In this paper we will examine the evolution of the HST data.

**Introduction**

The Hubble Space Telescope has provided us with unprecedented views of the distant universe. These observations have revolutionized our understanding of galaxy evolution and the large-scale structure of the universe. The HST has played a pivotal role in many of the most important discoveries in astrophysics, including the expansion of the universe, the nature of dark matter and dark energy, and the properties of black holes.

**Key Words:** Hubble Space Telescope, galaxy evolution, dark matter, dark energy.

**Abstract**

The HST has revolutionized our understanding of galaxy evolution and the large-scale structure of the universe. These observations have provided us with unprecedented views of the distant universe and have played a pivotal role in many of the most important discoveries in astrophysics, including the expansion of the universe, the nature of dark matter and dark energy, and the properties of black holes.
range of magnitudes, with possible relevant outcomes concerning the abundance of faint stars. For the analysis we will use observational data of field stars in the direction of NGC 6397 from the second epoch HST observations. Field stars have been obtained by an accurate separation from cluster stars based on proper motions analysis (KACP). The area covered by observations is of 6.6 arcmin² centered at the Galactic coordinates $l = 337^\circ 9, b = -11^\circ 2$. Figure 1 shows the $(V - I, V)$ color-magnitude diagram for the about 1000 stars whose proper motion is significantly different from that of the NGC 6397 stellar population (cf. Fig. 1 in KACP).

In Section 2 we will use these data to discuss the predictions of the Bahcall-Soneira Galaxy model (Bahcall 1986) which has been widely used by observers to predict the number of stars of different types, colors and apparent magnitude ranges in different fields of interest (see e.g. Boeshaar & Tyson 1985, Flynn & Freeman 1993, Lasker et al. 1990, Stuwe 1990, Basilio et al. 1996). Bahcall and Soneira assumed that the Galaxy is simply composed by a disk, with a scale height of a few hundred parsecs, and a more or less spherical spheroid. The Bahcall-Soneira model, as further improved mainly to account for interstellar reddening and obscuration, was proved to be in good agreement with available observations (see e.g. Bahcall 1986, Gould, Bahcall & Maoz 1993 and references therein) and the export code was made available on the Web to the scientific community in 1995. However at faint magnitudes such an agreement is strongly dependent on the assumption about the luminosity function (LF) of faint main sequence (MS) stars. As a result, in Sect.3, we will show that the above quoted agreement vanishes for several recent evaluations of the actual LF of either the disk or the halo components.

The need for more complex Galactic models has been advanced in 1983 by Gilmore & Reid, who firstly suggested the occurrence of an extended “thick disk” population consisting of stars with spatial and kinematic properties intermediate between disk and spheroid. Starcounts are not the best way to constrain this third component because similar results could be obtained either with a thick disk or without it in dependence of the adopted model parameters (disk/thick disk scale height, scale length and normalizations). However detailed analysis which, in addition to the star counts, take into account the velocity distribution of local stars, support the need for a thick disk component (Ojha et al. 1996, Wyse & Gilmore 1995, Norris & Ryan 1991, Casertano, Ratnatunga & Bahcall 1990.)

However up to now, sensitively different values for the thick disk structural parameters (density law, local density etc.) have been proposed (see e.g. Reid & Majewski 1993, Yamagata & Yoshii 1992, Ojha et al. 1996, Mendez & Guzman 1998). Thus the situation is not yet well defined and in Sect.4 we will use our data to test some three-component models currently available in the literature. In this context, one has to remind that with very faint observations it should be possible to resolve the Galaxy into its component populations. This because for $V>19$, at least for high Galactic latitudes, the various components are expected to contribute with different colors: the bluer colors are dominated by spheroid stars, the reddest objects do belong to the disk population, while the intermediate colors should be dominated by thick disk stars (see Basilio et al. 1996).

Unfortunately, our data refer to a sky region of some what low latitude, which gives a sensitive gain in the number of stars but which - as we will find - makes difficult a clear separation among the various populations. As a result we will find that either two-component or three component models give an equally satisfactorily agreement with magnitude and color observational counts. No one of the current Galactic models appears in agreement with observations for some extreme assumptions on the faint LF, whereas the data do not allow to discriminate between different three-component models within the current uncertainties on the scale height and on the density of stars in the solar neighborhood.

