Cosmological term as a source of mass

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In the spherically symmetric case the dominant energy condition, together with the requirement of regularity at the center, asymptotic flatness and finiteness of the ADM mass, defines the family of asymptotically flat globally regular solutions to the Einstein equations which includes the class of metrics asymptotically de Sitter as $r \to 0$. The source term corresponds to an $r$-dependent cosmological term $\Lambda g_{\mu\nu}$ invariant under boosts in the radial direction and evolving from the de Sitter vacuum $\Lambda g_{\mu\nu}$ in the origin to the Minkowski vacuum at infinity. The ADM mass is related to cosmological term by $m = (2G)^{-3} \int_0^\infty \Lambda^2 r^2 dr$, with de Sitter vacuum replacing a central singularity at the scale of symmetry restoration. Space-time symmetry changes smoothly from the de Sitter group near the center to the Lorentz group at infinity through radial boosts in between. In the range of masses $m \geq m_{\text{crit}}$, de Sitter-Schwarzschild geometry describes a vacuum nonsingular black hole (ABH), and for $m < m_{\text{crit}}$ it describes G-lump - a vacuum selfgravitating particelike structure without horizons. Quantum energy spectrum of G-lump is shifted down by the binding energy, and zero-point vacuum mode is fixed at the value corresponding (up to the coefficient) to the Hawking temperature from the de Sitter horizon.

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Introduction - In 1917 Einstein introduced a cosmological term into his equations describing gravity as spacetime geometry (G-field) generated by matter

$$G_{\mu\nu} = -8\pi GT_{\mu\nu}$$ (1)

to make them consistent with Mach’s principle which was one of his primary motivations [1]. Einstein’s formulation of Mach’s principle was that some matter has the property of inertia only because there exists also some other matter in the Universe ( [2], Ch.2). When Einstein found that Minkowski geometry is the regular solution to (1) perfectly describing inertial motion in the absence of any matter, he modified his equations by adding the cosmological term $\Lambda g_{\mu\nu}$ in the hope that modified equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}$$ (2)

will have reasonable regular solutions only when matter is present - if matter is the source of inertia, then in case of its absence there should not be any inertia [3].

The primary task of $\Lambda$ was thus to eliminate inertia in case when matter is absent by eliminating regular G-field solutions in case when $T_{\mu\nu} = 0$.

Soon after introducing $\Lambda g_{\mu\nu}$, in the same year 1917, de Sitter found quite reasonable solution with $\Lambda g_{\mu\nu}$ and without $T_{\mu\nu}$, whose nowadays triumphs are well known.

The story of abandoning $\Lambda$ by Einstein is also widely known, although typically with the accent on successes of FRW cosmology. This somehow left in shadow the basic sense of his idea of introducing $\Lambda$ as a quantity which has something in common with inertia. The question - can it be possible to find some constructive way connecting them? - seems to be related to the other Einstein’s profound proposal, suggested in 1950, to describe elementary particle by regular solution of nonlinear field equations as "bunched field" located in the confined region where field tension and energy are particularly high [4]. The possible way to such a structure of gravitational origin whose mass is related to $\Lambda$ and whose regularity is related to this fact, can be found in the Einstein field equations (1) and in the Petrov classification for $T_{\mu\nu}$ [5].

The aim of this paper is to show that in the spherically symmetric case with the requirements of A) asymptotic flatness and finiteness of the mass, B) regularity of metric and density at $r \to 0$, and C) the dominant energy condition for a source term, there exists the class of globally regular solutions, asymptotically Schwarzschild at infinity, with de Sitter vacuum replacing a singularity.

De Sitter-Schwarzschild geometry - A static spherically symmetric line element can be written in the standard form (see, e.g., [6], p.239)

$$ds^2 = e^{\mu(r)}dt^2 - e^{\nu(r)}dr^2 - r^2d\Omega^2$$ (3)

where $d\Omega^2$ is the metric of a unit 2-sphere.

