Lepton masses in a supersymmetric 3-3-1 model

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We consider the mass generation for both charginos and neutralinos in a 3-3-1 supersymmetric model. We show that R-parity breaking interactions leave the electron and one of the neutrinos massless at the tree level. However the same interactions induce masses for these particles at the 1-loop level. Unlike the similar situation in the MSSM the masses of the neutralinos are related to the masses of the charginos.

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I. INTRODUCTION

The generation of neutrino masses is an important issue in any realistic extension of the standard model. In general, the values of these masses (of the order of, or less than, 1 eV) that are needed to explain all neutrino oscillation data [1–3] are not enough to put strong constraints on model building. It means, several models can induce neutrino masses and mixing compatible with experimental data. So, instead of try to propose models built just to explain the neutrino properties it is more useful to consider what are the neutrino masses that are predicted in any particular model which has motivation other than the explanation of neutrino physics. For instance, the 3-3-1 model was proposed as a possible symmetry on the lightest lepton sector (ν,e−,e+)L [4]. Once assumed that symmetry it has to be implemented in the rest of the leptons and also in the quark sector. Like in the standard model if we do not introduce right-handed neutrinos and/or violation of the total lepton number neutrinos remain massless at any order in perturbation theory. In this vain it has been done some effort to produce neutrino masses in the context of that 3-3-1 model and some of its extensions [5].

In this work we consider the generation of neutrino masses in a supersymmetric 3-3-1 model with broken R parity. We show that, as an effect of the mixing among all leptons of the same charge, at the tree level only one charged lepton and one neutrino remain massless but they gain mass through radiative corrections. In order to compare this model we do the same calculations in the context of the minimal supersymmetric standard model (MSSM) with R broken parity also. In both cases we are not assuming that sneutrinos gain non-vanishing vacuum expectation values (VEVs) i.e., the only non-zero VEVs are those of the scalars of the non-supersymmetric models.

The outline of this work is as follows. In Sec. II we review the origin of the lepton masses in the MSSM context under the same assumptions that we will use in the case of the 3-3-1 supersymmetric model. In Sec. III we consider the supersymmetric version of a 3-3-1 model which has only three triplets of Higgs scalars. We explicitly show that leptons gain mass only as a consequence of their mixing with charginos and neutralinos. Our conclusions are found in the last section.

II. NEUTRINO MASSES IN THE MSSM

Let us consider in this section the lepton masses in the minimal supersymmetric standard model (MSSM) [9]. In this model the interactions are written in terms of the left-handed (right-handed) \( L \sim (2, -1) \) (\( l^c \sim (1, 2) \)) leptons, left-handed (right-handed) quarks \( \bar{Q} \sim (2, 1/3) \) (\( \bar{u}^c \sim (1, -4/3), \bar{d}^c \sim (1, 2/3) \)); and the Higgs doublets \( H_1 \sim (2, -1), H_2 \sim (2, 1) \). With those multiplets the superpotential which has R-parity conserving contributions is given by \( W_{2RC} + W_{3RC} + W_{2RC} + W_{3RC} \), where

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and we have suppressed $SU(2)$ indices, $\epsilon$ is the antisymmetric $SU(2)$ tensor. Above and below in the following, the subindices $a, b, c$ run over the lepton generations $e, \mu, \tau$ but a superscript $\tilde{c}$ indicates charge conjugation; $i, j, k = 1, 2, 3$ denote quark generations. We have also to add the soft terms that break the supersymmetry:

\[
\mathcal{L}_{\text{soft}} = -\frac{1}{2} \left( \sum_{p=1}^{3} m_p \lambda_p \hat{\lambda}_p + m' \lambda_B \hat{\lambda}_B + H.c. \right) - M_d^2 \tilde{l}^\dagger \tilde{l} - M_{d}^2 \tilde{\nu}^\dagger \tilde{\nu} - M_{\nu}^2 \hat{\nu}^\dagger \hat{\nu} - \left[ A_L H_1 \tilde{l} \tilde{c} + A_U H_2 \tilde{\nu}^c + A_D H_3 \hat{\nu}^c \right. \\
+ M^2_{\text{soft}} H_1 H_2 + B H_2 L + C_1 \tilde{l} \tilde{l}^c + C_2 \tilde{\nu} \tilde{\nu}^c + C_3 \hat{\nu} \hat{\nu}^c + H.c. \bigg] ,
\]

where $p$ is an $SU(2)$ index and $\lambda_p, \lambda_B$ are the supersymmetric partners of the respective gauge vector bosons but we have omitted generation indices and the gluinos masses.