2 THE B&S MODEL

As first step we compared color and magnitude distributions from our observational sample to the predictions of the Bahcall-Soneira model in its original export version (see Bahcall, 1986, for a complete description of the model). For convenience in the following we will briefly summarize the main parameters (see also Table 1):

i) Disk. The disk has a Wielen (1974) luminosity function and an exponential scale length of 3.5 kpc. The scale height for the later main sequence ($M_V>5$) is 325 pc. For the early main sequence ($M_V<5$) it is (82.5 $M_V = 97.5$) pc with a minimum of 90 pc. The scale height of the giant branch is 250 pc.

ii) Spheroid. The spheroid has the same LF as the disk implemented for $M_V \leq 4.5$ with the 47 Tuc luminosity function (Da Costa, 1982), shifted to fit the Wielen LF at $M_V \approx 4.5$. The Galactocentric distance is R=8 kpc and the axis ratio is 0.8. The density is normalized locally at 1/300 of the disk.

For each stellar population it is possible to associate intrinsic stellar magnitudes with colours, predicting in such a way the expected colour distribution of stars in each given interval of magnitudes. The original version of the B&S model adopts as relations between $(B-V)$ colors and absolute visual magnitudes, a disk main sequence (MS) from observations by Johnson (1965,1966) and Keenan (1963) and the color-magnitude (CM) diagram of M67 by Morgan & Eggen (1978) for disk giant stars. The program allows the choice of the spheroid giant branch from a number of
different color-magnitude diagrams (e.g. 47 Tic, M13, M15, M92) while the main sequence adopted for the disk is shifted to properly match the evolved stars of the selected diagram, reproducing in this way the different colors of the spheroid MS as a function of the assumed metallicity.

Figure 2 compares the distribution of apparent \( V \)-magnitudes of the sample stars with the corresponding predictions produced by a straightforward application of the B&K model. The observed magnitude distribution plotted in Fig. 2 has been corrected for incompleteness as in KACP (cf. Tab. 1 in KACP), restricting the investigation to the range of magnitudes with completeness larger or of the order of 80%, i.e. to stars with \( V \leq 26.5 \). As shown in Fig. 2, one finds a good agreement over the whole accepted range of magnitudes, with theoretical predictions almost always within the expected statistical fluctuation of the counts. As expected, at these low latitudes, counts are dominated by disk stars. Thus, as we will discuss better later, the total distributions are barely sensitive to the presence of a possible third component.

The observational data can also be used to test predicted color distributions. We used the \( M_V-(B-V) \) relations described above together with the relations from \( (B-V) \) to \( (V-I) \) colour bands for MS and giant stars obtained from Johnson (1965, 1966) observational data, as suggested in the original B&K code. Figure 3 shows that, in this case, the agreement is far from being satisfactory, the difference between predictions and observations becoming worse at fainter magnitude. Thus while \( V \) magnitudes are well reproduced, \( (V-I) \) colour distributions are not. The origin of such an occurrence is easily understood, since inspection of the B&K data disclosed that the adopted \( (M_V, B-V) \) diagram for the two stellar populations was still in excellent agreement with recent results from the Hipparcos satellite for nearby stars (Kovalevsky 1998), whereas this is not the case for the \( (M_V, V-I) \) distributions. To be quantitative, the upper panel in Fig. 4 compares the adopted B&K \( (M_V, V-I) \) distributions for disk and spheroid MS stars with recent data by Monet et al. (1992) and Dahn et al. (1995) for dwarf and subdwarf stars in the solar neighborhood; the need to update the \( M_V-(V-I) \) color distributions is obvious. In passing we note that the B&K model was never compared by the authors with \( (V-I) \) color distributions thus they had not the need to introduce in the code precise relations for this color band. We derived a new \( (M_V, V-I) \) relation for the disk population by best fitting the above-cited observational data for dwarf stars.

In order to keep our computations as close as possible to the original B&K model, we choose for the spheroid a \( Z = 0.0002 \) MS as defined by HST in the very metal-poor globular M30 (Piotto et al. 1997). The bottom panel of Fig. 4 shows the new sequences as compared with the same observational data in the upper panel. Here we notice that the excellent agreement recently found between theoretical predictions and HST observations of faint MS in Galactic globulars (Cassisi et al. 2000) supports the use of theoretical data in varying the adopted spheroidal metallicity. Thus in the lower panel of Fig. 4 the low MS at \( Z=0.002 \) is from evolutionary calculations by Cassisi et al. (2000). Finally, \( (M_V, V-I) \) relations for giants stars that have left the MS have been taken from the CM diagrams of M67 (Montgomery, Marshall & Janes 1993) and M92 (Johnson & Bolte 1998) for disk and spheroid stars, respectively.

