Lepton Masses from a TeV Scale in a 3-3-1 Model

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Abstract

In this work we develop a mechanism to generate mass for all leptons in a 3-3-1 model by replacing the usual scalar sextet by a neutral scalar singlet and considering effective operators of dimension five for the case of charged leptons and dimension seven for neutrinos. This mechanism requires a new physics at TeV scale whose most probable candidate is the 3-3-1 supersymmetric extension. We also generate in this context the bi-maximal mixing among the neutrinos involved in the solar and atmospheric oscillations.

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I. INTRODUCTION

The smallness of the neutrino masses and the pattern of their mixing, arising from atmospheric and solar neutrino data [1–4], suggest the extension of the standard model. According to those experimental data, the mixing involved in the atmospheric neutrino oscillation is maximal and the mixing involved in the solar neutrino oscillation has a possibility to be also maximal [5]. This is the bi-maximal mixing scenario which is realized in models with the global $L_e - L_\mu - L_\tau$ symmetry. From the theoretical point of view, we dispose already of well established ways where explanations to the smallness of those masses arise naturally. The most popular are the see-saw [6] and the radiative generation [7] mechanisms. However both of them require realistic extensions of the standard model and extra global and/or discrete symmetries. For instance, we can consider effective operators which naturally lead to light neutrinos. This approach has had success in accounting for the neutrino puzzle in the base of the standard model without resort to drastic fine tuning.

It was Weinberg [8] and independently Zee and Wilczek [9] who first pointed out in the context of the standard model that the dimension-five effective operator:

\begin{equation}
\frac{1}{\Lambda} \overline{\Psi}_{iaL} \Psi_{jbL} \varphi_k^{(m)} \varphi_l^{(n)} (f_{abmn} \epsilon_{ik} \epsilon_{jl} + f'_{abmn} \epsilon_{ij} \epsilon_{kl}),
\end{equation}

yields naturally light neutrino masses. The success of such effective operator approach is justified by the expression of the neutrino mass it generates:

\begin{equation}
M_{\nu ab} = \frac{f_{ab}}{\Lambda} \langle \varphi \rangle^2,
\end{equation}

which is a see-saw relation since $\langle \varphi \rangle \approx 246$ GeV and $\Lambda$ is a large effective mass. Particularly, in this case the realization of such operator turns to be important once it can guide us to all the possible realization of that mechanism using only the representation content of the standard model with operators of dimension five [10] or higher which were already considered [11]. Extensions of the scalar sector of the standard model have already been suggested [12].

In this work we address the problem of generating neutrino and charged lepton masses through effective operators in the context of 3-3-1 models [13,14]. In general in this model the minimal set of scalar multiplets required by the model to generate the fermion masses consists of three triplets: $\eta = (\eta^0, \eta^1, \eta^2)^T \sim (1,3,0)$, $\rho = (\rho^+, \rho^0, \rho^{++})^T \sim (1,3,1)$, $\chi = (\chi^-, \chi^-, \chi^0)^T \sim (1,3,-1)$ and a symmetric sextet, $S \sim (1,6,0)$. However some years ago Duong and Ma shown that it is possible to avoid the introduction of the sextet $S$ by adding a singlet charged lepton is added [15,16]. We will work in this scenario.

In the 3-3-1 models of Refs. [13,15] all leptons transform as a triplet under the electroweak gauge symmetry:

\begin{equation}
\Psi_{aL} = \begin{pmatrix}
\nu_a \\
l_a \\
l'^{\nu}_a
\end{pmatrix}_L \sim (1,3,0); \quad a = e, \mu, \tau.
\end{equation}

The outline of this work is as follows. In Sec. II we develop a formalism to address the issue of generating the charged lepton and neutrino masses by effective operators and
suggest what are the main ingredients a more fundamental theory has to have in order to realize such effective operators. Next, in Sec. III we use the formalism developed in Sec. II to generate a realistic scenario to accommodate leptons masses with bi-maximal mixing among neutrinos. We reserve the section IV for our conclusions.

