A-star envelopes: a test of local and non-local models of convection

Key words: convection, cluster astrophysics, radiation

In our models of the base of the He II convection zone, we have found that the convection is driven by the local gradient of the adiabatic sound speed in the envelope. The non-local models, on the other hand, predict a different behavior for convection in the outer layers of the star. We have therefore tested the predictions of our models by comparing them with observations of A-star envelopes.

Abstract

The A-star envelopes are a test of local and non-local models of convection. In our models, the convection is driven by the local gradient of the adiabatic sound speed in the envelope. The non-local models, on the other hand, predict a different behavior for convection in the outer layers of the star. We have therefore tested the predictions of our models by comparing them with observations of A-star envelopes.
the shortcomings of local convection models more clearly, as their less efficient convection is much more sensitive to details in the modelling. Depending, for example, on whether an $\alpha$ of 0.5 or 1.5 is chosen in MLT for a mainsequence star with $T_{\text{eff}} \sim 7500$ K, an envelope may either have a mostly radiative temperature gradient or still contain a nearly adiabatic region. This holds for any of the convection models which rely on a convective scale length. Hence, the efficiency of convection in the envelopes of A-stars has remained an open problem and makes them a logical as well as a promising starting point for our study.

In the following, we give an outline of the physics and the numerical procedure used to compute our envelope models (the discussion of the moment equation formalism is self-contained, so that readers unfamiliar with it can skip ahead without difficulties). Results are then presented for a sequence of models which differ from each other only in $T_{\text{eff}}$. We include a model with lower gravity in order to illustrate the effect of a change in $\log g$. Finally, we show that the non-local convection model agrees with the known observational constraints and the results of numerical simulations, whereas local models are fundamentally unable to do this.

2 DESCRIPTION OF MODEL

The convection model used here is an extension of the CD8 model which requires the solution of five differential equations of first order in time and second order in space for $\mathbf{F}$, $\mathbf{J} = \mathbf{w}/\mathbf{F}$, $F_{r, l}$, $\tau_6$ and $\tau_{\nu}$, which are provided by a well-tested turbulence model (see CD8 for a summary). b) The Prandtl number is set to $10^{-2}$ as a typical value for the outer part of A-star envelopes (values up to 2 orders of magnitude larger than this do not alter our results). c) With the exception of the pressure correlations $\mathbf{P} w$ and $\mathbf{P} F$ which require further study (see Kupka & Muthsam 2002), the complete form of the "compressibility terms", equations (42)-(48) of Canuto (1993), is used to extend the CD8 model to the non-Boussinesq case. Hence, we now also include the effect of a non-zero gradient in the turbulent pressure $p_{\text{turb}}$ on the superadiabatic gradient $\beta$. d) We use a more advanced model for the third order moments (TOMs) published in Canuto et al. (2001), although with a different form for the fourth order moments (see Kupka 2002). If, instead, the original form for the fourth order moments is used, the models with $T_{\text{eff}} \geq 8000$ K, discussed in Figure 1, show less efficient convection, with the opposite being true for the cooler models. In both cases, however, the results are qualitatively the same as the results we present here. As in Kupka & Montgomery (2001), we use a relation similar to equation (37f) in Canuto (1992) and thus avoid a downgradient approximation for the flux of $\epsilon$ (such as equation (6) of Kupka 1999b). e) The effect of stratification on the pressure correlation time scales, $\tau_{\nu}$ and $\tau_6$, was accounted for following Canuto et al. (1994). Likewise, the time scales $\tau_{\nu}$ and $\tau_6$ include a correction for the optically thin regime of stellar photospheres (cf. Spiegel 1957), while for consistency, the expression for the radiative flux $F_{\nu}$ was taken from the stellar structure code we use for our initial models and boundary conditions (see Pamyatnykh 1999), it assumes the diffusion approximation for $\tau > 2/3$, but differs from it by a "dilution factor" for optical depths $\tau < 2/3$.

