Noncommutativity and the Motion of $D_p$-brane along Itself

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Abstract

We consider open strings attached to a moving $D_p$-brane with motion along itself, in the presence of the backgrounds $B_{\mu \nu}$-field and a $U(1)$ gauge field $A_\alpha$. The effects of the motion of the brane on the open string propagator and on the open string variables are studied. We observe that some free parameters appear in the open string variables and in the propagator of it.

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There have been attempts to explain the noncommutativity on D-brane worldvolume through the study of open strings in the presence of background fields [1, 2]. It is known that Lorentz boosts act on background fields. This affects the noncommutativity parameter and the effective metric of open string. According to these facts, some effects of Lorentz boosts on the noncommutative string theory are studied [3].

Instead of Lorentz boosts, one can use the $\sigma$-model action of string with velocity terms [4, 5, 6]. These terms show the velocity of the brane that the ends of open string are on it. Some applications of this method are given in the Ref.[5, 7]. In this method, the velocity of the brane introduces spontaneously to all quantities extracted from the string action, for example the noncommutativity parameter, the open string metric, the boundary conditions of the open string and its propagator. In this note we consider the motion of the brane along itself. For example for membrane we observe that some arbitrary parameters appear in the open string variables and its propagator. As expected, for zero speed our results reduce to the known cases.

boundary conditions of open string

We begin with a $\sigma$-model action for string that contains $B_{\mu \nu}$ field and two boundary terms corresponding to the gauge field [4] and the velocity of the brane [5, 6]

$$S = \frac{1}{4\pi \alpha'} \int_{\Sigma} d^2 \sigma (g_{\mu \nu} \partial_a X^\mu \partial^a X^\nu - 2 \pi i \alpha' \epsilon^{ab} B_{\mu \nu} \partial_a X^\mu \partial_b X^\nu)$$
\[
+ \frac{1}{2\pi\alpha'} \int_{\partial \Sigma} d\tau (-2\pi i\alpha' A_\alpha \partial_\tau X^\alpha + V_\alpha X^0 \partial_\sigma X^\alpha),
\]

where \( \Sigma \) is the worldsheet of the open string and \( \partial \Sigma \) is the boundary of it that is at \( \sigma = 0 \). Here the open string worldsheet \( \Sigma \) has the Euclidean metric \( \delta^{ab} \). \( A_\alpha \) is a \( U(1) \) gauge field that lives in the worldvolume of the brane and \( V^\alpha \) is the velocity of the brane. The set \( \{X^\alpha\} \) specifies the directions of the worldvolume of the brane. Also let the set \( \{X^i\} \) show the directions perpendicular to the brane.

Assume that the background fields \( g_{\mu\nu} \) and \( B_{\mu\nu} \) to be constant, with the zero elements \( g_{\alpha i} = B_{\alpha i} = B_{ij} = 0 \). Furthermore let the field strength of the gauge field \( A_\alpha \) be constant. Vanishing the variation of the action with respect to \( X^\mu(\sigma, \tau) \) gives the equation of motion and the boundary conditions of the string

\[
(\partial^2_\tau + \partial^2_\sigma) X^\mu(\sigma, \tau) = 0 ,
\]

\[
\left( g_{0\beta} \partial_\sigma X^\beta + g_{\alpha\beta} V^\beta \partial_\sigma X^\alpha + 2\pi i\alpha' F_{0\beta} \partial_\tau X^\beta \right)_{\sigma=0} = 0 ,
\]

\[
\left( g_{\alpha\beta} \partial_\sigma (X^\beta - V^\beta X^0) + 2\pi i\alpha' F_{\alpha\beta} \partial_\tau X^\beta \right)_{\sigma=0} = 0 ,
\]

\[
(\delta X^i)_{\sigma=0} = 0 ,
\]

where \( \bar{\alpha} \) refers to the spatial directions of the brane, i.e. \( \bar{\alpha} \neq 0 \), and

\[
F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + B_{\alpha\beta} ,
\]

is total field strength.