II. THE MECHANISM

In this section we will build effective operators of dimension five and seven leading to small neutrino masses. In particular we will see that with dimension-seven operator we attain the desired order of the neutrino masses required by the recent experiments, i.e., at the eV scale. What is interesting is the fact that energy scale of energy required to obtain those neutrino masses is of the order of 5 TeVs.

A. Charged lepton masses

Even discarding the sextet we still dispose of the three scalar triplets \( \eta, \rho \) and \( \chi \) given above. According to the transformation properties under the symmetry \( SU(3)_C \otimes SU(3)_L \otimes U(1)_N \) of the last two triplets we can form with \( \bar{L} C L \) the following effective dimension-five operator:

\[
L = \frac{f_{ab}}{\Lambda} \bar{\Psi}_{aL} \Psi_{bL} \chi^* \rho^* + H.c.
\]

After the neutral components \( \chi^0 \) and \( \rho^0 \) develop their respective VEVs, \( \langle \chi \rangle \) and \( \langle \rho \rangle \), the effective operator above generates the following expression for the charged lepton mass matrix:

\[
M_{ab}^l = \frac{f_{ab}}{\Lambda} \langle \rho \rangle \langle \chi \rangle.
\] (5)

with \( a = e, \mu, \tau \). Let us discuss the values of the parameters presented in the expression above. None of them have already been fixed by the model. We just expect that they can be found inside some range of values. For example, \( \langle \chi \rangle \) must be inside the range : \( 300 \text{ GeV} < \langle \chi \rangle < 3 \text{ TeV} \) [17]. The constraint on \( \langle \rho \rangle \) comes from the mass of the gauge bosons \( W^\pm \) and \( Z^0 \) i.e., \( \langle \rho \rangle^2 + \langle \eta \rangle^2 = (246)^2 \text{ GeV}^2 \). Let us take, as an illustration, the following set of values:

\[
\langle \eta \rangle \simeq 22 \text{ GeV}, \langle \rho \rangle \simeq 245 \text{ GeV}, \langle \chi \rangle \simeq 10^3 \text{ GeV} \text{ and } \Lambda \simeq 5 \text{ TeV},
\] (6)

and the charged lepton mass matrix takes the following form:

\[
M_{ab}^l \simeq 49 f_{ab} \text{ GeV}.
\] (7)
If $f_{ab}$ is a diagonal matrix for obtaining the correct charged lepton masses we need $f_{ee} \sim 10^{-5}$, for the electron mass, $f_{\mu\mu} \sim 2 \times 10^{-3}$ for the muon mass and $f_{\tau\tau} \sim 3.6 \times 10^{-2}$ for the tau mass. In the case of the standard model where one has $f_{ee} \sim 10^{-6}$, $f_{\mu\mu} \sim 10^{-3}$ and $f_{\tau\tau} \sim 10^{-2}$.

### B. Neutrino mass

In order to generate neutrino masses we will consider only effective operators that explicitly conserve the total lepton number. The simplest way to obtain such operator is by adding an scalar singlet $\phi$, coming from new physics at the TeV scale, carrying total lepton number $L(\phi) = -1$ and forming with $\eta$ and $\Psi_{ab}$ the following effective dimension-seven operator:

$$
\mathcal{L} = \frac{f'_{ab}}{\Lambda^3} \overline{\Psi} aL \eta^* \eta \phi \phi + H.c.
$$

$$
= \frac{f'_{ab}}{\Lambda^3} \left\{ \left[ \bar{\nu}_{bL} \eta^0 \eta^0 + (l^c)_{bL} \eta^0 \eta_1 + (l^c)_{bL} \eta^0 \eta_2 \right] + \left[ \bar{\eta} \eta_1^+ + \bar{\eta} \eta_2^+ \right] \right\} + H.c.
$$