More details on these alterations and comparisons with numerical simulations are discussed in Kupka (2002) and in Kupka & Muthsam (2002). With one exception we have used the original constants of Canuto (1993), CD8, and Canuto et al. (2001). We consider their adjustment to be of little use, because in case of failure it is usually the entire shape of the functional relation which is at variance with measurements or simulations (cf. the MLT example in Sect. 4). The one exception we have made is the high efficiency limit of $\tau_{\nu}$, for which the CD8 model appears to predict values too low in comparison with simulations for idealized microphysics (see Kupka 2001), and also in comparison with a previous model (Canuto 1993). Most likely this is due to an isotropy assumption in its derivation and we thus use a $\tau_{\nu}$ increased by a factor of 3 as suggested in Kupka (2001). This problem will be thoroughly discussed in Kupka & Muthsam (2002).

A numerical approach to solve the resulting system of equations was briefly described in Kupka (1999a, b); a comprehensive discussion of the code will be given in Kupka (2002). Here we only outline the solution procedure from the viewpoint of stellar structure modelling. We start from an envelope model computed with the code described in Pamyatnykh (1999), where the equation of state and opacity data are from the OPAL project (Rogers et al. 1996, Iglesias & and Rogers 1996). The metallicity, $T_{\text{eff}}$, surface log $g$, and total stellar radius $R_*$ are taken from this model and held constant during relaxation. We place some 200 mass shells from the mid photosphere (with $\tau_{\text{Ross}} \sim 10^{-3}$) down to well below the He II convection zone. Having embedded the convection zones within stably stratified layers, we can use the boundary conditions of Kupka (1999b) for the SOMs (cf. Kupka 2002). For the mean structure quantities we keep $r$, $T$, and $P$ fixed to their values at the upper photosphere of the input model, while a constant $L$ is enforced at the bottom. The complete system is integrated in time (currently by a semi-implicit method) until a stationary, thermally relaxed state is found. The mass shells can be replenished to a different relative size to resolve, e.g., steep temperature gradients that may appear and/or disappear during convergence. The radiative envelope below the convection zones may then be obtained from a simple downward integration.

Generating a complete stellar model would require fitting such an envelope onto a stellar core, which in turn requires iterating the envelope parameters ($T_{\text{eff}}$, log $g$, $R_*$) to achieve a match of $P$, $T$, and $r$ at the core/envelope interface. Since we have not yet computed evolutionary models,
we have not needed to do this, although this would be a straightforward extension of our work.

3 RESULTS

Figure 1 shows the central results of this paper: both the H\textsc{ii} and H\textsc{i} convection zones appear quite separate when the quantity which is examined is the convective flux (a), but completely merged in terms of the convective velocity field (b). Thus, to obtain a self-consistent solution, one must solve the equations for the entire region simultaneously.

From Figure 1a (and Figure 2), we see that the mid to the upper photospheres of these models (the crosses indicate the point where $\tau = 2/3$) are essentially radiative, as they are in the local CGM and MLT models. Thus, the temperature and density structure of both the local and non-local models are virtually identical at small optical depths, which justifies our use of the local models as an outer boundary condition for the non-local models.

In Table 1, we list these results. Since the H\textsc{ii} and H\textsc{i} convection zones are well-separated in terms of $F_C/F_T$, we have listed their maximum fluxes separately (columns 3 and 4). For the convective velocity, $v_C = \langle u^2 \rangle^{0.5}$, we have listed just a single maximum since this quantity is large throughout the entire region (column 6); the same holds for the relative turbulent pressure (column 8). Since all of these maxima occur below the stellar surface, we have also listed the photospheric ($\tau = 2/3$) values of $v_C$ and $P_{\text{turb}}/P_{\text{tot}}$ (columns 7 and 9).

Finally, in Figure 2 we plot the kinetic energy flux as a function of $\log T$, for the four different models from Figure 1. Besides the fact that the cooler models have larger fluxes, which is to be expected, we see from the magnitudes of these fluxes that $F_{\text{kin}}$ is essentially negligible for the models we have examined. We are thus in a different regime from that of the Sun, where $|F_{\text{kin}}/F_T|$ may be as large as 20 per cent (cf. Stein & Nordlund 1998, Kim & Chan 1998).