With the interactions in Eq. (1) it is possible to give mass to all charged fermions in the model but neutrino masses generated by radiative corrections arisen from the interactions [12,14–16]. Here we will only consider the neutrino mass term in Eq. (2) must be set to zero in order to avoid a too fast proton decay. In Eqs. (1) and (2) we can define the basis $\Psi^0_{\text{MSSM}} = (\nu_{\tau}, \nu_{\mu}, \nu_{e}, -i\lambda^\dagger, -i\lambda_B, H^0_1, H^0_2)^T$, and the mass term is $-(1/2)[\Psi^T_{\text{MSSM}} Y^0_{\text{MSSM}} \Psi^0 + H.c.]$, where $Y^0_{\text{MSSM}}$ is the mass matrix

\[
Y^0_{\text{MSSM}} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -\mu_{0e} \\
0 & 0 & 0 & 0 & 0 & -\mu_{0\mu} \\
0 & 0 & 0 & 0 & 0 & -\mu_{0\tau} \\
0 & 0 & 0 & m_\lambda & 0 & M_Z s_{\beta} c_{\beta} \\
0 & 0 & 0 & m' & 0 & M_Z s_{\beta} s_{\beta} \\
-\mu_{0e} & -\mu_{0\mu} & -\mu_{0\tau} & -M_Z c_{\beta} c_{\beta} & M_Z s_{\beta} c_{\beta} & M_Z c_{\beta} s_{\beta} & M_Z s_{\beta} s_{\beta} & M_Z c_{\beta} s_{\beta} & M_Z s_{\beta} s_{\beta} & M_Z c_{\beta} s_{\beta} & M_Z s_{\beta} s_{\beta} & M_Z c_{\beta} s_{\beta} & M_Z s_{\beta} s_{\beta} & M_Z c_{\beta} s_{\beta} & M_Z s_{\beta} s_{\beta} & M_Z c_{\beta} s_{\beta}
\end{pmatrix}
\]

with $s_\beta = \sin \beta$, $s_W = \sin \theta_W$, etc are defined as $\tan \beta = v_2/v_1$ and $\theta_W$ is the weak mixing angle. The matrix in Eq. (4) is generated only by the two usual vacuum expectation values of the two scalars and by the $R$-parity breaking terms $\mu_{0a}$. The mass matrix is similar to that in Ref. [10–12] but we have included the three neutrinos and we are neither assuming that sneutrinos gain nonzero vacuum expectation values nor have introduced sterile neutrinos like in Ref. [13]. The mass matrix in Eq. (4) has two zero eigenvalues: it has determinant equal to zero and its secular equation which give the eigenvalues, $x$, has the form $x^2$ times a polynomial of five degree; thus there are two neutrinos $\nu_{1,2}$, which are massless at the tree level. Using $\tan \beta = 1$ and $M_Z = 91.187$ GeV, $s_W^2 = 0.223$, $\mu_{0e} = \mu_{0\mu} = 0$, $\mu_{0\tau} = 10^{-4}$ GeV (this value is consistent with that of Ref. [11]), $\mu = 100$ GeV, $m = 250$ GeV, $m' = -200$ GeV, we obtain besides the two massless neutrinos a massive one with $\mu_{0e} = -3 \times 10^{-3}$ eV, and four heavy neutralinos with masses 267.40, -199.99, -117.40, 100.0 GeV. These zero eigenvalues are a product of the matrix structure in Eq. (4) and there is not a symmetry to protect the neutrinos to gain mass by radiative corrections. On the other hand, if $\mu_{0a} = 0$, $a = e, \mu, \tau$, all neutrinos remain massless at the tree level. In this case it is the $R$-parity and total lepton number conservation that protect neutrinos of gain masses. The neutralino masses above are consistent with those of Ref. [11]: two states are massless and the other ones have masses of the order $O(M_Z)$. More realistic neutrino masses require radiative corrections [12,14–16]. Here we will only consider the neutrino masses generated by radiative corrections arisen from the interactions given in Eqs. (1) and (2) and only two VEVs. We have in this case the interactions

\[
-\frac{\lambda_{abc}}{3} \left( \bar{\nu}_{aL} \hat{b}_R \tilde{l}_c + \bar{\nu}_{aR} \hat{l}_R \tilde{b}_L \right)
\]
the 1-loop diagrams like those in Ref. [10] arise. Notice however that if we introduce a discrete symmetry (called $Z_3$ later on), $L_{e,\mu} \rightarrow -L_{e,\mu}$, and all other fields are even under this transformation, we have that

$$\mu_0 = 0;$$

$$\lambda_{abc} = 0, \ (b, c = \mu, \tau); \ \lambda_{\mu c} = 0, \ (b, c = \mu, \tau);$$

$$X'_{\mu ij} = X'_{ij}, \ (i, j = 1, 2, 3);$$

and the $\nu_{e, \tau}$ neutrinos will remain massless at all order in perturbation theory. It is also possible to choose the symmetry such as $L_{e, \tau} \rightarrow -L_{e, \tau}$ and all other fields being even. In this case we have that $\nu_\tau$ remain massless. However, if no discrete symmetry is imposed neutrinos gain mass through 1-loop effect like in Ref. [10].