Notice that this operator conserves the total lepton number since $L(\eta^\pm) = -2$ and $L(\eta^-) = 0$. After the scalars involved in it develop their respective VEV the neutrino masses are given by

$$
M^\nu_{ab} = \frac{f'_{ab}}{\Lambda^3} \langle \eta \rangle^2 \langle \phi \rangle^2.
$$

Inserting the values of the VEV given in Eq. (6) the expression above reads

$$
M^\nu_{ab} \approx f'_{ab} \left( \frac{\langle \phi \rangle}{1\text{ GeV}} \right)^2 3.9 \times 10^{-9} \text{ GeV}.
$$

According to this the VEV involved above, $\langle \phi \rangle$, has to be around $5 \times 10^{-2}$ GeV if $f_{ab} \approx O(1)$, in order to generate the expected order of magnitude of the neutrino masses, that is, of the $10^{-2}$ eV order. That value of the VEV for the scalar singlet implies a fine tuning since we expect $m_\phi \approx \Lambda$. However, in order to get such small VEV in a more natural way we can implement a type II see-saw mechanism with the scalar field $\phi$ [18].

In fact, it is possible to implement this mechanism as we will show in the following. Let us consider, for the sake of simplicity, the discrete symmetry $\eta \rightarrow -\eta$, $\phi \rightarrow -\phi$ (other fields are even under this symmetry). We also allow terms in the scalar potential that violate explicitly the total lepton number. In this case the most complete scalar potential presenting these terms is:

$$
V(\eta, \rho, \chi, \phi) = \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \mu_\phi \phi^\dagger \phi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 + \lambda_4 (\phi^\dagger \phi)^2
$$

$$
+ (\eta^\dagger \eta) \left[ \lambda_5 (\rho^\dagger \rho) + \lambda_6 (\chi^\dagger \chi) \right] + \lambda_7 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_8 (\rho^\dagger \eta) (\eta^\dagger \rho) + \lambda_9 (\chi^\dagger \eta) (\eta^\dagger \chi)
$$

$$
+ \lambda_{10} (\rho^\dagger \chi) (\chi^\dagger \rho) + (\phi^\dagger \phi) \left[ \lambda_{11} (\eta^\dagger \eta) + \lambda_{12} (\rho^\dagger \rho) + \lambda_{13} (\chi^\dagger \chi) \right]
$$

$$
+ [\lambda_{14} \eta \rho \chi \phi + \lambda_{15} \chi^\dagger \eta \rho^\dagger \eta + H.c.],
$$

(11)
where the last two terms violate explicitly the lepton number. From this scalar potential we find the following constraint equation over \( \langle \phi \rangle \):

\[
\langle \phi \rangle [\mu_\phi^2 + \lambda_{11} \langle \eta \rangle^2 + \lambda_{12} \langle \rho \rangle^2 + \lambda_{13} \langle \chi \rangle^2 + \lambda_4 \langle \phi \rangle^2] + \lambda_{14} \langle \eta \rangle \langle \rho \rangle \langle \chi \rangle = 0. \tag{12}
\]

Supposing that \( \mu_\phi^2 < 0 \) is the dominant parameter in the parenthesis term of the last equation we have,

\[
\langle \phi \rangle \simeq -\frac{\lambda_{14} \langle \eta \rangle \langle \rho \rangle \langle \chi \rangle}{\mu_\phi^2}. \tag{13}
\]

Using the values in Eq. (6) in Eq. (13), and assuming \( |\mu_\phi| \approx \Lambda \) we obtain \( \langle \phi \rangle \simeq 5 \times 10^{-2} \) GeV, if \( \lambda_{14} = 0.25 \). We recall that it was already shown in literature that it is possible to have a heavy scalar with small VEV [10,19] as in the present case.

Hence, from Eq. (10) we find that the neutrino mass matrix is given by the following expression:

\[
M^\nu_{ab} \simeq 10^{-2} f'_{ab} \text{eV}. \tag{14}
\]

In the next section we apply this mechanism to generate a realistic scenario in the neutrino physics. Before, however, we analyze what main ingredients an underlying theory should have to realize, in an economic way, the effective operators used above.

