We show how the application of a coupling field connecting the two lower metastable states of a Lambda system facilitates stoppage of light in a coherently driven Doppler broadened atomic medium via electromagnetic induced transparency.

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A recent paper shows how the pulse propagation could be subjected to a new control parameter so that the same system can exhibit both sub- and superluminal propagation \[12,13\]. The experiments of Budker et al. \[18\] demonstrated how the light could be stored in atomic coherences \[8,9\] and how it can be retrieved at will. All this has remarkable bearing on the pulse propagation. Harris and coworkers \[1,2\] first proposed how these control fields could be used to produce slow light. This was followed by a series of experiments in Bose condensates \[3\] as well as in hot atomic vapors \[4–6\] . These experiments demonstrated the production of ultraslow light. In a related development Wang et al. \[7\] demonstrated the production of superluminal propagation. Wang et al. used the novel idea of using a bichromatic pump to produce a region of anomalous dispersion with negligible absorption or gain. More recent experiments have demonstrated how the light could be stored in atomic coherences \[8,9\] and how it can be retrieved at will. All this depends on the temporal dispersion of the medium. In a recent work Kocharovskaya et al. \[10\] proposed how the spatial dispersion \[17\] can be used to stop light in a hot gas. They argued that the motion of atoms leads naturally to a refractive index or a susceptibility that is dependent on both the propagation vector and frequency. Explicit calculation for a Λ system shows that the stoppage of light occurs when the control fields is suitably detuned from the atomic transition and when the central frequency of the probe pulse satisfies the two photon resonant condition.

In the present communication we show how we can make the stoppage of light flexible by using another control field. We consider a Λ system as show in Fig. 1. We apply two control fields one on the transition \(|1\rangle \leftrightarrow |2\rangle\) and the other on the transition \(|2\rangle \leftrightarrow |3\rangle\). The probe pulse acts on the transition \(|1\rangle \leftrightarrow |3\rangle\) which is generally electric dipole forbidden transition. The states \(|2\rangle\) and \(|3\rangle\) are metastable states. We calculate the group velocity of the pulse for different strengths of the two control fields. We demonstrate existence of very wide region of parameters where the stoppage of light can be achieved while maintaining regions of very low absorption.

In general for a spatially dispersive medium \[17\] the response of the medium can be expressed by a susceptibility \(\chi(k, \omega)\) that is dependent on the propagation vector and frequency. Further the allowed wavevectors are given by the dispersion relation

\[
k^2 = \frac{\omega^2}{c^2} \left[ 1 + 4\pi \chi(k, \omega) \right].
\]  

It is well known that the above dispersion relation can lead to many real solutions for \(k\) for a fixed \(\omega\) and thus one has the possibility of additional waves in a spatially dispersive medium. In this paper we however consider only the case when \(|\chi|\) is much smaller than one. In this case we would basically have one wave. A simple analysis now shows that the group velocity is given by

\[
v_g = \left[ \frac{c \left( 1 - 2\pi k \langle \frac{\partial \chi}{\partial k} \rangle \right)}{1 + 2\pi \omega \langle \frac{\partial \chi}{\partial \omega} \rangle} \right].
\]  

This formula assumes weak spatial as well as temporal dispersion. Besides we assume that absorption is negligible. In a gas of atoms the \(\chi(k, \omega)\) is to be replaced by the average values \(\chi(\omega - kv)\) over the distribution of velocities. Then the expression for the group velocity becomes

\[
v_g = \text{Re} \left[ \frac{c \left( 1 + 2\pi k \langle \frac{\partial \chi}{\partial k} \rangle \right)}{1 + 2\pi \omega \langle \frac{\partial \chi}{\partial \omega} \rangle} \right].
\]  