$$X_{\text{MSSM}} = \begin{pmatrix}
-f_{\tau \mu}^1 v_1 & -f_{\tau e}^1 v_1 & -f_{\tau \tau}^1 v_1 & 0 & 0 \\
-f_{\tau \mu}^1 v_1 & -f_{\tau e}^1 v_1 & -f_{\tau \tau}^1 v_1 & 0 & 0 \\
-f_{\tau \mu}^1 v_1 & -f_{\tau e}^1 v_1 & -f_{\tau \tau}^1 v_1 & 0 & 0 \\
0 & 0 & 0 & m_\chi & \sqrt{2}M_{Wc_\beta} \\
\mu_{0e} & \mu_{0\mu} & \mu_{0\tau} & \sqrt{2}M_{Wc_\beta} & \mu
\end{pmatrix}. \tag{9}$$

With $f_{\tau e}^{1} = 2.7 \times 10^{-4}$, $f_{\tau \mu}^{1} = 3.9 \times 10^{-3}$, $f_{\tau \tau}^{1} = 1.6 \times 10^{-2}$, $f_{\mu \mu}^{f} = f_{\tau \tau}^{f} = 10^{-7}$ we obtain from Eq. (9) the masses 0.0005, 0.105, 1.777 in (GeV) for the usual leptons, and 4.3 and 81 TeV for the charginos. We see by comparing Eq. (4) with Eq. (9) that there is no relation between the charged lepton masses and the neutralino masses. Notice also that all charged leptons gain masses at the tree level. We will not consider this model (or some of its extensions) further since it has been well studied in literature [10–12,14–16].

### III. A SUPERSYMMETRIC 3-3-1 MODEL

In the nonsupersymmetric model the fermionic representation content is as follows: left-handed leptons $L = (\nu_0, l_0, \nu_3)_L \sim (1, 3, 0)$, $a = e, \mu, \tau$; left-handed quarks $Q_{1L} = (u_1, d_1, J) \sim (3, 3, 2/3)$, $Q_{2L} = (d_1, u_1, j_3) \sim (3, 3^*, -1/3)$, $i = 2, 3$, $a = 1, 2$; and in the right-handed components we have $u^c, d^c$ that transform as in the SM, and the exotic quarks $J^c \sim (3^*, 1, 5/3), j_3 \sim (3^*, 1, 4/3)$. The minimal scalar representation content is formed by three scalar triplets: $\eta \sim (1, 3, 0)$, $\rho \sim (1, 3, +1)$, $\chi \sim (1, 3, -1)$, and one scalar sextet $S \sim (1, 6, 0)$. In particular the $\eta$ triplet has the following electric charge assignment: $\eta = (\eta^0, \eta^1, \eta^2)^T$. We can avoid the introduction of the sextet by adding a charged lepton transforming as a singlet [18,19]. Notwithstanding, here we will omit both the sextet and the exotic lepton. A seesaw-type mechanism will be implemented by the mixing with supersymmetric partners, higgsinos or neutralinos. The complete set of fields in the 3-3-1 supersymmetric model has been given in Refs. [17,20]. For simplicity we will use the notation $(\tilde{\eta}^0) = u, (\rho^0) = v$ and $(\chi^0) = w$ and similarly for the primed fields. We will denote, like in the previous section, the respective superfields as $\tilde{L}$ and so on. We recall that in the nonsupersymmetric 3-3-1 model with only the three triplets the charged lepton masses are not yet the physical ones: 0, $m$, $-m$.

We will show how, in the present model it is a supersymmetry effect that gives the correct masses to these particles, $e, \mu, \tau$, even without a sextet or the charged lepton singlet, in the supersymmetric version we have the higgsinos $\tilde{\eta}, \tilde{\rho}, \tilde{\chi}$ and their respective prime fields. In particular the $\tilde{\eta}$ triplet has the following electric charge assignment: $\tilde{\eta} = (\tilde{\eta}^0, \tilde{\eta}^1, \tilde{\eta}^2)^T$. Due to the fact that in the supersymmetric model we have the higgsinos, (for details on the lagrangian of the model see [17]), when the $R$-parity is broken we have, in analogy with the MSSM, a mixture between the usual leptons and the higgsinos. The goal of this paper is to show that this is indeed the case.

One part of the superpotential is given by $W_2 + \bar{W}_2$ where

$$W_2 = \frac{\mu_{0a}}{2} \tilde{L}_a \tilde{\eta}^0 + \mu_0 \tilde{\eta}^0 + \mu_0 \tilde{\rho}^0 + \mu_0 \tilde{\chi}^0,$$  

$$a = e, \mu, \tau; \text{ and } W_3 + \bar{W}_3 \text{ where}$$
we have omitted the respective indices; the generation indices are as follows: a, b, c = e, μ, τ and i, j, k = 1, 2, 3 as in the previous section; on the other hand, since we have now two quark generations transforming differently under SU(3) L and also exotic charged -4/3 quarks, we have to extend the notation appropriately: α = 2, 3 and β = 1, 2. The gaugino masses come from the soft-terms shown in the Appendix, Eq. (A4). The μ0, λ1, λ4, λ1′ and λ1′′ terms break the R-parity defined in this model as \( R = (-1)^{3F+28} \) where F = B + L, B(L) is the baryon (total lepton) number; S is the spin. The λ2 term of the superpotential \( W_3 \) implies interactions like (see Eq. (42) below) \( ν_α L^2 R_{b}^T \) and we have also the interactions

\[
\mathcal{L}_η = \int d^4 \tilde{η} \tilde{η} c^2 V \tilde{η},
\]

where \( V \) is the superfield related to the \( V^a \) gauge boson of SU(3) L. This interaction mixes higgsinos with gauginos as showed in Ref. [20].

