A rigorous study of the $\eta$-nucleus interaction in the $pd \rightarrow ^3\text{He}\eta$ reaction near threshold

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Abstract

A detailed study of the effect of the $\eta$-nucleus final state interaction (FSI) in the $pd \rightarrow ^3\text{He}\eta$ reaction close to threshold is presented. The FSI is incorporated through a T-matrix for $\eta$-$^3\text{He}$ elastic scattering, constructed using few body equations. This T-matrix accounts for off-shell and binding effects in $\eta$-nucleus scattering. The energy dependence of the data on the $pd \rightarrow ^3\text{He}\eta$ reaction near threshold is reproduced only after including the FSI. The off-shell and binding effects in $\eta$-$^3\text{He}$ scattering are found to be important. Given the uncertainty in the knowledge of the elementary $\eta$-nucleon interaction, the sensitivity of the $pd \rightarrow ^3\text{He}\eta$ cross section to the $\eta N$ scattering length is also discussed.

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1 Introduction

Experimental data on the $pd \rightarrow ^3\text{He}\eta$ reaction close to threshold have revealed some surprising features \cite{1,2}. Inspite of the large momentum transfer involved in $\eta$ production as compared to that in pion production, the cross section for $pd \rightarrow ^3\text{He}\eta$ is large and comparable with that for $pd \rightarrow ^3\text{He}\pi^0$. The energy dependence of the two reactions near threshold is also very different. The $pd \rightarrow ^3\text{He}\eta$ reaction shows a much rapid variation, with the

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threshold amplitude falling by a factor of 3.75 over an $\eta$ centre of mass momentum of 75 MeV/c.

The observed features of the $pd \rightarrow ^3\text{He}\eta$ reaction have been attributed to the strong final state $\eta$-nucleus interaction. This reaction was studied in Ref. [3], where the final state interaction (FSI) was incorporated in an approximate way through an enhancement factor. The cross section was factorized in terms of the amplitude for the reaction $pd \rightarrow ^3\text{He}\eta$ with plane waves for the $^3\text{He}$ in the final state and an S-wave FSI factor written in terms of the on-shell $\eta$-nucleon amplitude. The energy dependence of the cross section data was reproduced, though its predicted absolute value was lower by a factor of 2.5. In yet another work [4], the scattering length of the $\eta$ meson on helium was calculated using multiple scattering theory, which was then used to calculate the FSI factor for the $pd \rightarrow ^3\text{He}\eta$ cross section. The FSI factor with an $\eta$-nucleon scattering length $a_{\eta N} = (0.291, 0.36)$ leading to $a_{\eta^3\text{He}} = (-0.89, 1.8)$ was found to give a good fit to the $pd \rightarrow ^3\text{He}\eta$ data.

The $pd \rightarrow ^3\text{He}\eta$ reaction at threshold and higher energies has also been studied theoretically to investigate the reaction mechanism involved [5, 6, 7]. Due to the large mass of the $\eta$ meson, the momentum transfer involved in this reaction is large. It starts with 900 MeV/c at the threshold becoming around 500 MeV/c at higher energies. As a result of this, it was shown in Ref. [5] that the three-body mechanism which allows the momentum transfer to be shared amongst three nucleons dominates. The one and two body mechanisms were found to underestimate the experimental cross sections by more than two orders of magnitude. The three nucleons share the large momentum transfer through a two step process where the incident proton interacts with a nucleon in the deuteron to produce a pion which then interacts with the other nucleon in the deuteron to produce an $\eta$ meson. The $pd \rightarrow ^3\text{He}\eta$ reaction, thus, proceeds via the $NN \rightarrow \pi d$ and $\pi N \rightarrow \eta N$ reactions. The three body mechanism requires only the low momentum components of the nuclear wave function. Considering the support for the three body mechanism in the existing literature [5, 8], at least at energies very close to threshold, it seems justified to consider this mechanism to be the only major reaction mechanism for the $pd \rightarrow ^3\text{He}\eta$ reaction.

In the present work we study the $pd \rightarrow ^3\text{He}\eta$ reaction near threshold using the three body mechanism mentioned above. Our main objective is to investigate the FSI in this reaction in a rigorous way. The above mentioned theoretical works in the literature either neglect the FSI or incorporate it in
an approximate way using on-shell amplitudes. We express the $\eta^3$He relative wave function in terms of the Lippmann Schwinger equation involving the T-matrix for $\eta^3$He elastic scattering. This T-matrix is evaluated using a method of few body equations [9, 10, 11] which will be described in detail in the next section.