The Einstein equations (1) reduce to ( [6], p.244)

$$8\pi G T^t_t = 8\pi G \rho(r) = e^{-\nu} \left( \frac{\mu'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2}$$ (4)

$$8\pi G T^r_r = -8\pi G p_r(r) = -e^{-\nu} \left( \frac{\mu'}{r} + \frac{1}{r^2} \right) + \frac{1}{r^2}$$ (5)

$$8\pi G T^\phi_\phi = 8\pi G T^\phi_\phi = -8\pi G p_\perp(r) =$$

$$-e^{-\nu} \left( \frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{(\mu' - \nu')}{2r} - \frac{\mu' \nu'}{4} \right)$$ (6)
Here \( \rho(r) = T^t_t \) is the energy density (we adopted \( c = 1 \) for simplicity), \( p_r(r) = -T^{\theta \theta}_r \) is the radial pressure, and \( p_\perp(r) = -T^\theta_\theta \) is the tangential pressure for a perfect fluid [6], p.243. A prime denotes differentiation with respect to \( r \).

To investigate this system in the case of different principal pressures we impose requirement of regularity at the center of both density and metric, and the energy dominant condition on the stress-energy tensor.

The dominant energy condition \( T^{ab} \geq |T^{ab}| \) for each \( a, b = 1, 2, 3 \), which holds if and only if \( \rho[7] \)

\[
\rho \geq 0; \quad -\rho \leq p_k \leq \rho; \quad k = 1, 2, 3 \tag{7}
\]

implies that the local energy density is non-negative and each principal pressure never exceeds the energy density.

Integration of Eq.(4) gives [8]

\[
e^{-\nu(r)} = 1 - \frac{2GM(r)}{r}; \quad M(r) = 4\pi \int_0^r \rho(x)x^2dx \tag{8}
\]

which has for large \( r \) the Schwarzschild asymptotics

\[
e^{-\nu} = 1 - 2Gm/r, \quad \text{where} \quad m = \frac{4\pi \int_0^\infty \rho(r)r^2dr}{1 - \frac{2Gm}{r}} \tag{9}
\]

In the limit \( r \to \infty \) the condition of finiteness of the mass (9) requires density profile \( \rho(r) \) to vanish at infinity quicker than \( r^{-3} \). The dominant energy condition \( p_k \leq \rho \), requires both radial and tangential pressures to vanish as \( r \to \infty \). Then \( \mu' = 0 \) and \( \mu = \text{const} at \infty \), and we impose the standard boundary condition \( \mu = 0 \) for \( r \to \infty \) to have asymptotic flatness needed to identify (9) as the ADM mass [8]. As a result we get the Schwarzschild asymptotics at infinity

\[
T_{\mu\nu} = 0; \quad ds^2 = \left( 1 - \frac{2Gm}{r} \right)dt^2 - \frac{dr^2}{1 - \frac{2Gm}{r}} - r^2d\Omega^2 \tag{10}
\]

From Eqs.(4)-(6) we derive the equation (see also [9])

\[
p_{\perp} = p_r + \frac{r}{2}p_r' + \left( \rho + p_r \right) \frac{GM(r) + 4\pi Gr^3p_r}{2(r - 2GM(r))} \tag{11}
\]

which is generalization of the Tolman-Oppenheimer-Volkoff equation ([8], p.127) to the case of different principal pressures, and the equation

\[
T^t_t - T^r_r = p_r + \rho = \frac{1}{8\pi G} e^{-\nu}(\nu' + \mu') \tag{12}
\]

From Eq.(8) we see that for any regular value of \( e^{\nu(r)} \) we must have \( M(r) = 0 \) at \( r = 0 \) [10], and that \( \nu(r) \to 0 \) as \( r \to \infty \). The dominant energy condition allows us to fix asymptotic behaviour of a mass function and of a metric at approaching the regular center. Requirement of regularity of density \( \rho(r = 0) < \infty \), leads, by the dominant energy condition \( p_k \leq \rho \), to regularity of pressures. Requirement of regularity of the metric, \( e^{\nu(r)} < \infty \), leads then, by (12), to \( \nu' + \mu = 0 \) and \( \nu + \mu = \mu(0) \) at \( r = 0 \) with \( \mu(0) \) playing the role of the family parameter.

The example of GR solution from this family is boson stars [12] (for review [13,14]) which are completely regular configurations without horizons generated by self-gravitating massive complex scalar field whose stress-energy tensor is essentially anisotropic, \( p_r \neq p_\perp \).