However, even if relying on observational data, one can not ignore that all the available theories unanimously predict that the location of the lower mass limit for H burning is strongly dependent on the star’s metallicity and, in particular, that for \( Z = 0.0002 \) this limiting magnitude is
$M_V = 14.2$ (see, e.g., again Cassisi et al. 2000). To be realistic, we thus used this magnitude as a cut-off of the luminosity function of the spheroid component. Numerical experiments, as reported in the previous Fig.2, shows that such a cut-off is predicted to affect the distribution of the observed luminosities only at magnitudes fainter than $V = 28$, thus beyond the limit of the present investigation. However, it appears interesting to notice that, at least in principle, observational data below $V \sim 28$ should constrain the magnitude of the cut-off, and, therefore, the mean metallicity of the spheroid component.

Before comparing theory and observation with the new color-magnitude relations, it is useful to discuss the adopted reddening correction. The original version of the code incorporates for intermediate-to-high Galactic latitudes ($|b| \geq 10°$) the “default” reddening/obscuration correction through “infinity” from Sandage (1972) based on Galaxy counts, which gives in the direction of NGC 6397 a reddening E(B-V)=0.36. However we prefer to adopt the more precise and extensively used reddening maps by Burnstein & Heiles (1982). For their reddening estimates the authors used together the results of surveys of neutral hydrogen column densities and of deep Galactic counts. The Burnstein & Heiles compilation gives E(B-V)$=0.18$ in the direction of NGC 6397, a value in good agreement with reddening estimates for the cluster from different methods: E(B-V)$=0.18 \pm 0.02$ (see e.g. Harris 1996, Anthony-Twarog et al. 1992, Drucker et al. 1993). Thus, in the following, we will adopt E(B-V)$=0.18$ transformed in E(V-I) by using E(V-I)=$1.25 E(B-V)$ by Bessel & Brett (1988). The reddening scale height adopted in the program is 100 pc (see e.g. Mendez & van Altena, 1998).

The difference between Sandage (1972) results and these estimates for the reddening/obscuration correction clearly affects both magnitude and colors distributions in a small but not negligible way; the magnitude distribution is shifted by about 0.2 mag, toward brighter magnitudes due to the reduced extinction, while the color distribution in the interval $23 < V < 25$ is shifted toward bluer colors by about 0.2 mag, and the total counts in this interval are increased due to the change in the extinction. In passing we note that the NGC 6397 field is just above the lower limit (|b| = 10°) of Galactic latitudes covered by the reddening treatment by Burnstein & Heiles (1982).

Figure 5 shows the effect of improving both the reddening and ($M_V, V - I$) relations for the predicted colour distributions in the same magnitude interval as in Fig.3. The agreement now appears rather satisfactory, showing that with such a minor modifications the B&S model appears able to fit all the observational constraints reasonably well. Numerical simulations show that results do not change significantly if we assumed Z=0.002 as spheroid mean metallicity.
3 THE FAINT STAR PROBLEM

The excellent agreement between theory and observation discussed in the previous section is not without problems. At present time one has much more information about the faint end of the disk luminosity function than at the time when the original B&S model was produced. As a matter of fact, below \( M_V = 12.5 \) the original version of the B&S model assumes a flat luminosity function both for the disk and the spheroid component. However, several lines of evidence have been found indicating that in both populations the luminosity functions undergo a noticeable decrease at the faintest luminosities. Figure 6 (upper panel) shows recent observational data for the faint end of the disk luminosity function together with the distribution originally assumed in the B&S model.

A decrease of the luminosity function beyond \( M_V = 13 \) has been firstly suggested by Wielen, Jahneiss and Krüger (1983, WJK), on the basis of parallaxes-star studies. An even more drastic decrease, starting at \( M_V = 12 \), was more recently suggested by Gould, Bahcall and Flynn (1996, 1997) from an analysis of HST data. The results from a large-area multicolor survey by Martini and Osmer (1998) and those from photometric parallax surveys by Reid & Gizis (1997) appear intermediate between the Wielen et al. and the Gould et al. results. All these luminosity functions fit the Wielen et al. LF at a visual magnitude of about 9.