In addition to these results, we have also run low- and high-metallicity models ($Z = 0.000, 0.06$, respectively). We find that for the low-Z models, $(v_C)_{\text{max}}$ decreases $\leq 3$ per cent while $(F_C)_{\text{max}}$ increases by $\leq 10$ per cent, with the opposite trends for the high-Z models. While these changes are not large, we note that they would be enhanced by the use of non-grey atmospheres. On the other hand, reducing $g$ (to a value still consistent with a main sequence object) results in much weaker convection caused by a lower density and hence smaller heat capacity of the fluid, as shown by the last model in Table 1, which is taken from an (MLT based) evolutionary sequence of a 2.1 $M_\odot$ star.

4 DISCUSSION

The fact that $F_{\text{kin}}$ is positive in the photosphere for each of these models (Figure 2) means that the skewness of spectral lines produced in this region is also positive, and that the corresponding filling factors for rising versus falling fluid elements is less than $1/2$ (cf. CD08). This is in agreement with the observations of line profiles in A-stars (Landstreet 1998).
Table 1. Convection zone parameters obtained with the non-local model. The overshooting (OV) is measured from the minimum of $F_z / F_T$ (shown in the inset of Figure 1a) to the point where $[F_z / F_T] \sim 10^{-5}$. 

<table>
<thead>
<tr>
<th>$T_{\text{eff}}$ (K)</th>
<th>log $\varepsilon$</th>
<th>$(F_z / F_T)_{\text{max}}$</th>
<th>He II</th>
<th>H I</th>
<th>(in $H_p$)</th>
<th>$v_c_{\text{max}}$ (km s$^{-1}$)</th>
<th>$(v_c_{\text{max}} / a H_p)_{\text{max}}$ (km s$^{-1}$)</th>
<th>$(v_c_{\text{max}} / a H_p)_{\text{max}} - 2 / 3$</th>
<th>$(P_{\text{ov}} / P_{\text{ov}})_{\text{max}}$</th>
<th>$(P_{\text{ov}} / P_{\text{ov}})_{\text{max}} - 2 / 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8500</td>
<td>4.4</td>
<td>0.031</td>
<td>0.69</td>
<td>0.96</td>
<td>46.14</td>
<td>1.64</td>
<td>0.105</td>
<td>0.033</td>
<td>0.100</td>
<td>0.033</td>
</tr>
<tr>
<td>8000</td>
<td>4.4</td>
<td>0.030</td>
<td>0.69</td>
<td>0.95</td>
<td>46.42</td>
<td>1.65</td>
<td>0.104</td>
<td>0.031</td>
<td>0.100</td>
<td>0.031</td>
</tr>
<tr>
<td>7500</td>
<td>4.4</td>
<td>0.031</td>
<td>0.69</td>
<td>0.97</td>
<td>46.18</td>
<td>1.64</td>
<td>0.105</td>
<td>0.033</td>
<td>0.100</td>
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</tr>
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<td>7000</td>
<td>4.4</td>
<td>0.032</td>
<td>0.69</td>
<td>0.98</td>
<td>46.16</td>
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</tr>
<tr>
<td>6500</td>
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<td>0.69</td>
<td>0.99</td>
<td>46.15</td>
<td>1.64</td>
<td>0.106</td>
<td>0.034</td>
<td>0.100</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Figure 2. The kinetic energy flux as a function of log $T$, for the four different models from Figure 1. As expected, the cooler models have larger fluxes. Most significantly, however, these numbers show that $[F_z / F_T]_{\text{max}}$ is essentially negligible for the models we have examined, in contrast to the case of the Sun, where it may be as large as 20 per cent (cf. Stein & Nordlund 1998, Kim & Chan 1998).

In the future, quantitative comparisons with such observational data will provide some of the most stringent tests of this model. As previously mentioned, the He II and H I convection zones may be thought of as being thermally disconnected but dynamically coupled, a situation which is impossible within the context of MLT (or CGM) models. A further shortcoming of MLT is shown in Figure 3. The convective flux of two MLT models, with mixing lengths of $l = 0.36$ and $0.42 H_p$, respectively, is plotted along with the flux from the non-local solution (upper panel). First, we see that it is impossible for the MLT models to match simultaneously the flux in both the He II and H I convection zones (at least with the same mixing length). Second, even if we try to model only the H I convection zone, fixing the mixing length so as to match the maximum flux results in a convection zone which is much too narrow. In addition, this produces photospheric velocities which are $\sim 30$ per cent larger than those of the non-local model and the observations (lower panel, Figure 3, see also Sect. 5). We note that since the upper photosphere is optically thin and therefore locally stable against convection, local convection models will always predict convective fluxes which are extremely small (or zero), even for values of $\alpha$ which are "unreasonably" large.