The weak energy condition, \( T_{\mu\nu}\xi^\mu\xi^\nu \geq 0 \) for any time-like vector \( \xi^\mu \), which is satisfied if and only if \( \rho \geq 0; \rho + p_k \geq 0; k = 1, 2, 3 \) and which is contained in the dominant energy condition [7], defines, by Eq.(12), the sign of the sum \( \mu' + \nu' \). In the case when \( e^{\nu(r)} > 0 \) everywhere, it demands \( \nu' + \mu' \geq 0 \) everywhere. In case when \( e^{\nu(r)} \) changes sign, the function \( T^t_t - T^r_r \) is zero, by Eq.(12), at the horizons where \( e^{-\nu} = 0 \). In the regions inside the horizons, the radial coordinate \( r \) is time-like and \( T^t_t \) represents a tension, \( p_r = -T^t_t \), along the axes of the spacelike 3-cylinders of constant time \( r = \text{const} \) [11], then \( T^t_t - T^r_r = -(p_r + \rho) \), and the weak energy condition still demands \( \nu' + \mu' \geq 0 \) there. As a result the function \( \mu + \nu = \text{a function growing from} \mu = \mu(0) \) at \( r = 0 \) to \( \mu = 0 \) at \( r \to \infty \), which gives \( \mu(0) \leq 0 \).

This range of family parameter includes the value \( \mu(0) = 0 \), which corresponds to \( \nu + \mu = 0 \) at the center. In this case the function \( \phi(r) = \nu(r) + \mu(r) \) is zero at \( r = 0 \) and at \( r \to \infty \), its derivative is non-negative, it follows that \( \phi(r) = 0 \), i.e., \( \nu(r) = -\mu(r) \) everywhere. The weak energy condition defines also equation of state and thus asymptotic behaviour as \( r \to 0 \). The function \( \phi(r) = \mu(r) + \nu(r) \), which is equal zero everywhere for \( 0 < r < \infty \), cannot have extremum at \( r = 0 \), therefore \( \mu'(r) + \nu'(r) = 0 \) at \( r = 0 \) (this is easily proved by contradiction using the Maclaurin rule for even derivatives in the extremum). It leads, by using L’Hopital rule in Eq.(12), to \( p_r + \rho = 0 \) at \( r = 0 \). In the limit \( r \to 0 \) Eq.(11) becomes \( p_\perp = -\rho - \frac{2}{r}\rho' \). The energy dominant condition (7) requires \( \rho' \leq 0 \), while regularity of \( \rho \) requires \( p_k + \rho < \infty \) and thus \( |\rho'| < \infty \). Then the equation of state near the center becomes \( p = -\rho \), which gives de Sitter asymptotics as \( r \to 0 \)

\[
ds^2 = \left( 1 - \frac{\rho^2}{r_0^2} \right) dt^2 - \frac{dr^2}{1 - \frac{\rho^2}{r_0^2}} - r^2d\Omega^2 \tag{13}
\]

\[
T_{\mu\nu} = \rho_0 g_{\mu\nu}; \quad \rho_0 = (8\pi G)^{-1} \Lambda; \quad r_0^2 = \frac{3}{\Lambda} \tag{14}
\]

where \( \Lambda \) is the value of cosmological constant at \( r = 0 \).

Summarizing, we conclude that if we require asymptotic flatness, regularity of a density and metric at the center and finiteness of the ADM mass

\[
e^{\nu(r \to 0)} < \infty; \quad \rho(r \to 0) < \infty; \quad m < \infty \tag{15}
\]
then the dominant energy condition defines the family of asymptotically flat solutions with the regular center which includes the class of metrics

\[ e^{u(r)} = e^{-v(r)} = g(r) = 1 - 2GM(r)/r; \quad T^t_t = T^r_r \quad (16) \]

with \( M(r) \) given by Eq. (8), whose behaviour in the origin - asymptotically de Sitter as \( r \to 0 \), is defined by the weak energy condition. Note, that we need the dominant energy condition \( p_k \leq \rho \) only to restrict principal pressures by density whose regularity is postulated. If we postulate regularity also for pressures, then the weak energy condition is enough to distinguish the class of metrics (16) asymptotically de Sitter in the origin, as the member of family of asymptotically flat solutions with the regular center.

For this class the equation of state is

\[ p_r = -\rho: \quad p_\perp = -\rho - (r/2)\rho' \quad (17) \]

The source term connects de Sitter vacuum \( T^\mu_\nu = \rho_0 g_{\mu\nu} \) in the origin with the Minkowski vacuum \( T^\mu_\nu = 0 \) at infinity, and generates de Sitter-Schwarzschild geometry \([15]\) asymptotically de Sitter as \( r \to 0 \) and asymptotically Schwarzschild as \( r \to \infty \).

