is probed. Moreover, in this mass range, the WD mass is only a very weak function of the progenitor mass (hence the peakiness of the WD mass function). Given both the astrometric errors and the errors in the photometric masses of the G star primaries, it seems unlikely that one could obtain much more detail than the total number of WDs as a function of period. Hence one would really obtain only a single point beyond the usual G dwarf ICMF (i.e., of secondaries that are fainter and lower-mass than the primary). Nevertheless, equal-mass is the only natural scale in this problem. Thus, if this direct determination of the time-averaged ICMF tracked the ICMF measured from present day companions of early-type stars across the equal-mass boundary, it would lend credence to the latter ICMF measurement at higher masses.

If \textit{GAIA} ultimately flies, then one could extend the orbits initially mapped out by \textit{FAME} to effectively cover periods of \( \sim 15 \) years, which would allow one to probe WDs that have much larger progenitor masses.
4. Conclusion

The notion that some massive stars undergo a collapse without producing a SN first appeared as a failure to produce SN explosions in hydrodynamical simulations, but later gained ground as a possible mechanism behind GRBs. These theoretical considerations were left without any empirical evidence. If in binary systems these failed SNe collapse into BHs without disrupting the binaries, one might be able detect such systems astrometrically. To this end we suggest using FAME since it will observe vast number of stars with great precision. To estimate the rate of SNe that fail, it is first necessary to estimate the number of binaries that were born consisting of a G dwarf (a typical star FAME will observe), and a massive, short-lived B star (NS/BH progenitor). We can derive this number using FAME detections of either currently existing G+B pairs, or by extrapolating from the number of G+WD binaries. FAME will be able to probe \( \sim 4 \times 10^7 \) stars for BH companions. The result of this experiment would either discover a new phenomenon (BH collapsar), or place stringent limits on SN and GRB models.

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Fig. 1.— The ratio of the error in the photocentric semimajor axis \( \sigma_{\alpha} \) to the parallax error \( \sigma_{\pi} \) as a function of period \( P \) for astrometric binaries with orbital solutions from the *Hipparcos* catalog. Plot shows 210 systems (4 lie outside of y-axis range.) As expected from general arguments, the ratio is typically unity. Note that *Hipparcos* mission lasted for 3.3 years.
Fig. 2.— Sensitivity of Hipparcos stars to dark binary companions in $P = 3.3$ yr orbits as a function of companion mass, according to stellar type. Types are broken down (in order of frequency in the catalog) into F-G dwarfs, giant stars, O-B-A stars, K-M dwarfs, and white dwarfs (WDs). A 5$\sigma$ signal is required for detection. Note that WD, NS, and BH companions of mass 0.6, 1.4 and 7 $M_{\odot}$, are respectively detectable among 39%, 68%, and 89% of all Hipparcos stars.
Fig. 3.— Signal-to-noise ratio $\alpha/\sigma_\alpha$ as a function of period $P$ for binaries detected in the \textit{Hipparcos} catalog. From the form of the distribution, we estimate that the catalog is complete down to roughly the 5 $\sigma$ detection level.
Fig. 4.— *FAME* sensitivity to binaries with dark (or very dim) companions in 5-yr periods as a function of the mass of the luminous star, for various spectral types. The calculation follows that of Salim, Gould, & Olling (2001), except that for early-type stars, the disk scale heights are rescaled in accordance with Miller & Scalo (1979).