hadronic models with $E/N \approx 2$ [10] will have greatly suppressed photon couplings because of a cancellation of two unrelated numbers, one being a function of the number of new quarks and their charges, the other a function of quark mass ratios $z \equiv m_u/m_d \approx 0.55$ and $w \equiv m_u/m_s \approx 0.029$ [11].

Cosmological and astrophysical considerations appear to restrict the PQ-breaking scale to two possible ranges, the so-called “hadronic axion window” of hot dark matter [12], $\lesssim f_{\text{PQ}} \lesssim 5 \times 10^5$ GeV [13, 14], and a window of cold dark matter mass, $\lesssim f_{\text{PQ}} \lesssim 10^{12}$ GeV [15]. The first range exists only for hadronic QCD axions which couple to photons and axion to electron couplings [13]. These arguments do not affect hadronic QCD axions which couple only to nucleons ($E/N = 2$) because their interactions with photons and electrons are strongly suppressed.

It should be noted that the hadronic axion window is indicated by the supernova (SN) 1987A cooling and axion burst arguments which suffer from statistical weakness, with only 10 neutrinos being observed, as well as from all uncertainties related to the axion emission from a hot and dense nuclear medium [13, 14]. It is therefore of crucial importance to experimentally measure or constrain the hadronic axion window.

In the following, the questions concerning the PQ-breaking scale related to the hadronic axions will be addressed.

**Limits on $f_{\text{PQ}}$ from $^{57}$Fe and $^7$Li experiments.**—Two new sources of near-monochromatic axions which might be emitted from the supernova core have been recently proposed: (i) thermally excited nuclei of $^{57}$Fe which is one of the stable isotopes of iron, exceptionally abundant among the heavy elements in the Sun [6], and (ii) excited nuclei of $^7$Li produced in the solar interior by $^7$Be electron capture and thus accompanying the emission of $^7$Be solar neutrinos of energy $384$ keV [5]. Since the axion is a pseudoscalar particle, one can expect the emission of near-monochromatic axions during $M_1$ transitions between the first excited level and the ground state in $^{57}$Fe and $^7$Li. The high temperatures in the center of the Sun ($\sim 1.3$ keV) symmetrically broaden the axion line to a full width at half maximum of about 5 eV and 0.5 keV owing to the motion of axion emitters $^{57}$Fe and $^7$Li, respectively.

As a result of Doppler broadening, these axions, approximately centered at the transition energy $E_0 = 14.4$ keV and $E_0 = 478$ keV, would be resonantly absorbed by the same nucleus $^{57}$Fe and $^7$Li, respectively, in a laboratory on the Earth. The detection of subsequent emission of gamma rays either of $14.4$ keV or $478$ keV would be a sign of axion existence.

Following the calculations in Refs. [5, 6], one can find that the rate of excitation per particular nucleus which is expected for solar-produced axions incident on a laboratory target of that metal is given by

$$P(m_{\text{PQ}}) = \frac{1}{\sigma_D(E_a) \Gamma_\gamma} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \frac{\omega}{\sqrt{\omega^2 - \omega_0^2}} e^{-\omega w},$$

where $\omega = \omega_{\text{PQ}}^2 / \omega_0^2$ is the wave number of the axion and $\sigma_D(E_a)$ is the effective cross section for resonant absorption of these axions by a particular nucleus on the Earth. Both $d\omega / d\omega_{\text{PQ}}$ and $\sigma_D(E_a)$ increase as $\Gamma_a / \Gamma_\gamma$. Here

$$\Gamma_a / \Gamma_\gamma = \left( k_3 / k_1 \right)^{3/2} \left( \frac{1}{2\pi} \right) \frac{1}{1 + \alpha^2} \left( \frac{g_1 \beta + g_3}{(\mu - \eta)^2 + \beta^2 + \beta \eta - \eta^2} \right)^2$$

represents the branching ratio of the $M_1$ axionic transition relative to the gamma transition [16] and contains the nuclear-structure-dependent terms $\beta$ and $\eta$ as well as the isoscalar and isovector nuclear magnetic moments $\mu_1$ and $\mu_3$. The momenta of the photon and the axion are denoted by $k_\gamma$ $\approx E_\gamma$ and $k_a$ $\approx \sqrt{E_a^2 - m_{\text{PQ}}^2}$, respectively, while $\alpha = 1/137$ is the fine structure constant, and $\eta$ is the $E2 / M1$ mixing ratio. The isoscalar and isovector axion-nucleon coupling constants, $g_1$ and $g_3$, are related to $f_{\text{PQ}}$ in the hadronic axion model [10, 17] by the expressions