Figure 6, lower panel, shows the situation for the spheroid component. Again there is agreement among the results of different authors. Dahn et al. (1995) from parallaxes studies of nearby subdwarf stars found a peak of the luminosity function at \( M_V \approx 11.5 \). However, Gould, Flynn and Bahcall (1998) investigated HST data and suggested a flat LF below \( M_V = 7 \), while very recently Gizis & Reid (1999), by using a sample of stars from the Palomar Sky Surveys, support the luminosity function results by Dahn et al. (1995). The original B&S choice for the spheroid LF is also shown.

An extensive investigation of the luminosity functions in Galactic globulars has recently shown the common occurrence of a maximum near \( M_V \sim 10 \) (Piotto, Cool & King 1997, Piotto & Zoccali 1999). In the same panel of Fig. 6 we show the luminosity function of the metal-poor Galactic globular M30, as obtained down to \( M_V = 8 \) by Sandquist et al. (1999) and extended below \( M_V \approx 8 \) with the LF of M30 provided by Piotto et al. (1997). M30 is a metal poor cluster that we will assume as possibly representative of the spheroid luminosity function. The rationale for this choice follows from the assumption of a substantial similarity between field and cluster spheroid populations. In this context, one may take note of the different LFs observed in Galactic globulars (see, e.g., Piotto & Zoccali 1999) as an evidence for a different efficiency of the evaporation of low-mass stars from the clusters, a mechanism which is obviously impoverishing the faint end of the original LFs. By adopting M30 we choose one of the globulars with the largest abundance of faint stars, thus with a LF which should be closer to the original one for spheroid stellar content.

Dahn et al (1995) and Gould et al. (1998) LFs join each other at \( M_V \approx 7.5 \) mag, to match the B&S LF. M30 LF has been normalized \( M_V = 10 \) at the spheroid LFs (see Gould et al. 1998). In passing we note that the difference between the M30 luminosity function and the original choice of B&S for \( M_V < 6 \) is mainly due to the normalization chosen by the authors: the luminosity function of 47 Tuc (for \( M_V < 4.5 \)) has been shifted to connect to the Wielen LF at \( M_V = 4.5 \). However numerical experiments show that, due to the poor population of the high luminosity LF regions, the quoted differences for \( M_V < 6 \) affects only slightly magnitude and color distributions, while difference in LF for \( M_V > 4.5 \) does not influence at all the results. In the following we will choose to adopt the M30 LF at bright magnitudes (\( M_V < 6 \)).

We are now in position to explore the role played on the predicted star counts by the faint portion of the luminosity function.
minimality function. Fig. 7 shows the predicted magnitude distribution of disk and spheroid stars when different LF's have been adopted. For comparison the histogram of the observed counts in the field of NGC 6397 is also shown. These results are easily understood on the basis of the shape of the adopted LFs.

From the results of Fig. 7 one also easily understand the magnitude distribution for the different combinations of disk and spheroid LFs shown in Fig. 8. As a first result, one finds that different assumptions about the faint end of the luminosity function affect star counts only for V > 23 (see Fig. 7 and Fig. 6). If one takes the already known capability of the B & S model to pass the observational tests down to V ~ 21 as an evidence that the model gives a satisfactory description of the Galactic distribution of the most luminous components of the stellar populations, one would conclude that the abundance of fainter stars depends only on the overall behaviour of the LFs.

By assuming the Gould et al. (1996, 1997) disk LF together with the Gould et al. (1998) spheroid LF (thin solid line in Fig. 8) there is no way to reconcile the predicted and observed V-magnitude distributions. Since star counts are governed mainly by the disk component, by assuming the Gould et al. (1997) disk LF one predicts a number of faint stars smaller than observations in the last two magnitude bins, even assuming for the spheroid component the M30 LF, which gives the largest counts at faint magnitudes (long dashed line in Fig. 8). Similarly by adopting the Gould et al. (1998) spheroid LF the model underestimates the observed counts in the last two bins of magnitude for any choice of the disk LF (dot-dashed line and thin solid line in Fig. 8).

However the reliability of star counts in the last magnitude bin: 20 < Mv < 26.5 is important to evaluate if the Gould et al. (1997) LF for the disk or the Gould et al. (1998) LF for the halo are separately acceptable for our field. We note that in the last bin (from Mv = 26 to Mv = 26.5) the completeness falls from about 90% to about 80%. If, very conservatively, we decide not to take into account the results in this bin of magnitude we cannot exclude the separate adoption either of the Gould et al. (1997) LF for the disk or of the Gould et al. (1998) LF for the halo, excluding however the contemporary adoption of both LFs.