**C. A possible underlying theory**

The minimal scenario we can imagine is the one where the effective dimension-five operator in Eq. (4) is realized at the tree level, while that effective dimension-seven operator in Eq. (8) is realized through the one loop level. For this we only need to add to the model the following heavy lepton singlets, \( E_{1L,R} \sim (1,1,-1) \) and \( E_{2L,R} \sim (1,1,0) \) [15,16]. Denoting \( L \) the total lepton number, i.e., \( L = L_e + L_\mu + L_\tau \), we will assume that these singlets have the following assignments: \( L(E_1) = 1 \) and \( L(E_2) = -1 \). The necessary interactions in order to realize the effective operators are the following:

\[
\mathcal{L} = G_{1a} \bar{\Psi}_{aL} E_{1R} \rho + G_{2a} \chi^T E_{1L} \Psi_{aL}^c + E_{1L} E_{2R} \phi^* + E_{2L} E_{1R} \phi + M_1 E_{1L} E_{1R} + M_2 E_{2L} E_{2R} + H.c., \tag{15}
\]

with \( M_1, M_2 \simeq \Lambda \). To realize the effective dimension-five operator only the interactions above are sufficient and the corresponding diagram is showed in the Fig. 1a, while for the realization of the effective dimension-seven operator the last term of the scalar potential in Eq. (11) is also important. Such realization is depicted in Fig. 1b.

We must point out that the fermions, \( E_1 \) and \( E_2 \), introduced here are do exist in the supersymmetric version of the 3-3-1 as part of the higgsino triplets \( \tilde{\rho} \) and \( \tilde{\eta} \) and they will appear as singlets of \( SU(2) \otimes U(1)_Y \) after the supersymmetry breaking [20].
III. A REALISTIC SCENARIO

We are interested in a scenario where the bi-maximal mixing among the neutrinos appears as a result of the symmetry \( L' = L_e - L_\mu - L_\tau \) in the leptonic sector \([12,21,23]\). Hence, the effective dimension-five operator in Eq. (4) must obey the symmetry \( L' \). For this, let us discriminate the components of the scalar triplets involved in that operator that carry lepton number. On the other hand, the total lepton number \( L \) has the following assignments:

\[
L(\chi^-, \chi^-) = +2, \quad L(\rho^{++}) = -2.
\]

If the \( L' \) symmetry is respected, the triplets \( \chi \) and \( \rho \) generate mass only for two charged leptons. Thus we need to add another pair of similar triplets. Hence we have four triplets denoted by: \( \chi_1, \rho_1 \) and \( \chi_2, \rho_2 \) with the following \( L' \) assignments:

\[
L'(\chi^-_1, \chi^-_2) = -2, \quad L'(\rho^{++}_1) = +2, \quad L'(\chi^-_1, \chi^-_1) = +2, \quad L'(\rho^{++}_1) = -2.
\]

With such assignments of \( L' \), the operators that obey \( L' \) and generate the charged lepton masses are the following:

\[
\mathcal{L} = \frac{f_{ee}}{\Lambda} \overline{L}_e \chi_1^* \rho_1 + \frac{f_{ii}}{\Lambda} \overline{L}_i \chi_2^* \rho_2 + H.c.,
\]

with \( i = \mu, \tau \). Taking the values of the parameters involved above as those displayed in Eq. (6), and also considering the charged lepton masses in a diagonal basis, the range of values of the parameters \( f's \) that yield the correct order of magnitude of the charged lepton masses as in Sec. II A:

\[
f_{ee} \simeq 1.4 \times 10^{-5}, \quad f_{\mu\mu} \simeq 3 \times 10^{-3}, \quad f_{\tau\tau} \simeq 5 \times 10^{-2}.
\]

In order to obtain the bi-maximal mixing among the neutrinos, we impose that \( L'(\eta) = L'(\phi) = 0 \), and from Eq. (14) directly give the following neutrino mass matrix at the tree level:

\[
M' = \begin{pmatrix}
0 & a & a \\
a & 0 & 0 \\
a & 0 & 0
\end{pmatrix},
\]

where \( a = 10^{-2} \) eV where we have used the approximation that \( f'_{ee} \approx f'_{e\tau} \approx O(1) \) and used the VEV from Eq. (6).