In section 2 we present the formalism for the calculation of the $pd \rightarrow ^3$He$\eta$ cross section including the FSI. The production mechanism for $pd \rightarrow ^3$He$\eta$ reaction within a two-step model is described in section 3. In section 4 we present the results. We reproduce the shape and magnitude of the experimentally measured scattering amplitude. The off-shell effects in $\eta^3$He scattering are found to be important in producing the energy dependence of the cross section.

2 Final state interaction

The transition matrix for the reaction $pd \rightarrow ^3$He$\eta$, which includes the interaction between the $\eta$ meson and $^3$He is given by,

$$T = < \Psi_{\eta^3He}^- (k_\eta) ; m_3 | T_{pd \rightarrow ^3He \eta} | k_p; m_1 m_2 >$$  \hspace{1cm} (1)

where $m_1$, $m_2$ and $m_3$ are the spin projections of the proton, deuteron and helium respectively. $k_p$ and $k_\eta$ are the momenta of the particles in the initial and final states. The final state $\eta^3$He wave function $\Psi_{\eta^3He}^-$ consists of a plane wave and a scattered wave, and can be written as,

$$< \Psi^-_{\eta^3He} | = < k_\eta | + \int \frac{d\vec{q}}{(2\pi)^3} \frac{< k_\eta | T_{\eta^3He}^- | \vec{q} >}{E(k_\eta) - E(q) + i\epsilon} < \vec{q} |$$  \hspace{1cm} (2)

where $T_{\eta^3He}^-$ is the T-matrix for $\eta^3$He elastic scattering. Replacing the above wave function in Eq. (1), we get,

$$T = < k_\eta^-; m_3 | T_{pd \rightarrow ^3He \eta} | k_p; m_1 m_2 > + \sum_{m_3'} \int \frac{d\vec{q}'}{(2\pi)^3} \frac{< k_\eta^-; m_3 | T_{\eta^3He}^- | \vec{q}' >}{E(k_\eta) - E(q') + i\epsilon} < \vec{q}' ; m_3' | T_{pd \rightarrow ^3He \eta} | k_p; m_1 m_2 >$$  \hspace{1cm} (3)

The matrix elements $< | T_{pd \rightarrow ^3He \eta} | >$ in the above equation correspond to the Born amplitude for the $pd \rightarrow ^3$He$\eta$ reaction. We calculate these matrix elements using a two step model which will be discussed in the next section.
The T-matrix, \( T_{\eta^3He} \), in Eq. (3) for \( \eta^3\text{He} \) elastic scattering is evaluated using four particle equations for the \( \eta(3N) \) system. For practical convenience, the evaluation of \( T_{\eta^3He} \) is done within a Finite Rank Approximation (FRA) approach. This means that the nucleus in the elastic meson-nucleus scattering is always in its ground state. At the low energies concerned in this work and for a light nucleus like \( ^3\text{He} \), it seems justified to use the FRA and write the target Hamiltonian \( H_A \) as,

\[
H_A \approx \varepsilon |\psi_0 > < \psi_0 |
\]  

(4)

where \( \psi_0 \) is the nuclear ground state wave function and \( \varepsilon \) the binding energy.

Within this approximation, the \( \eta^3\text{He} \) T-matrix is given as \( [9, 10, 11] \),

\[
T(\vec{k}', \vec{k}; z) = < \vec{k}'; \psi_0 | T^0(z) | \vec{k}; \psi_0 > + \varepsilon \int \frac{d\vec{k}''}{(2\pi)^3} \frac{< \vec{k}''; \psi_0 | T^0(z) | \vec{k}''; \psi_0 >}{(z - \frac{k''^2}{2\mu})(z - \varepsilon - \frac{k''^2}{2\mu})} T(\vec{k}'', \vec{k}; z)
\]  

(5)

where \( z = E - |\varepsilon| + i\delta \). \( E \) is the energy associated with \( \eta \)-nucleus relative motion and \( \mu \) is the reduced mass of the \( \eta \)-nucleus system. The operator \( T^0 \) describes the scattering of \( \eta \) meson from nucleons fixed in their space position within the nucleus. The matrix elements for \( T^0 \) are given as,

\[
< \vec{k}'; \psi_0 | T^0(z) | \vec{k}; \psi_0 > = \int d\vec{r} |\psi_0(\vec{r})|^2 T^0(\vec{k}', \vec{k}; \vec{r}; z)
\]  