The weak energy condition \( p_\perp + \rho \geq 0 \) gives \( \rho' \leq 0 \), so that it demands monotonic decreasing of a density profile. By Eq.(6) it leads to the important fact that, except the point \( r = 0 \) where \( g(r) \) has the maximum, in any other extremum \( g'' > 0 \), so that the function \( g(r) \) has in the region \( 0 < r < \infty \) only minimum and the metric (16) can have not more than two horizons.

Indeed, for the metric (16) the Eq.(6) reduces to

\[ 8\pi G p_\perp = -\frac{GM''}{r} \quad (18) \]

The derivative of the mass function \( M(r) \) is always positive since the density is positive (\( M' = 4\pi \rho r^2 \)); the function \( M'(r) \) can have only maximum and only one at the point \( r_c \) where \( p_\perp (r_c) = 0 \) and hence \( M''(r_c) = 0 \) (by Eq.(17) tangential pressure \( p_\perp (r) \) can change sign only once.) At the extremum \( r = r_m \) of the metric function \( g(r) \), Eq.(18) takes the form \( 8\pi G p_\perp (r_m) = g''(r_m)/2 \). In the region \( 0 \leq r < r_c \), the derivative \( M'' > 0 \), and hence for extremal \( r_m \) in this range \( g''(r_m) < 0 \), i.e. in this region there exists only maximum (and only one, this is the maximum at \( r = 0 \)). In the region \( r_c < r \leq \infty \), the second mass derivative is negative, \( M'' < 0 \), and for extremal from this region \( g''(r_m) > 0 \), i.e. metric function \( g(r) \) can have here only minimum (and only one).

To find explicit form of \( M(r) \) we have to choose the density profile leading to the needed behaviour of \( M(r) \) as \( r \to 0 \), \( M(r) \approx (4\pi r^3/3)\rho_0 \). The simplest choice \([16]\]

\[ \rho(r) = \rho_0 e^{-r^3/r_s} = \rho_0 e^{-4\pi r^3/3r_s^3} \quad (19) \]

can be interpreted \([15]\) as due to vacuum polarization in the spherically symmetric gravitational field as described semiclassically by the Schwinger formula \( w \sim e^{-F_{crit}/F} \) (see, e.g., \([17]\)) with tidal forces \( F \sim r_g/r^3 \) and \( F_{crit} \sim 1/r_0^2 \), in agreement with the basic idea suggested by Poisson and Israel that in Schwarzschild-de Sitter transition spacetime geometry can be self-regulatory and describable semiclassically down to a few Planckian radii by the Einstein equations with a source term representing vacuum polarization effects \([11]\). This density profile gives

\[ M(r) = m(1 - e^{-r^3/r_s^3}) \quad (20) \]

The key point is existence of two horizons, a black hole event horizon \( r_+ \) and an internal horizon \( r_- \). A critical value of a mass parameter exists, \( m_{crit} \), at which the horizons come together and which puts a lower limit on a black hole mass \([15]\). For the model (19)

\[ m_{crit} \approx 0.3 m_{Pl} \sqrt{\rho_{Pl}/\rho_0} \quad (21) \]

De Sitter-Schwarzschild configurations are shown in Fig.1.

![FIG. 1. The metric g(r) for de Sitter-Schwarzschild configurations plotted for the case of the density profile (19). The mass m is normalized to m_{crit}.](image-url)
moving through such a medium cannot in principle measure the radial component of his velocity with respect to it), i.e., vacuum with variable energy density and pressures, macroscopically defined by the algebraic structure (23) of its stress-energy tensor $T_{\mu\nu}^{\text{vac}}$ [16]. In the case of nonzero background $\lambda$ it connects in a smooth way two de Sitter vacua with different values of cosmological constant. This makes it possible to interpret $T_{\mu\nu}^{\text{vac}}$ as corresponding to the extension of the algebraic structure of the cosmological term $\Lambda_{\mu\nu}$ from $\Lambda_{\mu\nu} = \Lambda_{g\mu\nu}$ (with $\Lambda = \text{const}$) to an $r$-dependent cosmological term $\Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}^{\text{vac}}$, evolving from $\Lambda_{\mu\nu} = \Lambda_{g\mu\nu}$ as $r \to 0$ to $\Lambda_{\mu\nu} = \lambda_{g\mu\nu}$ as $r \to \infty$, and satisfying the equation of state (17) with $8\pi G \rho_\lambda = \Lambda_{\mu\nu}$, $8\pi G p_\lambda = -\Lambda_{\mu\nu}$, and $8\pi G p_\perp = -\Lambda_{\mu\nu}$ [19].