Figure 8 shows that it is possible to find several combinations of LFs for disk/spheroid which fit the observational magnitude distribution in a better way. However, due to the large uncertainties still present in the determination of disk and spheroid LFs this seems to us a meaningless numerical exercise. Figure 9 shows the predicted color distribution in the region 23 < V < 25 (the only affected by the changes in the adopted LFs) for the combinations of LFs shown in Fig. 8. All the results are in fair agreement with observational color distributions. Thus Fig. 9 does not allows to discriminate among different LFs.

4 MODELS WITH THICK DISK

In the previous sections we have shown that the two-component (disk + spheroid) B & S model, appears to reach a satisfactory agreement with the deep star counts...
provided by HST in the field of NGC 6397 only if LFs are larger than expected on the basis of Gould et al. (1997, 1998) indications. In this section we will include a thick disk component to check if such a conclusion is affected by the addition of a third component to the Galactic structure.

In 1993 Gould et al. discussed photometric data from the Hubble Space Telescope Snapshot Survey up to an average apparent magnitude of V = 24.4 to conclude that, mainly due to the lack of information about the colors of the observed stars, it was not possible to distinguish between the presence or absence of the thick disk. In 1996 Basili et al. used I magnitude and (V − I) color star counts from 17 HST deep fields data to discriminate between the results of the standard models by Bahcall & Soneira (1980, 1984) and by Gilmore & Reid (1983). They concluded that the two standard model predictions for the magnitude counts was similar and in good agreement with observations. However they obtained a reasonable agreement between model and data for colour distributions only with the introduction of a thick disk component. Mendez & Guzman (1998) by adopting a small data sample with V up to ≈25 found an equally good agreement between models with thick disk and observations for a thick disk with a local normalization of 2% and a scale height of 1300 pc (Reid & Majewski 1993) or a local normalization of 6% and a scale height of 750 pc (Ojha et al. 1996). More in general, one finds that a large variety of disk/thick disk parametrizations is available in literature (see e.g. Reid & Majewski 1993, Ojha et al. 1996).

For our first test we adopted a consistent set of disk/thick disk/spheroid parameters as given for the “Besancon model” by Haywood, Robin & Creze (HR&C, 1997, see also Haywood, Robin & Creze 1996, Ojha et al. 1996, Robin et al. 1996 for more details). The disk /thick disk luminosity function adopted by the authors is from Wienen et al. (1983) for M_L > 1 and an average of both LFs in the interval -1 ≤ M_L ≤ 5. The chosen parameters are: a disk scale height of 240±50 pc, a thick disk scale height of 700±50 pc, a disk scale length of 2500±300 pc, a thick disk scale length of 2500±500 pc, a thick disk/thick disk local density ratio of 5.6±1%, a spheroid/thick disk local density ratio of 0.10%, R_L = 8.5 Kpc (see HR&C). However the “Besancon” model explicitly allows for time evolution of vertical scaleheight whereas the other models implicit take into account this variation by assigning lower scaleheights to early-type dwarfs. Thus we will take a scale height of 90 pc for the youngest MS stars and 240 pc for the oldest ones.

To explore the range of possible results, we made tests by adopting either Wienen et al. (1983) or Gould et al. (1997) disk and thick disk LFs for selected assumptions about the spheroid LF and by adopting for the thick disk LF at bright magnitudes (M_L < -4.5) the LF of 47 Tuc (Hesser, Harris & Vandenberq, 1987), assumed more representative of high luminosity thick disk stars. However numerical results show that by replacing, for M_L < -4.5, the 47 Tuc LF with the Wienen et al. LF the differences in the resulting star counts, at least for our field, are small.

Figure 10 shows the theoretical magnitude distributions when different disk/thick disk and spheroid luminosity functions are used. The contribution of the thick disk component is also shown. The dashed line represents a model with Wienen et al. (1983) disk/thick disk luminosity function and Gould et al. (1998) spheroid LF at faint magnitudes (M_L > 8) implemented at brighter magnitudes by M30 LF (Sandquist 1995). The solid line represents a model with Gould et al. (1997) disk/thick disk LF and with Dahn et al. (1995) spheroid LF at low luminosities completed with M30 for (M_L < 8). As expected, one finds again that the distribution is affected by the change of the faint LF only for M_L ≥ 24. However now one finds that the three-component model with the parameters described above slightly overestimates the counts up to (M_L ≈ 24). Comparison with results in Fig.8 shows that predictions based on Gould et al. (1998) LF appear in better agreement with faint star counts (see e.g. the case of Wienen et al. disk LF and Gould et al., 1998, spheroid LF).