The parameters \( \mu_0,\tilde{\eta}_i \) and \( \mu_ρ \) are the equivalent of the \( \mu \) parameter in the MSSM [9]. The terms proportional to \( \lambda_2 \) and \( \mu_χ \) have no equivalent in the MSSM. The \( λ' \) and \( \lambda'' \) are such that

\[
λ''_{11} \lambda''_{22} < 10^{-24},
\]

assuming the superpartner masses in the range of 1 TeV [21].

### A. Charged lepton masses

Let us first considered the charged lepton masses. There are interactions like

\[
-\frac{λ_{4a}}{3} \left[ ω(l_a \tilde{ρ}^+ + \tilde{l}_a \tilde{ρ}^+) + u(l_a \tilde{χ}^- + \tilde{l}_a \tilde{χ}^-) \right] - \frac{1}{2} μ_{0a} [l_\tilde{η}_1^+ + l_\tilde{η}_2^- + l_\tilde{η}_2^- + l_\tilde{η}_2^-],
\]

which imply a general mixture in both neutral and charged sectors. Denoting

\[
\phi^+ = (e^c, μ^c, τ^c, -i l_\tilde{η}_1^c, -i l_\tilde{η}_2^c, \tilde{η}_1^c, \tilde{η}_2^c, \tilde{η}_2^c, \tilde{η}_2^c, \tilde{ρ}^+, \tilde{χ}^+)^T, \]

\[
\phi^- = (e, μ, τ, -i l_\tilde{η}_1^c, -i l_\tilde{η}_2^c, \tilde{η}_1^c, \tilde{η}_2^c, \tilde{η}_2^c, \tilde{η}_2^c, \tilde{ρ}^-, \tilde{χ}^-)^T,
\]

where all the fermionic fields are still Weyl spinors, we can also, as before, define \( Ψ^± = (φ^+φ^-)^T \), and the mass term \( -(1/2)[Ψ^+TY^±Ψ^± + H.c.] \) where \( Y^± \) is given by:

\[
Y^± = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix},
\]

with

\[
X = \begin{pmatrix} 0 & -\frac{λ_{4a}}{3} u & -\frac{λ_{4a}}{3} u & 0 & 0 & -\frac{μ_0}{2} & 0 & -\frac{λ_2}{2} w & 0 \\ \frac{λ_{4a}}{3} u & 0 & -\frac{λ_{4a}}{3} u & 0 & 0 & -\frac{μ_0}{2} & 0 & -\frac{λ_2}{2} w & 0 \\ \frac{λ_{4a}}{3} u & 0 & -\frac{λ_{4a}}{3} u & 0 & 0 & -\frac{μ_0}{2} & 0 & -\frac{λ_2}{2} w & 0 \\ 0 & 0 & 0 & m_λ & 0 & -g_u & 0 & g_u & 0 \\ 0 & 0 & 0 & g_v & 0 & -g_u & 0 & g_u & 0 \\ 0 & 0 & 0 & g_v & 0 & -g_u & 0 & g_u & 0 \\ -\frac{μ_0}{2} & 0 & 0 & 0 & 0 & -g'_u & 0 & -g'_u & 0 \\ -\frac{μ_0}{2} & 0 & 0 & 0 & 0 & -g'_u & 0 & -g'_u & 0 \\ -λ_2 & 0 & 0 & 0 & 0 & -g'_u & 0 & -g'_u & -\frac{μ_0}{2} \end{pmatrix},
\]

The chargino mass matrix is diagonalized using two unitary matrices, \( D \) and \( E \), defined by

\[
\tilde{χ}_i^+ = D_{ij} \Psi_j^+, \quad \tilde{χ}_i^- = E_{ij} \Psi_j^-; \quad i, j = 1, \cdots, 9,
\]

(\( D \) and \( E \) sometimes are denoted, in non-supersymmetric theories, by \( U_R \) and \( U_L \), respectively). Then we can write the diagonal mass matrix as

\[
M_{SCM} = E^* X D^{-1}.
\]

To determine \( E \) and \( D \), we note that

\[
M_{SCM}^2 = DX^* \cdot XD^{-1} = E^* X \cdot X^* (E^*)^{-1},
\]

and define the following Dirac spinors:

\[
Ψ(\tilde{χ}_i^+) = \begin{pmatrix} \tilde{χ}_i^+ \\ \tilde{χ}_i^- \end{pmatrix}^T, \quad Ψ(\tilde{χ}_i^-) = \begin{pmatrix} \tilde{χ}_i^- \tilde{χ}_i^+ \end{pmatrix}^T,
\]

where \( \tilde{χ}_i^+ \) is the particle and \( \tilde{χ}_i^- \) is the anti-particle [9,20].
We have obtained the following masses (in GeV) for the charged sector:

\[ m_e = 0, \quad m_\mu = 0.1052 \text{ and } m_\tau = 1.777. \]

