The interplay of soft and hard contributions in the electromagnetic pion form factor

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We consider various relativistic models for the valence Fock-state wave function of the pion. These models are obtained from simple instant-form wave functions by applying a Melosh rotation to the spin part and by imposing physical constraints on the parameters. We discuss how the soft and the hard (perturbative) parts of the electromagnetic form factor are affected by the choice of the model and by the Melosh rotation.

The knowledge of wave functions of the hadronic constituents allows to link hadronic phenomena in different kinematical regions. For several reasons it is advantageous to consider the composition of hadrons out of its constituents at fixed light-front time \( \tau = t + z \) rather than at ordinary time \( t \). The “time” evolution in \( \tau \) is then determined by front-form dynamics. One of the attractive features of such an approach is that the corresponding wave functions are direct generalizations of non-relativistic wave functions in the sense that they can be interpreted as probability amplitudes for finding a particular Fock state in the hadron under consideration with the constituents carrying certain momenta, spins, etc. Like in the non-relativistic case light-front wave functions can be expressed in terms of purely internal variables (momentum fractions \( x \) and transverse momenta \( k_\perp \)).

For a hadron form factor the, in principle, exact expression is just a sum (over all Fock states) of overlap integrals of incoming and outgoing light-front wave functions [1]. A widely used approximation is then to assume that the dominant contribution comes from the valence Fock state. For large momentum transfers \( Q \to \infty \) the analysis of the corresponding overlap integral reveals that the one-gluon-exchange tail of the wave function can be factored out, so that one ends up with a perturbative representation of the form factor in terms of a convolution integral [2]. The distribution amplitude \( \phi(x, Q) \) entering this convolution integral is again related to the valence-quark light-front wave function of the hadron. Its dependence on the factorization scale \( \tilde{Q} \) (which in turn depends on \( Q \)) is given by an evolution equation which is driven by one-gluon-exchange. Practically, this means that the knowledge of the soft part of the wave function suffices, since the high-momentum tail of the wave function is determined by perturbative evolution. What we therefore want to model is only the soft part of the pion wave function.

A commonly used ansatz for the quark-antiquark light-front wave function of the pion is of harmonic-oscillator type

\[
\psi(x, k_\perp) = A_\pi \chi(x, k_\perp) J(x, k_\perp) \exp \left( -\frac{M_0^2}{8\beta^2} \right) \tag{1}
\]

with \( M_0^2 \) denoting the front-form expression for the free two-particle mass

\[
M_0^2 = \frac{k_\perp^2 + m_q^2}{x(1-x)} \tag{2}
\]

\( A_\pi \) is a normalization constant, \( \chi(x, k_\perp) \) the (light-front) spin wave function of the \( q\bar{q} \) pair, and \( J(x, k_\perp) \) the square root of a Jacobian. Such a model for the pion wave function has, e.g., been proposed by Brodsky, Huang and Lepage (BHL) [3]. They took \( J = 1 \) and the usual (instant-form) expression for the spin-wave function \( \gamma \). Terent'ev and Karmanov (TK) [4], on the other hand, chose \( J(x) = \sqrt{1/2x(1-x)} \), i.e. the square root of the Jacobian relating

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Table 1
Parameters and properties of the wave-function models considered.

<table>
<thead>
<tr>
<th>wave function</th>
<th>β (MeV)</th>
<th>$A_\pi$ (GeV$^{-1}$)</th>
<th>$P_{q\bar{q}}$</th>
<th>$\sqrt{\langle k_F^2 \rangle}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{BHL}$ (full MR)</td>
<td>300</td>
<td>110.</td>
<td>1.00</td>
<td>282</td>
</tr>
<tr>
<td>$\psi_{BHL}$ (no MR)</td>
<td>300</td>
<td>78.2</td>
<td>0.51</td>
<td>282</td>
</tr>
<tr>
<td>$\psi_{BHL}$ (approx. MR [11])</td>
<td>300</td>
<td>65.8</td>
<td>1.85</td>
<td>323</td>
</tr>
<tr>
<td>$\psi_{TK}$ (full MR)</td>
<td>291</td>
<td>77.5</td>
<td>1.00</td>
<td>270</td>
</tr>
<tr>
<td>$\psi_{CCP}$ (full MR)</td>
<td>272</td>
<td>80.0</td>
<td>1.00</td>
<td>272</td>
</tr>
<tr>
<td>$\psi_{PL}$ (full MR)</td>
<td>1045</td>
<td>238.</td>
<td>1.00</td>
<td>266</td>
</tr>
</tbody>
</table>

$m_q = 330$ MeV and $f_\pi = 93$ MeV throughout.

The use of different wave functions does not have a significant effect on the electromagnetic pion form of constituent quarks (which can be obtained from much simpler dynamics). Therefore the mass $m_q$ occurring in the wave functions has to be interpreted as constituent-quark mass. We took $m_q = 330$ MeV. According to the finding in Ref. [7] we have also fixed the parameter $\alpha$ in $\psi_{PL}$ to be $\alpha = 3.5$. The normalization $A_\pi$ has then been adjusted such that the weak decay constant $f_\pi = 2\sqrt{3} \int dx d^2k_\perp \psi = 93$ MeV is reproduced. Finally, the parameter $\beta$ has been determined (for the full Melosh rotated wave functions) by means of the valence-quark dominance assumption $P_{q\bar{q}} = \int dx d^2k_\perp \psi^* \psi = 1$. This also leads to reasonable values for the mean intrinsic transverse momenta $\sqrt{\langle k_F^2 \rangle}$ of a quark (or an antiquark) inside the pion (cf. Table 1 where the model parameters are also summarized).

The dependence of the soft (overlap) contribution to $F_\pi$ on the choice of the quark-antiquark wave function is displayed in Fig. 1. At very small $Q^2$ (small figure) the curves nearly coincide. Figure 2, on the other hand, shows the interplay of soft and hard (perturbative) contributions to $F_\pi$ and the influence of the MR factor. Results are only considered for $\psi_{BHL}$, since the hard contribution to the form factor is nearly independent of the choice of the wave function. It does not even depend on whether the MR factor is fully taken into account or neglected at all. Marked differences are only observed if the MR factor is approximated like in Ref. [11]. Shortly summarized, the following conclusions can be drawn from the figures (and from Table 1):

- The use of different wave functions does not have a significant effect on the electromagnetic pion

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Figure 1. Soft contributions to $Q^2 F_\pi$ (large figure) and to $F_\pi^2$ (small figure) in different $Q^2$ ranges for $\psi_{BHL}$ (solid line), $\psi_{TK}$ (short-dashed line), $\psi_{CCP}$ (medium-dashed line), $\psi_{PL}$ (long-dashed line) with the full MR factor and for $\psi_{BHL}$ without MR factor (dotted line). Data are taken from Ref. [8] (filled triangles), Ref. [9] (filled circles), and Ref. [10] (open circles).

Figure 2. Soft (solid line) and hard (dash-dotted line) contributions to $Q^2 F_\pi$ for $\psi_{BHL}$ with the full MR factor. Results without (dotted line) and with approximate treatment [11] (dash double-dotted lines) of the MR factor are shown for comparison. Corresponding pion distribution amplitudes are displayed in the small figure along with a recently proposed model [12] (grey line).

REFERENCES