Mirror Matter as Self Interacting Dark Matter

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Abstract

It has been argued that the observed core density profile of galaxies is inconsistent with having a dark matter particle that is collisionless and alternative dark matter candidates which are self interacting may explain observations better. One new class of self interacting dark matter that has been proposed in the context mirror universe models of particle physics is the mirror hydrogen atom whose stability is guaranteed by the conservation of mirror baryon number. We show that the effective transport cross section for mirror hydrogen atoms, has the right order of magnitude for solving the “cuspy” halo problem. Furthermore, the suppression of dissipation effects for mirror atoms due to higher mirror mass scale prevents the mirror halo matter from collapsing into a disk strengthening the argument for mirror matter as galactic dark matter.

I. INTRODUCTION

It has recently been pointed out \cite{1} that the dark matter particles constituting the galactic halo need to satisfy a new constraint in order to avoid singular cusps. One way to quantify this constraint is to demand that the mean free paths of these particles be less than typical galactic sizes (say 0.1 Mpc) i.e. \( \lambda_{DM} \sim \frac{1}{\sigma_{DM} m_{DM}} \leq 0.1 \) Mpc. This equation implies that the typical cross section for the dark matter particle must be of order \( \sigma_{DM} \approx \frac{m_{DM}}{10^{24}} \) cm\(^2\). This cross section is large and it grows with the mass of the dark matter particle linearly. Some favorite long standing candidates like the neutralino LSP of 50-100 GeV mass \cite{2} would then need to have a scattering cross section of \( 10^{-22} \) cm\(^2\), a requirement which is not met by any of the existing supersymmetric models. Barring some new long range interactions, such a large cross section would be in conflict with unitarity bounds \cite{3} for pointlike dark matter particle.

The growth of cross sections with mass is generic to solitonic structures and it has been noted that \( Q \)-balls originally suggested in a different context \cite{4} can, for certain range of
An alternative and natural candidate arises in mirror matter models where it is postulated that there is a parallel standard model which duplicates all the matter and forces and coexists, in our universe, with the familiar standard model. The mirror and familiar particles in such models are only connected by gravity [6–9]. In particular, the asymmetric mirror model [7,10] where both the weak scale as well as the QCD scale in the mirror sector are about 20-30 times the corresponding scales in the the familiar sector has been studied extensively in connection with neutrino physics and explanation of the microlensing events [10]. A particularly interesting feature of these models is that the lightest mirror baryon \( p' \) (in the form of mirror hydrogen atom) is ideally suited to be the dark matter of the universe and as such would be the dominant constituent of the dark halo of the galaxies. If the QCD scale parameter in the mirror sector \( \Lambda' \approx 30 \Lambda \) the corresponding scale in the familiar sector, then \( m_{p'} \approx 30 m_p \) and using the slightly lower reheat temperature of the mirror sector (required to satisfy the BBN constraints arising from \( \gamma', \nu_e', \mu', \tau' \)), we find that \( \frac{\Omega_{p'}}{\Omega_B} \approx \left( \frac{T'}{T} \right)^3 \frac{m_{p'}}{m_p} \).

The BBN constraint requires that \( T'/T \approx (1/10.75)^{1/4} \approx 0.5 \). Using this we get \( \Omega_{p'} \approx 4 \Omega_B \). For \( \Omega_B \sim 0.05 \) this would lead to 20% dark matter and about 75% dark energy. This is of the right order of magnitude for the required fraction of the dark matter in the universe.

A very important property that distinguishes mirror baryon from other dark matter candidates is that mirror matter has self interaction. It was suggested in a recent paper by two of the authors (R. N. M. and V. L. T.) [12], that this might help resolve the core density problem. We further pursue this question in the brief note. Specifically taking account of the important distinction between total and transport cross sections, we show that the parameters of the model suggested by considerations of \( \Omega_{DM} \) yield scattering of mirror hydrogen atoms in the right range suggested in Ref.1.

We then address the question of the shape of the dark halo if it is made up of mirror dark matter particles. Mirror symmetry requires that the coupling parameters in the mirror sector be identical to those of the familiar sector. This has led to a suspicion that if halos were to be made up of mirror baryons, they would collapse due to dissipation of their transverse energy and become disk shaped, in contradiction to observations. The point however is that even though the couplings are identical due to mirror symmetry, the masses are different i.e. the mirror matter masses are a factor of 30 or so higher. As a result, the processes such as brehmsstrahlung responsible for dissipation of transverse energy are down, preventing the collapse of mirror halo to a disk. All this makes mirror baryons a viable dark matter candidate.