Since the transverse coordinates \( \{X^i\} \) can be treated trivially, we concentrate only on the brane directions. The boundary conditions (3) and (4) can be written as the following

\[
(S_{\alpha\beta} \partial_\sigma X^\beta + 2\pi i\alpha' F_{\alpha\beta} \partial_\tau X^\beta)_{\sigma=0} = 0 ,
\]

where the matrix \( S \) is

\[
S_{\alpha\beta} = g_{\alpha\beta} + \omega_{\alpha\beta} ,
\]

and the antisymmetric matrix \( \omega_{\alpha\beta} \) is

\[
\omega_{0\alpha} = -\omega_{\alpha 0} = g_{\alpha\beta} V^\beta ,
\]

\[
\omega_{\alpha\beta} = 0 .
\]

Now we use the equation of motion (2) and the boundary condition (7), to obtain the open string propagator.
The propagator for the open string can be obtained by solving the equations of motion

$$\partial \overline{\partial} G^\alpha \gamma (z, z') = -\frac{\pi}{2} \alpha' G^\alpha \delta^{(2)}(z - z') ,$$

(10)

where $z = \tau + i \sigma$, and $G^\alpha \gamma$ is the open string metric. Propagator also should satisfy the boundary conditions

$$\left( (\partial - \overline{\partial}) G^\alpha \gamma (z, z') + 2\pi \alpha' (S^{-1} \mathcal{F})^\alpha \gamma (\partial + \overline{\partial}) G^\gamma\beta (z, z') \right)_{z = \bar{z}} = 0 .$$

(11)

The solution of the equations of the propagator can be written as

$$G^\alpha \gamma (z, z') = -\alpha' \left[ \frac{1}{2} Q^\alpha \gamma \ln(z - z') + \frac{1}{2} Q^\alpha \gamma \ln(\bar{z} - \bar{z}') \right.
\left. + ( - \frac{1}{2} Q^\alpha \gamma + G^\alpha \gamma + \frac{\theta^\alpha \gamma}{2\pi \alpha'} ) \ln(z - z') \right.
\left. + ( - \frac{1}{2} Q^\alpha \gamma + G^\alpha \gamma - \frac{\theta^\alpha \gamma}{2\pi \alpha'} ) \ln(\bar{z} - \bar{z}') - \frac{i}{2\alpha'} \theta^\alpha \gamma \right] .$$

(12)

The equations (11) and (12) give the open string metric and the noncommutativity parameter as the following

$$G^\alpha \gamma = \left( (S + 2\pi \alpha' \mathcal{F})^{-1} S(S - 2\pi \alpha' \mathcal{F})^{-1} S Q \right)^\alpha \gamma ,$$

(13)

$$G_{\alpha \beta} = \left( Q^{-1} S^{-1} (S - 2\pi \alpha' \mathcal{F}) S^{-1} (S + 2\pi \alpha' \mathcal{F}) \right)_{\alpha \beta} ,$$

(14)

$$\theta^\alpha \gamma = - (2\pi \alpha')^2 \left( (S + 2\pi \alpha' \mathcal{F})^{-1} \mathcal{F}(S - 2\pi \alpha' \mathcal{F})^{-1} S Q \right)^\alpha \gamma .$$

(15)

The open string metric $G$ should be symmetric and the noncommutativity parameter $\theta$ should be antisymmetric. These imply that the matrix $Q$ should satisfy the following equation

$$SQ \mathcal{F} - \mathcal{F}(SQ)^T = \frac{1}{2\pi \alpha'} S(S^T - Q) S^T .$$

(16)

The propagator (12) under the exchanges $\alpha \leftrightarrow \beta$ and $z \leftrightarrow z'$, should be symmetric. Therefore the matrix $Q$ is symmetric, i.e.

$$Q^T = Q .$$

(17)

Since the equations (16) and (17) do not give a unique solution for the matrix $Q$, some arbitrary parameters appear in the open string variables and the propagator of it. In fact these free parameters show a class of solutions.
According to the condition (17), $Q = S^{-1}$ is not an available solution of the equation (16), because $S$ is not a symmetric matrix. For zero speed we have $Q = g^{-1}$. In this case the propagator (12) and the equations (13)-(15) reduce to the known cases.

Since the ordinary DBI action and the noncommutative description of it at zero speed and zero field strength are equal the effective open string coupling $G_s$ is as previous, i.e. it is independent of the speed of the brane.