Before discussing such an evidence, as an alternative choice, let us now explore the model by Gilmore & Reid (1983) by adopting their most frequently used parameters (G&R, 1983, 1984, see also Colless et al. 1991, Basili et al. 1996). The chosen parameters are: an disk scale height of 325 pc, a thick disk scale height of 1300 pc, a thick disk scale length of 3500 pc, a thick disk/thick disk local density ratio of 2%, a spheroid/thick disk local density ratio of 0.125% with, as in Reid & Majewski (1993), similar LFs for both disk and thick disk. Reid & Majewski (1993) adopted for disk/thick disk luminosity function the Wienen et al. (1983) LF for M_L < 11 implemented at fainter magnitudes with the photometric-parallax-based LF by Reid (1987) which is not too much different from the Reid & Gizis one, shown in Fig.6 and thus it is intermediate among the Wienen et al. (1983) and Gould et al. (1997) one. For reader’s convenience, table 1 summarizes the main disk/thick disk/spheroid parameters of the selected galactic models.

For our second test we adopted a consistent set of disk/thick disk/spheroid parameters as given for the “Besancon model” by Haywood, Robin & Creze (HR&C, 1997, see also Haywood, Robin & Creze 1996, Ojha et al. 1996, Robin et al. 1996 for more details). The disk /thick disk luminosity function adopted by the authors is from Wienen et al. (1983) for M_L > 1 and an average of both LFs in the interval -1 ≤ M_L ≤ 5. The chosen parameters are: a disk scale height of 240±50 pc, a thick disk scale height of 700±50 pc, a disk scale length of 2500±300 pc, a thick disk scale length of 2500±500 pc, a thick disk/thick disk local density ratio of 5.6±1%, a spheroid/thick disk local density ratio of 0.10%, R_L = 8.5 Kpc (see HR&C). However the “Besancon” model explicitly allows for time evolution of vertical scaleheight whereas the other models implicit take into account this variation by assigning lower scaleheights to early-type dwarfs. Thus we will take a scale height of 90 pc for the youngest MS stars and 240 pc for the oldest ones.

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Fig.11 (upper panel) shows the comparison between the predicted magnitude distribution for a model with the pa-
Table 1. Parameters of the two components Bahcall-Soneira (B&S) model and of the models with thick disk by Haywood, Robin & Creze (HR&C) and by Gilmore & Reid (G&R), see text for more details.

<table>
<thead>
<tr>
<th>Models</th>
<th>B&amp;S</th>
<th>G&amp;R</th>
<th>HR&amp;C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk</td>
<td>scale height (pc)</td>
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<td>325</td>
</tr>
<tr>
<td>scale length (pc)</td>
<td>3500</td>
<td>3500</td>
<td>2500 ± 300</td>
</tr>
<tr>
<td>Thick Disk</td>
<td>scale height (pc)</td>
<td>—</td>
<td>1300</td>
</tr>
<tr>
<td>scale length (pc)</td>
<td>—</td>
<td>3500</td>
<td>2500 ± 300</td>
</tr>
<tr>
<td>thick disk/disk density ratio</td>
<td>—</td>
<td>2%</td>
<td>5.6% ± 1%</td>
</tr>
<tr>
<td>Spheroid</td>
<td>spherical/disk</td>
<td>0.200%</td>
<td>0.125%</td>
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<tr>
<td>density ratio</td>
<td>—</td>
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</table>

Figure 10. The histogram of the observed magnitudes (as in Fig. 2) as compared with theoretical predictions by the B&S model with a thick disk component with parameters as in Haywood et al. (1997) (HR&C). The results for the two labelled different combinations of disk (thick disk)/spheroid LFs are shown (see text). The contribution of the thick disk component to the total counts is also shown.