(6)

where,

\[
T^0(\vec{k}', \vec{k}; \vec{r}; z) = \sum_{i=1}^{A} T^0_i(\vec{k}', \vec{k}; \vec{r}_i; z)
\]  

(7)

\( T^0_i \) is the t-matrix for the scattering of the \( \eta \)-meson from the \( i^{th} \) nucleon in the nucleus, with the rescattering from the other (\( A-1 \)) nucleons included. It is given as,

\[
T^0_i(\vec{k}', \vec{k}; \vec{r}_i; z) = t_i(\vec{k}', \vec{k}; \vec{r}_i; z) + \int \frac{d\vec{k}''}{(2\pi)^3} \frac{t_i(\vec{k}'', \vec{k}''; \vec{r}_i; z)}{z - \frac{k''^2}{2\mu}} \sum_{j \neq i} T^0_j(\vec{k}'', \vec{k}; \vec{r}_j; z)
\]  

(8)

The t-matrix for elementary \( \eta \)-nucleon scattering, \( t_i \), is written in terms of the two body \( \eta N \) matrix \( t_{\eta N} \) as,

\[
t_i(\vec{k}', \vec{k}; \vec{r}_i; z) = t_{\eta N}(\vec{k}', \vec{k}; z) \exp[i(\vec{k} - \vec{k}') \cdot \vec{r}_i]
\]  

(9)
The $^3$He wave function $\psi_0$, required in the calculation of $T_{\eta^3He}$ is taken to be of the Gaussian form.

We assume that the $\eta N \to \eta N$ scattering is dominated by the formation of the S-wave resonance, $N^*(1535)$ and write the $\eta N$ t-matrix in a standard Breit-Wigner form as,

$$t_{\eta N}(k', k; z) = \frac{1}{(k'^2 + \Lambda^2)} \frac{\alpha}{(z - E_0 + \frac{i}{2})} \frac{1}{(k^2 + \Lambda^2)}$$

where the range parameter $\Lambda$ is a measure of the off-shell extrapolation of the coupling at the $\eta NN^*$ vertex. It is taken to be 2.357 fm$^{-1}$ as obtained in the coupled channel fit [12] to $\pi N \to \pi N$ and $\pi N \to \eta N$ scattering data. The parameter $\alpha$ is fixed from the scattering length using the following relation:

$$t_{\eta N}(0, 0; 0) = -\frac{2\pi}{\mu_{\eta N}} a_{\eta N}$$

Since there exists a lot of uncertainty in the knowledge of the $\eta$-nucleon interaction, the scattering length $a_{\eta N}$ is not accurately known. In Ref. [12] an off-shell model for pionic $\eta$ production was presented. The $\pi N$, $\eta N$ and $\pi \Delta$ channels were treated in a coupled channels formalism and data on $\pi N \to \pi N$ and $\pi N \to \eta N$ were reproduced very well. This model predicted an $\eta N$ scattering length, $a_{\eta N} = (0.28, 0.19)$. In another calculation involving three coupled channels and multiple resonances [13], the $\eta N$ scattering length was given as, $a_{\eta N} = (0.717 \pm 0.03) + i(0.263 \pm 0.025)$. The values of $a_{\eta N}$ obtained from other analyses [14-17] range between 0.25 and 1.05 for the real part of $a_{\eta N}$ and 0.16 to 0.49 for the imaginary part. A compilation of different values of the $\eta N$ scattering length, $a_{\eta N}$, as evaluated by different models can be found in Ref. [18]. In the present work, we vary the values of $a_{\eta N}$ and discuss the sensitivity of the results to the variation in $a_{\eta N}$. Once the value of $a_{\eta N}$ is fixed, we evaluate the elementary t-matrix $t_{\eta N \to \eta N}$ using this $a_{\eta N}$ as input. This $t_{\eta N \to \eta N}$ is then used in the evaluation of the T-matrix for $\eta^3He$ scattering. As we shall see later, the value of $a_{\eta N} = (0.62, 0.19)$ gives the best fit to the data on the $pd \to ^3He \eta$ reaction.