In this paper we concentrate on de Sitter-Schwarzschild geometry (16) generated by a cosmological term $\Lambda_{\mu\nu}$ evolving from the de Sitter vacuum $\Lambda_{\mu\nu} = \Lambda_{g\mu\nu}$ at $r = 0$ to the Minkowski vacuum $\Lambda_{\mu\nu} = 0$ at infinity.

For $m \geq m_{\text{crit}}$ de Sitter-Schwarzschild geometry describes the vacuum nonsingular black hole (\text{ABH}) [16], and global structure of spacetime, shown in Fig. 2 [15], contains an infinite sequence of black and white holes whose future and past singularities are replaced with regular cores $\mathcal{R}$ asymptotically de Sitter as $r \to 0$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Penrose-Carter diagram for $\Lambda$ black hole.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Temperature-mass diagram for $\Lambda$ black hole.}
\end{figure}

A \text{ABH} emits Hawking radiation from both horizons (see Fig. 3), and configuration evolves towards a self-gravitating particlelike structure without horizons (see Fig. 1). While a \text{ABH} loses mass, horizons come together and temperature drops to zero [15]. The Schwarzschild asymptotics requires $T_+ \sim m^{-1} \to 0$ as $m \to \infty$. The temperature $T_+$ on BH horizon $r_+$ is positive by general laws of BH thermodynamics [8]. As a result the temperature-mass diagram has a maximum between $m_{\text{crit}}$ and $m \to \infty$ [15]. In a maximum a specific heat is broken and changes sign testifying to a second-order phase transition in the course of Hawking evaporation (and suggesting symmetry restoration in the origin [20]).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{G-lump in the case $r_0 = 0.1r_0$ ($m \simeq 0.06m_{\text{crit}}$).}
\end{figure}

For masses $m < m_{\text{crit}}$ de Sitter-Schwarzschild geometry describes a self-gravitating particlelike vacuum structure without horizons, globally regular and globally neutral. It resembles Coleman’s lumps - non-singular, non-dissipative solutions of finite energy, holding themselves together by their own self-interaction [21]. Our lump is regular solution to the Einstein equations, perfectly localized (see Fig. 4) in a region where field tension and energy are particularly high (this is the region of the former singularity), so we can call it G-lump.
It holds itself together by gravity due to balance between gravitational attraction outside and gravitational repulsion inside of zero-gravity surface \( r = r_c \) beyond which the strong energy condition of singularities theorems \([7]\), \((T_{\mu\nu} - T g_{\mu\nu}/2)\xi^\mu\xi^\nu \geq 0\), is violated \([15]\). The surface of zero gravity is defined by \( 2\rho + r\rho' = 0 \).

It is depicted in Fig.5 together with horizons and with the surface \( r = r_s \) of zero scalar curvature \( R(r_s) = 0 \) which in the case of the density profile (19) is given by

\[
r_s = \left( \frac{4}{3}\pi^2 r_g^3 \right)^{1/3} = \left( \frac{m}{\pi\rho_0} \right)^{1/3}
\]

and confines about 3/4 of the mass \( m \).

The mass of both G-lump and ABH is directly connected to cosmological term \( \Lambda_{\mu\nu} \) by the ADM formula (9) which in this case reads

\[
m = (2G)^{-1} \int_0^\infty \Lambda_{\mu\nu}^i(r)r^2 dr
\]

and relates mass to the de Sitter vacuum at the origin.

The Minkowski geometry allows existence of inertial mass as the Lorentz invariant \( m^2 = p_\mu p^\mu \) of a test body. High symmetry of this geometry allows both existence of inertial frames and of quantity \( m \) as the measure of inertia, but geometry tells nothing about this quantity.