Note that

\[
\langle v \frac{\partial \chi}{\partial k} \rangle \neq \langle v \rangle \frac{\partial \chi}{\partial k} \\
\langle v \frac{\partial \chi}{\partial \omega} \rangle \neq 0,
\]  

and hence the stoppage of light takes place if the numerator in Eq. (3) vanishes. The susceptibility \(\chi(\omega)\) will depend strongly on the intensities and the frequencies of the two control fields. The susceptibility \(\chi(\omega)\) is obtained by solving the density matrix equations for the Λ system of Fig. 1, i.e, by calculating the density matrix element \(\rho_{13}\) to first order in the applied optical field on the transition \(|1\rangle \leftrightarrow |3\rangle\) but to all orders in the two control fields. By making a unitary transformation from the density matrix \(\rho\) to \(\sigma\) via

\[
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we have the relevant density matrix equations
\[
\begin{align*}
\dot{\sigma}_{12} &= iG\sigma_{21} + i\alpha_{s21} - 2(\gamma_{1} + \gamma_{2})\sigma_{12}, \\
\dot{\sigma}_{22} &= i\alpha_{s12} + i\alpha_{s22} + iG\sigma_{21} + i\gamma_{2}\sigma_{22} + i\gamma_{1}\sigma_{21}, \\
\dot{\sigma}_{13} &= -\gamma_{1}\sigma_{13} - \Gamma_{12}\sigma_{12} + i\Delta_{2}\sigma_{12}, \\
\dot{\sigma}_{33} &= -\gamma_{2}\sigma_{33} - \Gamma_{12}\sigma_{32} - i\Delta_{2}\sigma_{32}.
\end{align*}
\] (6)

Here Γ’s give collisional dephasing terms; γ’s give the radiative decay of the state |1⟩; Δ1’s are the detunings
\[
\Delta_{1} = \omega_{1} - \omega_{13}, \quad \Delta_{2} = \omega_{2} - \omega_{12}, \quad \Delta_{3} = \omega_{3} - \omega_{23}, \quad \Delta_{4} = \Delta_{1} - \Delta_{2} - \Delta_{3}.
\] (7)

The coupling constants 2g and 2G are the Rabi frequencies of the probe and the optical control field G = \vec{d}_{13} \cdot \vec{E}_{p}/h, G = \vec{d}_{12} \cdot \vec{E}_{c}/h. The parameter Ω characterizes the coupling between two lower levels. We will refer to this field as LL coupling field. The susceptibility χ can be obtained by considering the steady state solution of (6) to first order in the field on the transition |1⟩ → |3⟩. For this purpose we assume γ1 = γ2 = γ and write the solution as
\[
\sigma = \sigma^{0} + \frac{g}{\gamma} e^{-i\Delta_{1}t} \sigma^{+} + \frac{g^{*}}{\gamma} e^{i\Delta_{1}t} \sigma^{-} + ....
\] (8)

The 13 element of σ+ will give the susceptibility at the frequency ω1 as can be seen by combining Eqs. (5) and (8) and using the definition of the induced polarization
\[
\chi(\omega_{1}) = \frac{n|d_{13}|^{2}}{h\gamma} \sigma_{13}^{+},
\] (9)

where n is the density of the atoms. To obtain the probe response in a Doppler-broadened medium σ+ should be averaged over the Maxwell Boltzmann velocity distribution of the moving atoms. For a single atom, moving with a velocity v along the z axis, the probe frequency ω1(v) and frequencies ω2(v), ω3(v) of the two control fields as seen by the atom are given by
\[
\omega_{1}(v) = \omega_{1} - k_{1}v, \quad \omega_{2}(v) = \omega_{2} - k_{2}v, \quad \omega_{3}(v) = \omega_{3} - k_{3}v.
\] (10)

Thus susceptibilities for moving atoms are obtained by using the replacement (10) in the solution of Eqs. (6). Note that the velocity dependence of ω3 is insignificant and can be dropped. For simplicity we can also set k1 ≡ k2. These susceptibilities are to be averaged over the Maxwell-Boltzmann distribution for the atomic velocities, defined by
\[
P(k_{1}v)d(k_{1}v) = \frac{1}{\sqrt{2\pi D^{2}}} e^{-(k_{1}v)^{2}/2D^{2}} d(k_{1}v), \quad D = \sqrt{K_{B}T/4\pi M m}.
\] (11)