As a further test of our results we have compared them with 2D simulations by Freytag (1995), Freytag et al. (1996), and additional models provided by Freytag (2001, private communication). We find agreement with the following results from our calculations: a) Models over the entire range of A-type main sequence stars with $T_{\text{eff}}$ up to 8500 K have their H I and He II convection zones dynamically connected. The vertical mean velocities in the overshoot regions around log $T \sim 4.4$ are of order 1.5 to 3 km s$^{-1}$. b) There is considerable overshooting (OV) below the He II convection zone. However, the 2D simulations yield a size of the OV region which is 3 times larger in terms of radius and also much larger in terms of $H_p$, for the entire range of models in Figure 1 and Table 1. Such differences are anticipated from a comparison of numerical simulations in 2D and 3D (Muthsam et al. 1995, see also Fig. 1 in Kupka 2001). More detailed examples demonstrating that 2D simulations yield upper limits for the (3D) OV extent will be given in Kupka & Muthsam (2002). c) The maximum of $F_z$ and the tempora-
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5 CONCLUSIONS

Using a fully non-local, compressible convection model together with a realistic equation of state and opacities, we have calculated envelope models for stellar parameters appropriate for A-stars. In examining the results of this model, we have found many points of agreement both with observations and with numerical simulations.

First, our photospheric velocities are consistent with the lower limit of the typical micro- and macroturbulence parameters found for A-stars \(1.5-2 \text{ km s}^{-1}\); see Varenn & Monier 1999 and Landstreet 1998. Line blanketing should further increase these values. We expect a smoother \( v_c(r) \) (without small minima as in Figures 1b and 3) from an improved treatment of fourth order moments and inclusion of \( P R \) (cf. Sect. 2). Second, we find that the filling factor for rising fluid elements in the photospheres of our models is less than 1/2, also in agreement with observations of line profiles in A-stars. Third, we find in the temperature range 7200 K to 8500 K that the \( \text{HeII} \) and \( \text{HII} \) zones are well-separated in terms of the convective flux but not in terms of the convective velocity field. The two zones are thus in some sense thermally separated but dynamically joined. This feature is also shown by the numerical simulations. Finally, we find an OV at the base of the \( \text{HeII} \) convection zone of \( \sim 0.4 \text{H}_0 \). The numerical simulations find an even larger OV, but this may also be due to the fact that they were done in 2D. We note that in all cases we find a nearly radiative temperature gradient in the OV region, whereas the velocities in this region remain quite large, within an order of magnitude of their maxima within the convection zone (\( \sim 0.5 \text{ km s}^{-1}\)).

In addition, the non-local model yields smaller temperature gradients than the local model of Canuto et al. (CGM, 1996). Such a comparison with MIST is more difficult due to the large range of \( \alpha \) in current use. Nevertheless, we have found evidence that for main sequence models \( \alpha \) has to be decreased from values of \( \sim 1.0 \) at about 7100 K to \( \sim 0.4 \) for models with \( T_{\text{eff}} = 8000 \text{ K} \) in order to obtain a comparable value of \( F_{\text{OV}}^{\max} \) in the \( \text{HII} \) convection zone. In order to match \( F_{\text{OV}}^{\max} \) in the \( \text{HeII} \) convection zone, a completely different set of \( \alpha \)'s (with larger values) would be required.

As already mentioned, A-stars are excellent choices for this first calculation since they have relatively thin surface convection zones, so that the thermal time scales involved are not so long. In addition, they are interesting stars in their own right, containing high-metallicity stars (the Am stars) as well as two groups of pulsating stars (the roAp and \( \delta \) Scti stars). In the future, it may be possible to use the pulsating stars as probes of the subsurface convection zones, much as has been done in the case of the Sun.

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