In the Schwarzschild geometry the parameter \( m \) is responsible for geometry, it is identified as a gravitational mass of a source by asymptotic behavior of the metric at infinity (see, e.g., \([8]\), p.124). By the equivalence principle, gravitational mass is equal to inertial mass (see, e.g., \([2]\), Ch.4). The inertial mass is represented thus by a purely geometrical quantity, the Schwarzschild radius \( r_g \) which is just geometrical fact \([22]\), but which does not tell yet anything about origin of a mass.

In de Sitter-Schwarzschild geometry the parameter \( m \) is identified as a mass by Schwarzschild asymptotics at infinity. The geometrical fact of this geometry is that a mass is related to cosmological term, since Schwarzschild singularity is replaced with a de Sitter vacuum. The operation of introducing mass by the ADM formula (9) is impossible in the de Sitter geometry. The reason is that symmetry of the source term \( T_{\mu\nu} = \rho_0 g_{\mu\nu} = (8\pi G)^{-1} \Lambda_{\mu\nu} \) is too high. It implies \( \rho_{vac} = \text{const} \) by virtue of the Bianchi identities \( G^\mu_{\nu} = 0 \). In the case of geometry generated by the cosmological term \( \Lambda_{\mu\nu} \), symmetry of a source term is reduced from the full Lorentz group to the Lorentz boosts in the radial direction only. Together with asymptotic flatness this allows introducing a distinguished point as the center of an object whose ADM mass is defined by the formula (25). The reduced symmetry of a source and the asymptotic flatness of geometry are responsible for mass of an object given by (25).

This picture seems to be in remarkable conformity with the basic idea of the Higgs mechanism for generation of mass via spontaneous breaking of symmetry of a scalar field vacuum from a false vacuum (where \( T_{\mu\nu} = V(0)g_{\mu\nu} \), and \( p = -\rho \)), to a true vacuum \( T_{\mu\nu} = 0 \). In both cases de Sitter vacuum is involved and vacuum symmetry is broken. Even graphically the gravitational potential \( g(r) \) resembles a Higgs potential (see Fig.6).

The difference is that in case of a mass coming from \( \Lambda_{\mu\nu} \) by (25), the gravitational potential \( g(r) \) is generic, and de Sitter vacuum supplies a particle with mass via smooth breaking of space-time symmetry from the de Sitter group in its center to the Lorentz group at its infinity. This leads to the natural assumption \([23]\) that whatever would be particular mechanism for mass generation, a fundamental particle (which does not display substructure, like a lepton or quark) may have an internal vacuum core (at the scale where it gets mass) related to its mass and a geometrical size defined by gravity. Such a
core with de Sitter vacuum at the origin and Minkowski vacuum at infinity can be approximated by de Sitter-Schwarzschild geometry. Characteristic size in this geometry is given by (24). It depends on vacuum density at \( r = 0 \) and presents modification of the Schwarzschild radius \( r_g \) to the case when singularity is replaced by de Sitter vacuum. While application of the Schwarzschild radius to elementary particle size is highly speculative since obtained estimates are many orders of magnitude less than \( l_{Pl} \), the characteristic size \( r_s \) gives reasonable numbers (e.g., \( r_s \sim 10^{-18} \) cm for the electron getting its mass from the vacuum at the electroweak scale) close to estimates obtained in experiments (see Fig.7 [23] where they are compared with electromagnetic (EM) and electroweak (EW) experimental limits).

A spherical bubble can be described by the minisuperspace model with a single degree of freedom [25]. The momentum operator is introduced by \( \hat{p} = -i\hbar d/d\tau \), and the equation (27) transforms into the Wheeler-DeWitt equation in the minisuperspace [25] which reduces to the Schrödinger equation

\[
\frac{\hbar^2}{2m_{Pl}} \frac{d^2 \psi}{dr^2} - (V(r) - E)\psi = 0
\]

with the potential (in the Planckian units)

\[
V(r) = -\frac{GM(r)}{r}
\]

depicted in Fig.8.