We next give the results of our calculations for the model systems shown in the Fig. 1. We show a number of numerical results in Figs. 2 and 3. In Fig. 2(a) and Fig. 2(b) we show the behavior of the susceptibility as a function of the detuning of the probe when the control field ω2 is detuned, Δ2 = −50γ. It is clear from the Fig. 2(a) that with increase of the microwave field intensity results in decrease of the probe absorption in presence of collisional dephasing. At two photon resonance condition i.e., Δ1 = Δ3 = −50γ, the absorption of the probe is very small. Therefore the feasible transparency window is attained only if only if Δ1 = Δ2. Note that the transparency dip that appears in the absorption spectrum is typically very narrow and its width depends on coupling fields G and Ω. The probe pulse spectrum should be contained with in this narrow dip. However, the transparency dip is a accompanied by a steep variation of \(\langle Re[\chi]\rangle\) with probe detuning. We notice from the Fig. 3(d) that at resonance condition (Δ1 = Δ2 = 0) light can not be stopped. We find that if two control fields are suitably detuned then the light can be stopped. We show in the Fig. 3(a) how the group velocity \(v_{g}\) as defined in Eqs. (3), changes from negative values to large positive values as the intensity of the LL coupling field is increased. The group velocity \(v_{g}\) become zero at the value of Ω = 2.45 × 10^{−6}γ when the control field G is out of one photon but satisfies the two photon resonance condition (Δ1 = Δ2 = −50γ). Note that for Rb, a Rabi frequency of 10^{−5}γ implies a magnetic field of the order of .993μG. The slope of \(\langle Re[\chi]\rangle\) with respect to central frequency of the probe pulse depends on the intensity of the two control fields and density of atoms. The group velocity becomes zero [Fig. 3(b)] as the numerator in Eq. (3) changes sign.
when the LL coupling field is increased. The Fig. 3(c) gives the group velocity in the absence of spatial dispersion. A comparison of the Figs. 3(a) and 3(c) shows the important role played by spatial dispersion.

Thus in conclusion we have demonstrated how the application of an LL coupling field in the Λ system helps one to change the group velocity of the pulse inside the medium from a negative to positive value, and thereby helps in stopping light. Thus for a suitable detuning of the pump and probe fields, one can stop light by just changing the intensity of the LL coupling field.

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FIG. 1. Schematic diagram of three level Λ-system; the probe pulse is applied on the transition $|1\rangle \leftrightarrow |3\rangle$; other fields are cw.
FIG. 2. (a) and (b) The imaginary and real parts of susceptibility $\langle |\chi| \rangle$ at the probe frequency $\omega_1$ in the presence of control field $G$ and LL coupling field $\Omega$. Detuning of the control field $\Delta_2$ is chosen as $-50\gamma$. The common parameters of the above three graphs for $^{87}\text{Rb}$ vapor are chosen as: Doppler width parameter $D=1.33 \times 10^9$ rad/sec, density $n=10^{12}$ atoms/cc, $G=0.3\gamma$, $\Delta_3=0$, $\Gamma_{12}=\Gamma_{13}=0$, $\Gamma_{23}=0.001\gamma$, $\gamma=3\pi \times 10^6$ rad/sec.
FIG. 3. (a) shows variation of group velocity, in meter per sec, of Eq. (3) with the strength $\Omega$ of the LL coupling field. The group velocity becomes zero because the numerator in Eq. (3) becomes zero as shown in the Fig. 3(b). The Fig. 3(c) gives the behavior of the group velocity if the spatial dispersion of the susceptibility were ignored. The common parameters of the above three graphs for $^{87}\text{Rb}$ vapor are chosen as: Doppler width parameter $D=1.33 \times 10^9$ rad/sec, density $n=10^{12}$ atoms/cc, $G=0.3\gamma$, $\Delta_3=0$, $\Gamma_{12}=\Gamma_{13}=0$, $\Gamma_{23}=0.001\gamma$, $\gamma = 3\pi \times 10^6$ rad/sec, $\Delta_1=\Delta_2=-50\gamma$. For comparison we also show in the Fig. 3(d) the result for $\Delta_1=\Delta_2=0$. 