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1One can show that the asymmetry between the two QCD scales owes its origin to the asymmetry between the weak scales [11]. The main reason the first asymmetry follows from the second is that the mirror quarks are much heavier than the familiar quarks and therefore decouple earlier from the evolution of the QCD couplings in the mirror sector. This helps to speed up the rise of the mirror QCD fine structure constant.

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II. EFFECTIVE SCATTERING CROSSSECTION FOR MIRROR HYDROGEN

For small relative velocities of atoms of order $\beta_{\text{virial}} \sim 10^{-3}$, the total atom-atom elastic scattering crosssections are of order $\pi R_{\text{atom}}^2$. For $H$ or $He$ atoms, $R_{\text{atom}} \sim 0.55$ angstroms leading to $\sigma_{HH} \simeq 10^{-16}$ cm$^2$. If we take the mirror scale factor to be about $30 - 100$, then the Bohr radius of the corresponding hydrogen atoms will scale inversely with it and will give $\sigma_{H'H'} \simeq 10^{-19} - 10^{-20}$ cm$^2$. This value is higher than the value apparently required for solving the core density problem by a factor of 100-1000. The new observation in this note is that the naive use of the cross section is not adequate for our discussion and there is indeed a substantial suppression factor which arises from a more careful analysis.

The main point is that the cross section relevant for avoiding the catastrophic accumulation of dark matter particle particles is not the total elastic cross section, $\sigma_{el}$ but the transport cross section, $\sigma_{tr}$, to which large angle scattering contributes more strongly i.e.

$$\sigma_{tr} = \frac{1}{4\pi} \int d\Omega (1 - \cos \theta) \frac{d\sigma}{d\Omega}$$  \hspace{1cm} (1)

For isotropic (say S-wave) or slightly backward hard sphere scattering, $\sigma_{el}$ and $\sigma_{tr}$ are roughly the same. This is however not the case for H-H or H'-H' scattering at relative velocity of $\beta \approx 10^{-3}$. Here many partial waves up to $\ell_{\text{eff}} = m_H v_{\text{Bohr}} \approx m_{H'} v'_{\text{Bohr}} \approx 200$ contribute allowing for strongly forward peaked elastic differential cross sections. To estimate this cross section, note that: (i) large number of partial waves suggest a quasi-classical WKB treatment; (ii) the collision virial velocity is smaller than the velocity of the electron in the atom i.e. $10^{-3}c \approx \beta_{\text{virial}} c < \alpha_{\text{em}} c$, where $\alpha_{\text{em}}$ is the velocity of the electron in the atom. Hence we can adopt an adiabatic Born-Oppenheimer type approximation. The interatomic potential can be computed for each atom-atom configuration denoted by the impact parameter $b$ and the position of the $H'$ along its path (assumed to be a strightline) $z(t)$. The interatomic distance is then given by: $R(t) = \sqrt{z^2(t) + b^2}$. The magnitude of the interatomic potential $V_{HH}(R) \approx m_e \alpha_{\text{em}}^2 \approx 27$ eV is about 20 times smaller than the kinetic energy of the collision $\frac{1}{2}m_H \beta^2 \approx 500$ eV (the same ratio applies to the mirror sector since both terms get scaled by a common factor). Hence for such velocities, atoms are “soft” and interpenetrate quite a bit. As we indicate, the scattering angle $\Delta p_\rho$ is approximately given by $\frac{\Delta p_\rho}{p} \sim \frac{V}{1.2m_e \beta^2} \approx \frac{1}{20}$. The classical deflection angle which may be appropriate here is

$$\theta \approx \frac{\Delta p_y}{p} = \frac{\int F_y(z(t), b) dt}{p} \approx 2 \int_0^\infty \left[ \frac{\partial V(\sqrt{z^2 + b^2})}{\partial y} \right] \frac{dz}{pv} \approx \frac{2V}{mv^2} \sim \frac{V}{T}. \hspace{1cm} (2)$$