Near the minimum \( r = r_m \) the potential takes the form \( V(r) = V(r_m) + \frac{4\pi G\rho_{\perp}(r_m)}{r_m}(r - r_m)^2 \). Introducing the variable \( x = r - r_m \) we reduce Eq.(28) to the equation for a harmonic oscillator

\[
\frac{d^2 \psi}{dx^2} - \frac{m_{Pl}^2\omega^2 x^2}{\hbar^2} \psi + \frac{2m_{Pl}\tilde{E}}{\hbar^2} \psi = 0
\]

where \( \tilde{E} = E - V(r_m) \), \( \omega^2 = \Lambda c^2 \tilde{\rho}_{\perp}(r_m) \), and \( \tilde{\rho}_{\perp} \) is the dimensionless pressure normalized to vacuum density \( \rho_0 \) at \( r = 0 \); for the density profile (19) \( \tilde{\rho}_{\perp}(r_m) \approx 0.2 \). The energy spectrum

\[
E_n = \hbar \omega \left(n + \frac{1}{2}\right) - \frac{GM(r_m)}{r_m} E_{Pl}
\]

is shifted down by the minimum of the potential \( V(r_m) \) which represents the binding energy. The energy of zero-point vacuum mode

\[
\tilde{E}_0 = \frac{\sqrt{3}\tilde{\rho}_{\perp} \hbar c}{2r_0}
\]
In the spherically symmetric case the ADM mass represents the energy of virtual particles which could become real in the presence of the horizon. In the case of G-lump which is structure without horizons, kind of gravitational vacuum exciton, they are confined by the binding energy $V(r_m)$.

The question of stability of de Sitter-Schwarzschild configurations is currently under investigation. De Sitter-Schwarzschild black hole configuration obtained by direct matching of the Schwarzschild metric outside to de Sitter metric inside of a spacelike three-cylindrical short transitional layer is a stable configuration in a sense that the three-cylinder does not tend to shrink down under perturbations. De Sitter-Schwarzschild configurations considered above represent general case of a smooth transition with a distributed density profile. The heuristic argument in favour of their stability comes from comparison of the ADM mass with the proper mass $\mu = 4\pi \int_0^\infty \rho(r) \left(1 - \frac{2GM(r)}{r}\right)^{-1/2} r^2 dr$.

In the spherically symmetric case the ADM mass represents the total energy, $m = \mu +$-binding energy. In de Sitter-Schwarzschild geometry $\mu$ is bigger than $M$. This suggests that the configuration might be stable since energy is needed to break it up. Analysis of stability of a ABH as an isolated system by Poincare’s method, with the total energy $m$ as a thermodynamical variable and the inverse temperature as the conjugate variable, shows immediately its stability with respect to spherically symmetric perturbations. The analysis by Chandrasekhar method is straightforward for a ABH stability to external perturbations, in close similarity with the Schwarzschild and Reissner-Nordström cases. The potential barriers in one-dimensional wave equations governing perturbations, external to the event horizon, are real and positive, and stability follows from this fact.

Preliminary results suggest stability also for the case of G-lump. In the context of catastrophe-theory analysis, de Sitter-Schwarzschild configuration resembles high-entropy neutral type in the Maeda classification, in which a non-Abelian structure may be approximated as a uniform vacuum density $\rho_{vac}$ within a sphere whose radius is the Compton wavelength of a massive non-Abelian field, and self-gravitating particle approaches the particle solution in the Minkowski space.

Discussion - The main result of this paper is that there exists the class of globally regular solutions to the minimally coupled GR equations (4)-(6), with the algebraic structure of a source term (23), interpreted as spherically symmetric vacuum with variable density and pressure $T_{\mu\nu}^{vac}$ associated with a variable cosmological term $\Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}^{vac}$, whose asymptotics in the origin, dictated by the weak energy condition, is the Einstein cosmological term $\Lambda g_{\mu\nu}$. For this class the mass defined by the standard ADM formula (9) is related (generically, since matter source can be any from the considered class) to both de Sitter vacuum trapped in the origin and to breaking of space-time symmetry.

In the inflationary cosmology which is based on generic properties of de Sitter vacuum $\Lambda g_{\mu\nu}$ independently on where $\Lambda$ comes from, several mechanisms are investigated relating $\Lambda g_{\mu\nu}$ to matter sources (for review see [33]). Most frequently considered is a scalar field

$$S = \int d^4x \sqrt{-g} \left[R + (\partial \phi)^2 - 2V(\phi)\right]$$

where $R$ is the scalar curvature, $(\partial \phi)^2 = g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi$, with various forms for a scalar field potential $V(\phi)$.