$$g_1 = \frac{m_N}{f_{\text{PQ}}} \left( \frac{2S + (3F - D)}{1 + z + 2w} \right)$$

and

$$g_3 = \frac{m_N}{f_{\text{PQ}}} \left( \frac{D + F}{1 + z + w} \right),$$

where $m_N$ is the nucleon mass, the constants $F$ and $D$ are the invariant matrix elements of the axial current, determined from hyperon semi-leptonic decays, and $S$ is the flavor-singlet axial-vector matrix element extracted from polarized structure function data.

The experimental methods as described above are based on axion to nucleon coupling, both at the source as well as at the detector, and therefore favorable for investigating the hadronic axions. First experiments performed along this new line of solar axion searches set an upper limit on hadronic axion mass of $745$ eV ($^{57}$Fe experiment) [4] and of $32$ keV ($^7$Li experiment) [5] at the 95% confidence level. Translating these results into limits on the PQ-breaking scale, one obtains $f_{\text{PQ}} \gtrsim 5 \times 10^3$ GeV and $f_{\text{PQ}} \gtrsim 1.9 \times 10^3$ GeV for the experiments with $^{57}$Fe and $^7$Li, respectively.

**Axions in large extra dimensions.**—In Ref. [1], it was noted that with $n$ compact extra dimensions, and factorizable geometry with volume $V_n$, the relation between the familiar Planck scale $M_P = 1.22 \times 10^{19}$ GeV and the higher-dimensional gravitational scale $M_H$ is given by the formula

$$M_H^n = M_P^{n+2} V_n,$$
where $V_n \equiv R^n$ was considered to be exponentially large, such that $M_H \sim M_W$. Similarly, one can place the QCD axion in the “bulk” of $\delta$ extra dimensions [1, 3, 18], by considering a relation of the type Eq. (5),

$$f_{\rho Q} = \alpha_{\rho Q} M_S^{\delta} V_{\delta}, \tag{6}$$

now connecting the four-dimensional PQ-breaking scale $f_{\rho Q}$ with a higher-dimensional PQ-breaking scale $\alpha_{\rho Q}$, and $M_S$ is the string scale, $M_S \sim M_H$. The most restrictive limits on the compactification scale $M_S$ for $n = 2$ and 3 extra dimensions come from astrophysics. For the most stringent constraints, see the recent work [19].

Since the astrophysical limits on $f_{\rho Q}$ [13, 14, 15] are many orders of magnitude larger than $M_H \sim M_W$, one should account for such a large mass scale by introducing the axion field in higher dimensions, with $f_{\rho Q}$ that could even be much smaller than $M_H$ [20]. On the other hand, as $f_{\rho Q} \ll M_H$, it is natural to assume that $\delta \leq n$ [3, 20]. The full generalization of the PQ mechanism to higher dimensions can be found in Ref. [3].

Another feature of the higher-dimensional axion field important to us is its Kaluza-Klein decomposition. These four-dimensional modes with an almost equidistant mass-splitting of order $1/R$ will be emitted from excited nuclei of $^{35}$Fe and $^7$Li up to their kinematical limits. Only the zero mode transforms under the PQ transformation as the true axion, and therefore is only required to have a derivative coupling to fermions, thereby playing a role of the ordinary QCD axion. It is, however, to the higher-dimensional structure of the axion field that each KK mode has identical derivative couplings to fermions, with strength set by $f_{\rho Q}$ (the coupling strength of the full linear superposition of the KK states is set by $f_{\rho Q}$).

Now, we calculate the rate from Eq. (1) as a function of the KK axion mass, with $k_{\delta} \approx \sqrt{E_\delta^2 - m_\delta^2}$. The masses of the KK modes are given by

$$m_\delta = \frac{1}{R} \sqrt{n_1^2 + n_2^2 + \ldots + n_\delta^2} \equiv \frac{\sqrt{\delta}}{R}, \tag{7}$$

where we assume that all $n$ extra dimensions are of the same size $R$. As a next step we need to calculate a sum due to contributions of the massive KK modes. Because of the smallness of the mass splitting for the size $R$ large enough ($\sim 1/R$), the justification is the use of integration instead of summation [21]. The rate of excitation per particular nucleus from Eq. (1) therefore reads

$$P = \frac{2 \pi \delta^{3/2}}{\Gamma(\delta/2)} R^{\delta} \int_0^{E_\delta} \frac{dE}{E} m_\delta^{-1} P(m). \tag{8}$$

For $n = 2, 3, 4$ extra dimensions ($\delta \leq n$), our limits on $f_{\rho Q}$ for both experiments are summarized in Tables I and II.