By using the equations (13)-(17) and the identity

$$\partial\bar{\partial}(\ln|z - z'| + \ln|z - \bar{z}'|) = \frac{\pi}{2}\delta^{(2)}(z - \bar{z}') , \quad (18)$$

one can verify that the propagator (12) also satisfies the equation (10).

$\alpha' \to 0$ limit

The zero slope limit ($\alpha' \to 0$) of this open string system depends on the speed of the brane. Since in this limit there are $\alpha' \sim \epsilon^2$ and $g_{\alpha\beta} \sim \epsilon$ (where $\epsilon \to 0$), the term $\omega_{\alpha\beta}$ in the equation (8) is zero (for zero elements) or goes to zero like $\epsilon$ (for non-zero elements). Therefore in this limit there is $S_{\alpha\beta} \sim \epsilon$. From the equations (13)-(15) we have

$$G^{\alpha\beta} = -\frac{1}{(2\pi\alpha')^2}(F^{-1}SF^{-1}SQ)^{\alpha\beta} , \quad (19)$$

$$G_{\alpha\beta} = -(2\pi\alpha')^2(Q^{-1}S^{-1}FS^{-1}F)_{\alpha\beta} , \quad (20)$$

$$\theta^{\alpha\beta} = (F^{-1}SQ)^{\alpha\beta} . \quad (21)$$

From the equations (16) and (17) we see that the matrix $SQ$ in this limit is finite. Therefore the open string metric $G$ and the noncommutativity parameter $\theta$ also are finite.

When the speed of the brane is non-zero, the equations (16) and (17) impose some conditions on the matrices $S$ and $F$. To see this clearly, we study the following examples. Since for the Minkowski spacetime, again the equations (16) and (17) hold, we consider the following examples in the Minkowski spacetime.

**Example 1 : $D_1$ - brane**

For $D_1$-brane with speed $v$ along itself, $Q$ is non-zero if there is $v^2\det S = 0$. Zero speed $v = 0$, is a known case. The case $\det S = 0$, means that the matrix $S$ is not invertible, and therefore the open string metric (13) is not invertible. In this case the speed of the $D_1$-brane is

$$v = \pm \sqrt{g''^2 + g_0g_1} , \quad (22)$$

where the matrix \( \begin{pmatrix} -g_0 & g' \\ g' & g_1 \end{pmatrix} \) is the closed string metric. Since $-1 \leq v \leq 1$, the speed (22) is available if the elements of the closed string metric obey the condition $g_0g_1 + g''^2 \leq 1$. 
Example 2: $D_2$ - brane

Consider a $D_2$-brane parallel to the $X^1X^2$-plane, with the speed $v$ along $X^1$-direction. Let the closed string metric be $\text{diag}(-g_0, g_1, g_2)$. The matrix $Q$ is

$$Q = \begin{pmatrix} q & q_1 & q_2 \\ q_1 & q' & q_3 \\ q_2 & q_3 & q'' \end{pmatrix}.$$  \hfill (23)

If there is an electric field on the brane parallel to the velocity, the element $q''$ is arbitrary. The other elements of $Q$ are zero. Therefore $G^{\alpha\beta}$, $G^{\alpha3}$ and $\theta^{\alpha\beta}$ depend on the arbitrary parameter $q''$.

If the electric field is perpendicular to the velocity, the element $q'$ is arbitrary and $q_2 = q_3 = 0$. Also there are the following relations between $q_1$, $q$ and $q''$

$$q_1 = qv,$$

$$q'' = \frac{-g_0 + g_1v^2}{g_2}q.$$ \hfill (24)

In this case the free parameters $q$ and $q'$ appear on the variables $G^{\alpha\beta}$, $G^{\alpha3}$ and $\theta^{\alpha\beta}$.

For magnetic field perpendicular to the brane, $q$ is arbitrary and

$$q_2 = q_3 = 0,$$

$$q_1 = \frac{g_1}{g_0}qv',$$

$$q'' = (1 - \frac{g_1}{g_0}v^2)\frac{g_1}{g_2}q'. $$ \hfill (25)

These equations imply the variables $G^{\alpha\beta}$, $G^{\alpha3}$ and $\theta^{\alpha\beta}$ depend on the parameters $q$ and $q'$, which again are arbitrary. For branes with higher dimensions, more free parameters appear in the open string variables and in the propagator.

References