parameters of the “Besancon model” and the one with the Gilmore & Reid model parameters. The adopted luminosity functions are Wielen et al. (1983) for disk and thick disk and Gould et al. (1998) + M30 for the spheroid. One sees that the predictions of the two models are coincident in the whole region of interest (Mv ≤ 26.5). This result confirms that different combinations of disk/thick disk/spheroid parameters can give the same results for predicted star counts (see e.g. the discussion in Mendez & Guzman 1998) and thus there is no an unique solution when the results of different models are compared with observational magnitude and color counts. Mendez & Guzman (1998) concluded that to discriminate among different possible parameters one should need kinematical data in addition to star counts as done e.g. by Ojuka et al. (1996) who obtained the parameters adopted in the “Besancon” model. Fig. 11 also shows the contribution of the thick disk components to the total counts. Note that while the total counts are very similar for the two sets of parameters (G&R and HR&C) the contribution of the thick disk is different, due to the different normalizations, scale heights and scale lengths. For comparison the total star counts of the two components B&S model with the same disk/spheroid LFs is also shown. As expected, due to the lack of the thick disk component the total counts of the B&S model are lower. Fig. 11 lower panel shows the contribution to the total counts of the upper panel of disk and spheroid components for the three selected models. The B&S disk overlaps the G&R one because the two models adopt the same disk parameters (see Table 1), while the HR&C model adopts a lower scale height for the disk and thus its thick disk contribution is larger (see upper panel). The spherical contribution of the B&S model is larger with respect to the ones of the models with the thick disk (see Table 1).

From Figs. 10 and 11 one should conclude that, for the field of NGC 6397, both the adopted three-component models overestimate star counts at bright magnitudes. However, we remind that Robin et al. (1996) evaluate for the thick disk density and scale height an uncertainty of, respectively, ≈20% and ≈7%. Within this uncertainty the thick disk density and scale height for the “Besancon” model can thus be decreased down to 4.6% and 7.10 pc, respectively. The results of such a decrease are shown in Fig. 12 for various combinations of disk/thick disk/spheroid LFs at faint magnitudes.

One finds that just by tuning the thick disk parameters within their uncertainties it is possible to obtain a good fit in the high luminosity region of the magnitude distribution, while the fit at faint magnitudes depends on the choice of the combination of disk/thick disk/spheroid faint LFs. If relying on such a model, one finds again that the combination of Gould et al. (1997) disk/thick disk LF and Gould et al. (1998) halo LF (completed with M30 LF) appears in agreement with observations (Fig. 13), whereas the separate adoption of Gould et al. (1997) LF for the disk/thick disk or the Gould et al. (1998) LF for the halo cannot be excluded.

Fig. 14 shows the predicted (V – I) colour distribution in the labelled intervals of magnitude with the “Besancon” parameters tuned as in Fig. 12 with the Wielen et al. LF for the disk and Dahn et al. 1995 (+ M30) LF for the spheroid (heavy long dashed line in Fig.12). Following Gilmore (1984) (see also Reid & Majewski 1993) we used the CMD of the metal rich globular cluster 47 Tucanae to derive the color-magnitude relation for evolved thick disk stars.
For MS thick disk stars we used theoretical predictions with $Z=0.006$ (Cassisi et al. 2000) which, as already discussed, have shown to be in very good agreement with observational data. In all cases $E(B-V)=0.18$. One sees that the agreement is reasonable in all the magnitude intervals. Figure 15 shows the contribution of disk, thick disk and spheroid to the colour-number plots of Fig.14. Due to the HR&C choice of normalizations, scale heights and scale lengths, the thick disk contribution in the selected intervals of magnitude is slightly larger than the disk contribution (see Fig. 11). Moreover the thick disk colors are bluer than the disk ones due to the lower metallicity ($Z=0.006$) of the thick disk.

Figure 16 compares observed and predicted profile counts for $23<V<25$ for different assumptions about the faint LFs for disk/thick disk and spheroid (see Fig.12 for the corresponding magnitude distributions). As in the two-component model, all the LF combinations taken into account (even Gould et al. 1997 - Gould et al. 1998) show an acceptable fit to observational data; thus the color distributions do not allow to discriminate among different LFs at faint magnitudes.

Figure 11. Upper panel: comparison between observational data and predicted magnitude distributions for a three-component model with Gilmore & Reid (G&R, 1983) parameters and Haywood et al. (HR&C, 1997) parameters (see text). The adopted LFs are Wielen et al. (1983) for the disk/thick disk (completed in this last case with the LF of 47 Tuc) and Gould et al. (1998), completed with M30 for $M_V<8$, for the spheroid. The contribution to the total counts of the two thick disk components are also shown. For comparison the two components B&S model with the same disk/spheroid LFs is also shown. Lower panel: contribution to the star counts of the disk/spheroid components for the three selected models.