These values have been obtained by using the following values for the dimensionless parameters

\[ \lambda_{2e\mu} = 0.001, \quad \lambda_{2e\tau} = 0.001, \quad \lambda_{2\mu\tau} = 0.393, \]

\[ \lambda_{3e} = 0.0001, \quad \lambda_{4\mu} = 1.0, \quad \lambda_{4\tau} = 1.0, \]

\[ f_1 = 0.254, \quad f_3 = f'_1 = f'_3 = 1.0, \] \hspace{1cm} (23)

and for the mass dimension parameters (in GeV) we have used:

\[ \mu_e = \mu_\mu = 0.0; \quad \mu_\tau = 10^{-6}, \quad \mu_\eta = 300, \]

\[ \mu_\rho = 500, \quad \mu_\chi = 700, \quad m_\lambda = 3000. \] \hspace{1cm} (24)

We also use the constraint \( V_n^2 + V_\rho^2 = (246 \text{ GeV})^2 \) coming from \( M_W \), where, we have defined \( V_\eta^2 = v_\eta^2 + v_0^2 \) and \( V_\rho^2 = v_\rho^2 + v_\rho^2 \). Assuming that \( v_\eta = 20, v_\chi = 1000, v_\rho = v_\rho = 1 \) and \( v_\chi = 2000 \) in GeV, the value of \( v_\rho \) is fixed by the constraint above.

Notice, from Eq. (22), that the electron is massless at the tree level. This is again a result of the structure of the mass matrix in Eq. (9) and there is not a symmetry that protects the electron to get a mass by loop corrections. Hence, it can gain mass trough radiative corrections like that shown in Fig. 1. The interactions of the leptons with the sleptons written in term of Dirac fermions (although we are using the same notation) are given by (and the respective hermitian conjugate)

\[
\mathcal{L}^i = \frac{\lambda_{2ab}}{3} \left[ \bar{\nu}_R(l_b \nu_i - l_a \nu_b + l_a \nu_b - l_b \nu_i) \right] + \frac{\lambda_{3ab}}{3} \left[ \bar{\nu}_R(l_a \nu_i - l_i \nu_a - l_i \nu_a + l_a \nu_i) \right] + \frac{\lambda_{4ab}}{2} \left[ \bar{\nu}_R l_a \nu_i + \bar{l}_a \nu_i l_a \nu_i \right] + \frac{\lambda_{5ab}}{2} \left[ \bar{\nu}_R l_a \nu_i + \bar{l}_a \nu_i l_a \nu_i \right],
\] \hspace{1cm} (25)

The \( \lambda' \) interactions generate the low vertices in Fig. 1. On the other hand, the interactions between the squarks, sleptons and scalars, see the Appendix, are given by the scalar potential. The soft part contributes only through the trilinear interactions

\[ V_{soft} = \epsilon_{1ab}(\tilde{l}_a \tilde{l}_b - \tilde{l}_b \tilde{l}_a)\eta^0, \] \hspace{1cm} (26)

while the \( D \)-terms have only quartic interactions

\[ V_D = \frac{g^2}{4} \sum_i (X_0^0 X_i^0 + X_a^0 X_i^0) \sum_a \left( \tilde{l}_a \tilde{l}_a + \frac{1}{2} \tilde{l}_a \tilde{l}_a \tilde{l}_a \tilde{l}_a \right), \] \hspace{1cm} (27)

where \( X_i^0 = \chi_i^0, \rho_i^0 \) which will contribute to the upper quartic vertex in Fig. 1. Due to the interactions given in Eq. (25)–(29), we can generate the appropriate mass to the electron. The dominant contributions, assuming the mass hierarchy \( m_{\text{fermion}} \ll m_{\text{scalar}} \) where fermion means a fermion different from \( j_{2,3} \) and scalar means \( \nu, l, H \) (\( H \)

\[
V_{3F} = \left( \frac{\mu_\rho \lambda_{1abc}}{2} + \frac{\mu_\chi \lambda_{2abc}}{6} + \frac{\mu_\eta \lambda_{3abc}}{9} \right) \left( \tilde{l}_a \tilde{l}_b - \tilde{l}_b \tilde{l}_a \right) \eta^0 + H.c.,
\] \hspace{1cm} (28)

\[
V_{4F} = \left( \frac{\lambda_{4ab} \lambda_{1abc}}{3} + \frac{f_1 \lambda_{2abc}}{9} \right) \left( \tilde{l}_a \tilde{l}_b - \tilde{l}_b \tilde{l}_a \right) \eta^0 + \frac{4 \lambda_{5ab} \lambda_{2abc}}{9} \left( \tilde{l}_a \tilde{l}_b + \tilde{l}_b \tilde{l}_a \right) \eta^0 \] \hspace{1cm} (29)