The form of the mass matrix above yields the bi-maximal neutrino mixing pattern \([21]\):

\[
U_{BM} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

This pattern of mixing has been studied in the literature and it gives a splitting between \( \nu_1, \nu_2 \) and \( \nu_3 \) with \( \Delta m^2_{atm} = m^2_{1,2} - m^2_3 \) and the neutrino mass spectrum has an inverted hierarchy \([21]\).

Now we should address the splitting between \( \nu_1 \) and \( \nu_2 \) in order to explain the solar neutrino oscillation. For getting such a split we have to consider terms in the scalar potential that break explicitly \( L' \). This will be done by the last term in the scalar potential in Eq. (11). That term together with the effective operator in Eq. (16) will generate corrections to the
diagonal entries in the mass matrix in Eq. (18) through the one loop diagrams, providing then the splitting between \( \nu_1 \) and \( \nu_2 \).

The loop diagram which will generate the diagonal entries in Eq. (18) is depicted in Fig. 2. It gives, up to logarithmic corrections, the following expression for such entries:

\[
M^{\nu}_{aa} \simeq \frac{\lambda_{15} f_{aa}^3 \langle \eta \rangle^2 \langle \rho \rangle^2 \langle \chi \rangle^2}{m_\chi^2 L^3}.
\] (20)

All the parameters above, but \( m_\chi \) and \( \lambda_{15} \), were already previously fixed in this work. Assuming now that \( m_\chi = \langle \chi \rangle \), and inserting Eq. (6) in Eq. (20), we have:

\[
M^{\nu}_{aa} \simeq 1.1 \times 10^{-8} \lambda_{15} f_{aa}^3 \text{GeV}.
\] (21)

The LMS MSW solutions to the solar neutrino anomaly require a neutrino mass scale of the order of \( 2.8 \times 10^{-3} \text{ eV} \) [22]. The only free parameter in Eq. (21) is \( \lambda_{15} \), while the diagonal \( f \) parameters are already fixed by the charged lepton masses and their values are expressed in Eq. (17). We easily obtain this mass scale for the neutrinos by considering \( \lambda_{15} \sim 2.5 \times 10^{-4} \). Hence to generate the \( \nu_1-\nu_2 \) mass splitting we must to fine tuning \( \lambda_{15} \).

IV. CONCLUSIONS

In this work we developed a simple mechanism based on effective operators in the context of a 3-3-1 model which generates masses for the neutrinos and for the charged leptons as well. The charged lepton masses are generated in this mechanism by an effective dimension-five operator, while the neutrino masses require an effective dimension-seven operator in conjunction with a type II see-saw mechanism applied on a scalar singlet. Then, we use the effective operator mechanism to generate the bi-maximal mixing among the neutrinos involved in the solar and the atmospheric processes. For this we consider the symmetry \( L' = L_e - L_\mu - L_\tau \). In this case, without resort to large fine tuning we obtain neutrino masses compatible with some solutions to the solar and atmospheric neutrino anomalies. Finally, we should remark that this mechanism works in an energy scale of the order of few TeVs and that the probable underlying theory is a supersymmetric version of the 3-3-1 model once it disposes of the ingredients which are necessary to realize the effective operators of this mechanism [24]. Although we have not considered the quark masses recently an interesting mechanism in the context of a 3-3-1 model for generating the top and bottom masses at the tree level and the masses of the other quarks and charged leptons arising at the 1-loop level has been pointed out in Ref. [25].

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REFERENCES


FIG. 1. Diagrams contributing to the effective operators.
FIG. 2. One loop contribution to the diagonal entries for the neutrino mass matrix.