Figure 12. The histogram of the observed magnitudes (as in Fig. 2) as compared with theoretical predictions by the B&S model with a thick disk component with parameters as in Haywood et al. (1997) (HR&C) for the labelled combinations of disk/thick disk/spheroid LFs at faint magnitudes. The thick disk parameters are: scale height = 710 pc and thick disk/disk local density ratio of 4.6% (see text). The contribution of the thick disk component to the total counts is also shown.

Figure 13. The histogram of the observed magnitudes (as in Fig. 2) compared with theoretical predictions by the B&S model with a thick disk component with parameters as in Haywood et al. (1997) (HR&C). The adopted luminosity functions are Gould et al. (1997) (completed for $M_V<9$ with Wielen et al. 1983) for the disk/thick disk and Gould et al. 1998 (completed with M30) for the spheroid. The contribution to the total counts of the three components is shown. The thick disk scale height is 710 pc and the thick disk/disk local density ratio is 4.6% (see text).
Figure 14. Comparison between the observed (shaded region) and predicted distribution of $V-I$ colours in the labelled intervals of magnitude for the three-component model with Haywood et al. parameters. The thick disk scale height and density are respectively 710 pc and 4.6% of the local disk (see text). The adopted LFs are Wielan et al. (1983) for disk and thick disk and Dahn et al. (1995), completed at bright magnitudes with the LF of M30, for the spheroid component.

Figure 15. Contributions of the three different populations for the colour-number plots of Fig. 14.

For the Gilmore & Reid model too the thick disk parameters can be tuned to reconcile the model with observations. In this case one expects that due to the lower contribution of the thick disk to the counts (see Fig. 11) the effect of varying the thick disk parameters will be lower. In fact, taking as representative the estimate of errors by Robin et al. (1996), to obtain a good agreement with observational data, one needs to reduce the thick disk scale height and normalization by $2\sigma$ (scale height=1100 pc, normalization=1.25%). Figure 17 shows the magnitude distribution for the model with original Gilmore & Reid (1983) parameters, (with Wielan et al. disk/thick disk LF and Gould et al., 1998, +M30 spheroid LF) together with the corresponding results when the thick disk parameters are lowered by $2\sigma$. For comparison the model with the HR&C parameters tuned as in Fig. 12 is also provided. With this modification the results of the G&R model appear as good as the HR&C one. We do not show the color distributions for the “modified” G&R model because as disclosed by Fig. 17, the results are practically coincident with those of Fig. 14 and Fig. 16.

Figure 16. Comparison between the observed (shaded region) and predicted distribution of $(V-I)$ colours for $23 < V < 25$ for the three component model with Haywood et al. parameters and for different combinations of disk/thick disk/spheroid LFs (as labelled). The thick disk scale height and density are respectively 710 pc and 4.6% of the local disk (see text).

Figure 17. Comparison among theoretical magnitude distribution for the Gilmore & Reid model with original (heavy dashed line) and tuned parameters (heavy solid line, see text). The adopted LFs are Wielan et al. for the disk/thick disk and Gould et al. 98 (+M30) for the spheroid. For comparison the model with the HR&C parameters tuned as in Fig. 12 is also shown (dotted line); the adopted LFs are Wielan et al. for the disk/thick disk and Dahn et al. (+M30) for the spheroid. The thick disk contribution to the total counts is also shown.

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5 CONCLUSION

In this paper we have discussed star counts in the field of the globular NGC 6397. As expected, we found that the predicted distribution of stars fainter than Mv ≈ 24 is critically dependent on the assumptions about the LF of faint MS stars. According to the discussion given in the previous sections one finds that neither the two-component B&K model nor the more sophisticated three-component models by HR & C and G&R can satisfactorily fit star counts in the field of NGC 6397, when assuming for disk and spheroid the low LF suggested by Gould et al. (1997), Gould et al. (1998), respectively. A satisfactory fit of the data can be achieved, either for a two-component or for a three component model, with other suitable combinations of LFs suggested in the current literature, provided that density and scale height of the two discussed three-component models are decreased within the predicted existing uncertainties. Nor the color distributions appear to discriminate among the various possible solutions. However, we regard the previous conclusions as suggestions to be further tested in deep surveys at different Galactic locations. In this context, and before closing the paper, one has to notice that from the comparison of Figs. 2 and 6 one derives the tantalizing evidence that going deeper than V = 25.5 by a couple of magnitudes one would derive much more stringent and precise constraints on the LF of faint stars and, in turn, on the mean metallicity of the spheroid. Such an evidence should be taken into account in future researches on this matter.

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