\[ \text{ denote the heaviest scalar} \) and using the values of the masses and the parameters given in Eqs. (22), (23) and (24) we obtain that the dominant contribution to the electron mass is (up to logarithmic corrections)

\[ m_e \propto \lambda_{2e\tau} \lambda_{2e\mu} \left( v_\chi^2 + 4 v_\chi^2 \right) \frac{m_{\tilde{\nu}_L}}{9 m_{\tilde{\nu}_L}^2}. \] \hspace{1cm} (30)
where $m_\nu = 0.0005 \text{ GeV}$ results if $m_{Z'} \approx m_5 \sim O(1 \text{ TeV})$, $v_\chi \sim 10^3 \text{ GeV}$ and $v_\chi' \sim 4 \times 10^3 \text{ GeV}$, with $\lambda_{2 \nu}^1 \lambda_{2 \nu}^1 \approx 10^{-3}$.

B. Neutral lepton masses

Like in the case of the charged sector, the neutral lepton masses are given by the mixing among neutrinos, induced by the $\mu_{ba}$ term in Eq. (10), and the neutral higgsinos and gauginos [20], and also by $\lambda_2$ and $\lambda_4$ in Eq. (11). The first two terms in Eq. (10) give the interactions between neutrinos and higgsinos:

$$ \Psi^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \[0.5cm] 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \[0.5cm] 0 & 0 & 0 & m_\lambda & 0 & 0 & 0 & 0 & 0 & 0 \[0.5cm] 0 & 0 & 0 & 0 & m_\lambda & 0 & 0 & 0 & 0 & 0 \[0.5cm] 0 & 0 & 0 & 0 & 0 & m_\lambda & 0 & 0 & 0 & 0 \[0.5cm] 0 & 0 & 0 & 0 & 0 & 0 & m' & 0 & 0 & 0 \[0.5cm] -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} & -\frac{\mu_{ba}}{2} \end{pmatrix} . \tag{34} $$

The neutralino mass matrix is diagonalized by a $12 \times 12$ rotation unitary matrix $N$, satisfying

$$ M_{NMD} = N^T \Psi^0 N^{-1} , \quad (35) $$

and the mass eigenstates are

$$ \tilde{\chi}^0_i = N_j \Psi_j^0 , \quad j = 1, \ldots, 12 . \quad (36) $$

We can define the following Majorana spinor to represent the mass eigenstates

$$ \Psi(\tilde{\chi}_i^0) = \begin{pmatrix} \tilde{\chi}_i^0 \\ \tilde{\chi}_i^0 \end{pmatrix} . \quad (37) $$

As above the subindices $a, b, c$ run over the lepton generations $e, \mu, \tau$.

With the mass matrix in Eq. (34), at the tree level we obtain the eigenvalues (in GeV),

$$ -4162.22, 3260.48, 3001.11, 585.19, -585.19, 453.22, -344.14, 283.14, -272.0 , \quad (38) $$

and for the three neutrinos we obtain (in eV)

$$ m_1 = 0, \quad m_2 \approx -0.01, \quad m_3 \approx 1.44 . \quad (39) $$

We have got the values in Eqs. (38) and (39) by choosing, besides the parameters in Eqs. (23) and (24), $m' = -3780.4159 \text{ GeV}$, Notice that the coupling constant $g'$ and the parameter $m'$ appear only in the mass matrix of the neutralinos, all the other parameters in Eq. (34) have already been fixed by the charged sector, see Eq. (17), (23) and (24). The neutrino masses in Eq. (39) are of the order of magnitude for LSND and solar neutrino data. However, by choosing $m' = -3780.4159 \text{ GeV}$ and $\mu_{ba} = 2 \times 10^{-8} \text{ GeV}$, we obtain (in eV)

$$ m_1 = 0, \quad m_2 \approx -5.47 \times 10^{-5}, \quad m_3 \approx 1.32 \times 10^{-2} , \quad (40) $$

which are of the order of magnitude required by the solar and atmospheric neutrino data. Notice also the sensitivity of the neutrino masses in $\mu_{0r}$ and $m'$, and that if $\mu_{0a} = 0$, $a = e, \mu, \tau$, all neutrinos remain massless.
We have obtained numerically the unitary matrices $E,D$ and $N$ which diagonalize the mass matrices in Eq. (17) and (34) but we will not write them explicitly. The charged current is written in the mass-eigenstate basis as $\hat{L}^\mu V_{MNS} \nu L W^-_\mu$ with the Maki-Nakagawa-Sakata matrix [22] defined as $V_{MNS} = \mathcal{E}_L^T \mathcal{N}$, where $\mathcal{E}$ and $\mathcal{N}$ are the $3 \times 3$ submatrices of $E$ and $N$, respectively. Hence, we have:

$$V_{MNS} \approx \begin{pmatrix}
1.000 & -0.004 & -0.001 \\
0.001 & 0.000 & 0.003 \\
-0.004 & -0.979 & -0.199
\end{pmatrix}. \quad (41)$$

Notice that this leptonic mixing matrix is not orthogonal as it must be since we are omitting the mixture with the heavy charginos and neutralinos and that one neutrino remains massless at the tree level. We can always rotate the neutral fields in such a way that the electron neutrino is the one which remains massless; or we can also assume $\mu_{\nu e} = \lambda_{\nu e} = 0$ so that the electron neutrino decouples from the other neutrinos and neutralinos. In this case, diagonal and non-diagonal mass terms in Eq. (34) will be induced by loop corrections like that in Fig. 2. Thus, a $3 \times 3$ non-orthogonal mixing matrix will appear in Eq. (41). Here we will only consider the order of magnitude of a mass generated by the process of Fig. 2.