**Table I:** Limits on $f_{\rho Q}$, $m_\rho Q$, and $m_\rho$ derived from the experiment with $^{35}$Fe [4] when the QCD axion is placed in the bulk of two and three extra spacetime dimensions.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$M_H$ (TeV)</th>
<th>$M_H$ (1000 TeV)</th>
<th>$R$</th>
<th>$1/2 R^{-1}$</th>
<th>1/2 R^{-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.22 \times 10^7 keV^{-1}</td>
<td>1.22 \times 10^7 keV^{-1}</td>
<td>0.4 eV</td>
<td>41 eV</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9 \times 10^7 \times 10^7 \times 10^7</td>
<td>9 \times 10^7 \times 10^7 \times 10^7</td>
<td>1/2 R^{-1}</td>
<td>1/2 R^{-1}</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2 \times 10^7 \times 10^7 \times 10^7</td>
<td>2 \times 10^7 \times 10^7 \times 10^7</td>
<td>43 eV</td>
<td>439 eV</td>
<td></td>
</tr>
</tbody>
</table>

**Table II:** Limits on $f_{\rho Q}$, $m_\rho Q$, and $m_\rho$ derived from the experiment with $^7$Li [5] when the QCD axion is placed in the bulk of two, three and four extra spacetime dimensions.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$M_H$ (TeV)</th>
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</table>

**Discussion.—** One notices from Tables I and II that our lower limits on $f_{\rho Q}$ are always much more stringent than those obtained in conventional cases. Obviously, this is due to the fact that for $E_\delta > R^{-1}$ the multiplicity of states which can be produced is large. Going to higher $n$ the mass splitting of the spectrum becomes larger, thereby decreasing the multiplicity and the bound on $f_{\rho Q}$ is less stringent. Such a behavior is clearly displayed in Tables I and II.

Another feature visible in our Tables I and II represents a practical demonstration of the effect found in Ref. [3] that the mass of the axion can become independent of the energy scale associated with the breaking of the PQ symmetry. Such an effect can be used to decouple the mass of the axion from its couplings to ordinary matter, thereby providing a sought-for method of rendering the
axion invisible in higher-dimensional scenarios. The effect of KK modes on the axion mass matrix is such that the zero-mode axion mass is strictly bounded by the radius of the extra dimensions, $m_a \lesssim (1/2) R^{-1}$. Thus, in higher dimensions the mass of the axion is approximately given as

$$m_a \approx \min \left( \frac{1}{2} R^{-1}, m_{PQ} \right).$$

We see that for most combinations of $n, A, \delta$, and $R$, the upper limit on $m_{PQ}$ is considerably higher than $(1/2) R^{-1}$, and therefore cannot be considered as a genuine limit on the mass of the axion.

Summary.—We have interpreted data from two recent experiments aimed to search for solar, near-monoenergetic axions, assuming KK axions coming from the Sun. Within the context of conventional hadronic models with $E/N \gtrsim 2$, both experiments set a stringent upper limit on the axion mass. We have shown that data, when interpreted in the higher-dimensional framework, cannot be used, in most cases, to set any relevant limit on the axion mass. On the other hand, our lower limits on the four-dimensional PQ-breaking scale turned out to always be a few orders of magnitude more stringent than the ordinary QCD limit. Finally, we stress that the most restrictive bounds we have derived from the $^{57}$Fe experiment ($f_{PQ} \gtrsim 1 \times 10^{15}$ GeV in theories with two extra dimensions and $M_H \sim 100$ TeV as well as in theories with three extra dimensions and $M_H \sim 1$ TeV) and from the $^7$Li experiment ($f_{PQ} \gtrsim 1.4 \times 10^{15}$ GeV and $3.4 \times 10^{12}$ GeV, respectively) fall into the parameter space of hot dark matter interest.