Hydrogen-Hydrogen scattering at KeV energies can be measured experimentally and calculated with high accuracy. We believe that the qualitative features of strong forward peaking and correspondingly reduced transport cross section will still be manifest. Thus the transport cross section, which is $\frac{1}{2} < \theta >^2 \sigma_{el}$ will be about $10^3$ times smaller than the naive geometric value. The transport cross section for mirror hydrogen then is of order $10^{-22}$ cm$^2$, which is close to the required value for self interacting dark matter. These approximations are commensurate with the data on cross section for H-H scattering [13].
We next examine the dissipation time scale which is important for understanding the shape of the mirror dark matter halo. Mirror symmetry implies that mirror particles like ordinary ones are dissipative, namely that energy can be lost by $\gamma'$ emission. For the baryonic matter in galaxies, it is this process of energy loss that causes the collapse to a galactic disc, which provides a lower energy configuration with same total angular momentum. However if mirror matter is to form a realistic, roughly spherical galactic halo, such disc formation should not be allowed. The time scale for the disc formation in our galaxy has been estimated to be [14] to be equal the dynamical free fall time $1/(G_N \rho)^{1/2} \approx 10^8$ years (using density of one proton per cm$^3$)$^2$ The dominant dissipative process is thermal brehmstrahlung. Since the latter scales like $m^{-2}$, for the mirror baryons, the corresponding time scale would be shorter by a factor $(m^2/m'^2) \sim 10^{-3}$. This slows the relaxation time required to form a disc to about 100 billion years, which is way beyond the age of the universe.

Mirror star formation, as discussed in [10] does not depend on brehmstrahlung, but rather on molecular cooling and is not affected by the present discussion. We do however note, in that connection, that one can derive from rather general principles an additional scaling rule (with $\zeta \equiv m'/m$) for compact objects made of mirror matter, for the mass of the minimum cloud that will further fragment, as shown in the appendix.

In conclusion, we have pointed out that, in mirror matter models, the mirror hydrogen atom has all the right properties to be a self interacting dark matter of the universe. In particular, we note that due to near forward nature of the H-H scattering, the effective, relevant transport cross section is around $10^{-22}$ cm$^2$, and is adequate to damp the core density of the dark matter in galactic halos. We also point out that due to reduced bremsstrahlung cross section of mirror matter, dissipation processes get weakened enough so that the dark matter does not become disc shaped.

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Appendix

In this appendix, we discuss the scaling with $\zeta \equiv \frac{m'}{m_p} = \frac{m_e}{m_e}$ for stellar masses. The first point to note that for compact objects, the free fall time is much shorter than the

$^2$This accident is crucial to the formation of the disc. If the dissipation time was much larger, galaxy clusters would form prior to discs and if it were much shorter, the galaxy would likely fragment.
corresponding one for galaxies. Therefore, we can expect compact objects to form in the mirror sector of the universe. Let us now estimate the maximum mass of the mirror cloud that will not further fragment. We follow the treatment in Carroll and Ostlie [15]. They ask that the cloud luminosity needed to permit loss of potential energy $\Delta E \sim \frac{3}{10} \frac{GM^2}{R}$ in the free fall time $t_{ff} \sim (G\rho)^{-1/2}$, (to be called $L_{ff} \equiv \frac{\Delta E}{t_{ff}}$) be less than the black body luminosity $L_{bb} = 4\pi R^2 \sigma T^4$. This gives

$$G^3 M^5 \approx T^8 R^9$$

where we have omitted all non-dimensional constants. We can combine it with the equation that follows from the virial theorem i.e. $\frac{Gm_p M}{R} = kT$ to get

$$M = \left( \frac{T}{m} \right)^{1/4} \cdot \frac{1}{G^{3/2} m_p^2}$$

This shows that the size of the minimum fragmenting cloud scales like $\zeta^{-2}$ (assuming that the mirror temperature rises with $\zeta$ as is the case). In practice, we do not expect the cooling processes to support fragmentation to such a low mass, in view of cross section decrease with $\zeta$. Rather as pointed out in [10], we expect suppression of the processes that limit accretion in familiar stars so that the mirror stars will tend to cluster in mass near the maximum allowed stellar mass.

The point we believe should be noted is that the $\zeta^{-2}$ scaling from these considerations (i.e. $L_{bb} = L_{ff}$) is the same scaling as that for the maximum mass of a mirror star [10] which comes from finding the mass of a star such that radiation pressure dominates matter pressure thereby giving rise to instability. It is also the same scaling that one gets from computing the minimum mass of a mirror star for which it will burn as well as for the Chandrasekhar mass. It is intriguing that, even though the physical inputs to these four calculations are quite different, the resulting scaling law for the stellar mass at issue (maximum, minimum and Chandrasekhar) is still the same, i.e. $\zeta^{-2}$.
REFERENCES