The massless neutrino can get a mass from the loop correction like that in Fig. 2 as a consequence of the Majorana mass term of the neutral lepton in the triplet. This is equivalent to the mechanism of Ref. [23] but now with a triplet of leptons instead of a neutral singlet. For instance, the $\lambda_2$ interactions will contribute in the left- and right vertices in Fig. 2:

$$\mathcal{L}' = \frac{\lambda_{2ab}}{3} \left[ \tilde{\eta}_R (\nu_{aL} \tilde{b} - \nu_{aL} \tilde{l}_a) + \tilde{\eta}_R (\nu_{bL} \tilde{l}_a - \nu_{aL} \tilde{l}_b) \right] + \frac{\lambda_{4a}}{3} \left[ \tilde{\chi}_R^+ \nu_{aL} \rho^{++} + \tilde{\rho}_R \nu_{aL} \chi^{--} - \tilde{\chi}_R \nu_{aL} \rho^0 - \tilde{\rho}_R \nu_{aL} \chi^0 \right] + H.c.,$$

(42)

these interactions generate the lower vertices of Fig. 2. The upper vertex are given in Eqs. (26)–(29). With these interactions we can generate the following small mass to the electron neutrino. In fact, assuming a hierarchy of the interations we can generate the following small mass to the electron neutrino. In fact, assuming a hierarchy of the interations we can generate the following small mass to the electron neutrino. In fact, assuming a hierarchy of the interations we can generate the following small mass to the electron neutrino. In fact, assuming a hierarchy of the interations we can generate the following small mass to the electron neutrino. In the present model it happens the same: the $R$-parity breaking terms in Eqs. (10) and (11) are forbidden. Notwithstanding the $Z_3$ symmetry [24],

$$\hat{L}, \hat{e} \to \hat{L}, \hat{e}; \quad \hat{H}_1 \to \hat{H}_1, \hat{H}_2 \to \hat{H}_2;$$

$$\hat{Q} \to \omega \hat{Q}, \quad \hat{\tilde{c}} \to \omega^{-1} \tilde{c}, \quad \hat{d} \to \omega^{-1} \tilde{d},$$

(45)

where $\omega = e^{2i\pi/3}$, forbids the $B$ violating terms but allow the $L$ violating ones. This also happens in the present model. However, if we introduce an extra discrete $Z_3'$-symmetry, such that $\hat{L}_e \to -\hat{L}_e$, and all other fields being even under this transformation, we have that $\mu_{\nu e} = \lambda_{\nu e} = \lambda_{\nu e} = 0$, at all orders in perturbation theory. This does not modify the mass matrix in the charged sector in Eqs. (16) and (17), but forbids the electron neutrino to get a mass, at all orders in perturbation theory.

In summary, we have analyzed the charged lepton and neutrino masses in a $R$-parity breaking supersymmetric 3-3-1 model. Unlike the MSSM model the electron and its neutrino remain massless at the tree level but gain masses at the one loop level. The resulting leptonic mixing matrix $V_{MNS}$ is non-orthogonal.

IV. CONCLUSIONS

Although in the nonsupersymmetric 3-3-1 model [4] only three scalar triplets, and with the same lepton sector of the standard model, are not $e$ enough to produce the observed charged lepton masses and neutrinos remain massless, when we supersymmetrized the model and allow $R$-parity breaking interactions we can give to all known charged leptons and neutrinos the appropriate masses. The correct values for the lepton masses can still be obtained if the new VEVs $u', v'$ and $w'$ are zero, however it was shown in Ref. [17] that in order to give mass to all the quarks in the model all these VEVs have to be different from zero.