The question whether a regular black hole can be obtained as a false vacuum configuration described by (33), has been addressed in the paper [34], where “the no-go theorem” has been proved: Asymptotically flat regular black hole solutions are absent in the theory (33) with any non-negative potential $V(\phi)$. This result has been extended to the case of any $V(\phi)$ and any asymptotics and then generalized to the case of a theory with the action $S = \int d^4x \sqrt{-g} \left[R + F[(\partial \phi)^2, \phi]\right]$, where $F$ is an arbitrary function, to the multi-scalar theories of sigma-model type, and to scalar-tensor and curvature-nonlinear gravity theories. It has been shown that the only possible regular solutions are either de Sitter-like with a single cosmological horizon or those without horizons, including asymptotically flat ones. The latter do not exist for $V(\phi) \geq 0$, so that the set of causal false vacuum structures is the same as known for $\phi = const$ case, namely Minkowski (or anti-de Sitter), Schwarzschild, de Sitter, and Schwarzschild-de Sitter and does not include de Sitter-Schwarzschild configurations. However, as it was noted three years before formulating “no-go theorems”, in the case of complex massive scalar field the regular structures can be obtained in the minimally coupled theory with positive $V(\phi)$.

In a recent paper on A-variability, Overduin and Cooperstock distinguished two basic approaches to $\Lambda g_{\mu\nu}$ existing in the literature. In the first approach $\Lambda g_{\mu\nu}$ is shifted onto the right-hand side of the field Einstein equations (2) and treated as a dynamical part of the matter content. This approach, characterized by Overduin and Cooperstock as being connected to dialectic materialism of the Soviet physics school, goes back to Gliner who interpreted $\Lambda g_{\mu\nu}$ as vacuum stress-energy tensor, to Zeldovich who connected $\Lambda$ with the gravitational interaction of virtual particles inhabiting vacuum, and
to Linde who suggested that $\Lambda$ can vary in principle [41]. In contrast, idealistic approach prefers to keep $\Lambda$ on the left-hand side of Eqs.(2) as geometrical entity and treat it as a constant of nature [42].

This classification suggests that any variable $\Lambda$ must be identified with a matter, in such a case the best fit for $T_{\mu \nu}^{\text{vac}} = (8\pi G)^{-1}\Lambda_{\mu \nu}$ would be gravitational vacuum polarization in the spirit of Zel’dovich’s idea [40] and Poisson and Israel self-regulatory picture [11]. On the other hand nothing prevents from shifting $\pi G T_{\mu \nu}^{\text{vac}}$ from the right-hand to the left-hand side of Eqs.(1) and treating $\Lambda_{\mu \nu} = 8\pi GT_{\mu \nu}^{\text{vac}}$ as evolving geometrical entity. (The Einstein field equations (1) can be written in the four-indices form $G_{\alpha \beta \gamma \delta} = -8\pi GT_{\alpha \beta \gamma \delta}$ as the equivalence relations which put the matter and geometry in direct algebraic correspondence [43]).

The considered connection between $\rho$-dependent cosmological term $\Lambda_{\mu \nu}$ and the ADM mass seems to satisfy Einstein’s version of Mach’s principle - no matter, no inertia - if we explicitly separate two aspects of the problem of inertia: existence of inertial frames and existence of inertial mass. In empty space, $T_{\mu \nu} = 0, \Lambda_{\mu \nu} = 0$, inertial frames exist due to high symmetry of Minkowski geometry, but to prove it we need a measure of inertia, a region in space where Minkowski vacuum is a little bit disturbed. When a mass comes from cosmological term, it is disturbed by $\Lambda_{\mu \nu} \neq 0$. In other words, full symmetry of Minkowski spacetime is responsible for existence of inertial frames, while its breaking to Lorentz boosts in the radial direction only is responsible for inertial mass.

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[34] D.V.Gal'tsov and J.P.S.Lemos, Class. Quant. Grav. **18** 1715 (2001)
[37] F.E.Schunck, astro-ph/9802258
[42] It looks that the question of where to put $\Lambda_{\mu \nu}$ is kind of philosophical question of what is primary. If we remind that dialectic materialism is nothing but application of Hegel’s dialectic idealism to matter, then one is tempted to approach $\Lambda_{\mu \nu}$ by the Hegel’s laws of a new triad, as a new quality ether appearing in the new turn of cognitive helix, a Lorentz-invariant ether with respect to which one cannot measure velocity in principle.