It is interesting to note that in the context of MSSM a $Z_2$ symmetry [10,24],

$$M \to -M, \quad V \to V, \quad X \to X,$$

(44)

where $M,V,X$ is a matter, vector and scalar superfields, respectively, forbids the $R$-parity breaking terms in Eq. (2). In the present model it happens the same:

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APPENDIX A: THE SCALAR POTENTIAL

The interactions between the scalars of the theory is given by the scalar potential that is written as

\[ V_{331} = V_D + V_F + V_{\text{soft}}, \]  

where the \( V_D \) term is given by

\[ V_D = -L_D = \frac{1}{2} (D^a D^a + DD), \]

\[ = \frac{g^2}{2} \left( 2 \frac{Q_1}{3} \hat{Q}_1 - \frac{1}{3} Q_1 \hat{Q}_1 + \frac{2 \lambda_2}{3} \epsilon_{ijk} \eta^c \hat{L}_j \hat{L}_k + \frac{\lambda_4}{3} \epsilon_{ijk} \chi_j \rho_k - \frac{1}{3} \hat{Q}_1 \hat{Q}_1 \right)^2 \]

\[ + \frac{g^2}{2} \sum_{i,j} (\tilde{L}_i^c \chi_j^a \tilde{L}_j + \hat{Q}_1^c \chi_j^a \hat{Q}_1 + \eta_j^a \lambda_j^a \eta_j + \rho_j^a \lambda_j^a \rho_j - \chi_j^a \lambda_j^a), \]

\[ - \hat{Q}_1^c \chi_j^a \xi \tilde{Q}_a - \eta^a \lambda^a \eta_j - \rho^a \lambda^a \rho_j - \chi^a \lambda^a, \]  

(A2)

the \( F \) term is

\[ V_F = -L_F = \sum_m F_m^* F_m \]

\[ = \sum_{i,j,k} \left[ \left( \frac{\mu_0}{2} \eta_i^c + \frac{2 \lambda_2}{3} \epsilon_{ijk} \eta^c \hat{L}_j \hat{L}_k + \frac{\lambda_4}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 \right] + \left[ \frac{\mu_2}{2} \eta_i^c + \frac{2 \lambda_2}{3} \epsilon_{ijk} \eta^c \hat{L}_j \hat{L}_k + \frac{\lambda_4}{3} \epsilon_{ijk} \chi_j \rho_k \right]^2 \]

\[ + \frac{\mu_3}{2} \eta_i^c + \frac{2 \lambda_2}{3} \epsilon_{ijk} \eta^c \hat{L}_j \hat{L}_k + \frac{\lambda_4}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 \]

\[ + \frac{\mu_4}{2} \eta_i^c + \frac{2 \lambda_2}{3} \epsilon_{ijk} \eta^c \hat{L}_j \hat{L}_k + \frac{\lambda_4}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 \]

(Finally, the soft term is (the following soft-terms do not included the exotic quarks)

\[ V_{\text{soft}} = -L_{\text{soft}} \]

\[ = \frac{1}{2} \left( m_\omega \sum_{u=1}^8 \lambda^u \xi^u + m'_\omega \lambda_B \chi_B + H.c. \right) + m_{Q_1} \hat{L}_1 \hat{L}_1 + m_{Q_2} \hat{Q}_1 \hat{Q}_1 + \sum_{\alpha=2}^3 m_{\tilde{Q}_a} \tilde{Q}_a \tilde{Q}_a \]

\[ + \sum_{i=1}^{3} m_{u_i} \tilde{u}_i + m_{d_i} \tilde{d}_i + m_{u', i} \tilde{u}_i' + m_{d', i} \tilde{d}_i' + m_{\eta} \eta \tilde{\eta} + m_{\rho} \rho \tilde{\rho} + m_{\chi} \chi \tilde{\chi} + m_{\eta'} \eta' \tilde{\eta'} + m_{\rho'} \rho' \tilde{\rho'} + m_{\chi'} \chi' \tilde{\chi'} \]

\[ + \left[ M^2 \sum_{i=1}^{3} \tilde{L}_i \tilde{L}_i + \epsilon_0 \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{ijk} \tilde{L}_i \tilde{L}_j \tilde{L}_k + \epsilon_1 \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{ijk} \tilde{L}_i \tilde{L}_j \rho_k + \epsilon_2 \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{ijk} \tilde{L}_i \chi_j \rho_k \right. \]

\[ + k_1 \epsilon_{ijk} \rho_i \chi_j \eta_k + k'_1 \epsilon_{ijk} \rho_i' \chi_j' \eta_k' + \sum_{i=1}^{3} \tilde{Q}_1 (\xi_{ij} \eta_i' \tilde{\eta}_j' \eta_j + \zeta_{ij} \rho_i' \tilde{\rho}_j' \rho_j) + \sum_{\alpha=2}^3 \tilde{Q}_\alpha \left( \sum_{i=1}^{3} \omega_{\alpha i} \eta_i' \tilde{\eta}_i + \omega_{\alpha i} \rho_i' \tilde{\rho}_i \right) \]
\[ + \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \varsigma_{ijk} \tilde{d}_i \tilde{d}_j \tilde{d}_k + H.c. \]  

(A4)


FIG. 1. Diagram generating the electron mass. There are also a contribution with \( v_\chi \to v_\chi' \). The left- and right-side vertices are proportional to \( \lambda_{a\alpha i}/3 \) and \( \lambda_{a\alpha j}/3 \), respectively.

FIG. 2. Diagram generating the mass for the lightest neutrino. There is another dominant contribution with \( v_\chi \to v_\chi' \). Each vertex on the left- and right-side are proportional to \( \lambda_{2e\tau}/3 \) and \( \lambda_{2e\mu}/3 \), respectively.