3 Production mechanism

As mentioned in the Introduction, we assume the $\eta$ production in $pd \to ^3He \eta$ to proceed through a two-step process via the $NN \to \pi d$ and $\pi N \to \eta N$
Figure 1: Diagram of $\eta$ production in the $pd \rightarrow ^3\text{He}\eta$ reaction with a two step process. The ellipse indicates the final state interaction of $^3\text{He}$ and $\eta$.

reactions as shown in Fig. 1. The amplitude for the $pd \rightarrow ^3\text{He}\eta$ reaction appearing in Eq. (3) can be written within this model as,

$$< |T_{pd \rightarrow ^3\text{He}\eta}| > = i \int \frac{dp_1}{(2\pi)^3} \frac{dp_2}{(2\pi)^3} \sum_{int/m'/s} < pn | d > < \pi^+ d | T_{pp \rightarrow \pi^+ d} | pp > (12)$$

where the sum runs over the spin projections of the intermediate particles. The spin projections and momenta of the interacting particles are as shown in Fig. 1. $k_\pi$ is the four momentum of the intermediate pion which could either be $\pi^+$ or $\pi^0$. In the case of an intermediate $\pi^0$, the matrix element for $pd \rightarrow ^3\text{He}\eta$ is half of that written above for $\pi^+$. Hence we calculate the T-matrix as in Eq. (12) and multiply it by a factor of 3/2 to account for the intermediate $\pi^0$. Each of the individual matrix elements in the above equation is expressed in terms of partial wave expansions. The matrix elements for the $pp \rightarrow \pi^+ d$ reaction, parametrized in terms of the available experimental data are taken from Ref. [19]. For the $\pi^+ n \rightarrow \eta p$ reaction, we use the coupled channel t-matrix of Ref. [12], mentioned in the previous section. The matrix elements $< pn | d >$ and $< ^3\text{He} | pd >$ consist of the deuteron and helium wave functions in momentum space. We use the deuteron wave function from Ref. [20] where an analytical parametrization of it was done with a Paris potential. This wave function reproduces the known low energy properties and the
electromagnetic form factor of the deuteron well. For the $^3$He wave function, we use the parametrization given in Ref. [21]. The values of the parameters in [21] were obtained by fitting the wave function to the variational calculations of Schiavilla et al. [22] using the Urbana force. The details of Eq. (12) are given in the appendix.

4 Results and Discussion

The reaction $pd \rightarrow ^3$He$\eta$ has been studied at Saturne [1, 2] for proton energies between 0.2 and 11 MeV above threshold. Taking out the phase

\begin{center}
\begin{tabular}{c c c}
| $a_{\eta N}$ & \quad & \\
| \quad & Mayer et al. & Berger et al. \\
| \quad & $a_{\eta N} = (0.28, 0.19)$ & $a_{\eta N} = (0.62, 0.3)$ \\
| \quad & $a_{\eta N} = (0.62, 0.19)$ & No FSI \\
\end{tabular}
\end{center}

Figure 2: The square of the $pd \rightarrow ^3$He$\eta$ amplitude defined in Eq. (13) as a function of the $\eta$ momentum in the centre of mass. The data is from Refs [1, 2]. The thin solid line is the calculation of the present work without including the FSI. The dashed, dash-dotted and thick solid lines are the calculations including FSI with different values of the $\eta$-nucleon scattering length $a_{\eta N}$.
space factor, the spin averaged amplitude can be defined as,

\[ |f|^2 = \frac{k_p}{k_\eta} \frac{d\sigma}{d\Omega_{cm}} \]  \hspace{1cm} (13)

where \( k_p \) and \( k_\eta \) are the proton and \( \eta \) momenta in the centre of mass system. The data on \(|f|^2\) (see Fig. 2) drops rapidly (by about a factor of 3.75) from threshold to 0.4 fm\(^{-1}\) momentum (corresponding to 11 MeV energy) above threshold. In Fig. 2 we compare our calculations of \(|f|^2\) (at \( \theta_\eta = 180^\circ \)) with and without the inclusion of the \( \eta^3\)He final state interaction (FSI), with the data from Refs [1, 2]. The thin solid line in Fig. 2 is our calculation without FSI and can be seen to be a constant as a function of energy. Since the value of the \( \eta N \) scattering length, \( a_{\eta N} \), is not well known, we vary \( a_{\eta N} \) in our calculation (within the limits of recommended values available from different analyses in literature) of the FSI and find \( a_{\eta N} = (0.62, 0.19) \) to give the best agreement with data. The dashed and dash-dotted curves are the results obtained by changing the real and imaginary part of \( a_{\eta N} \) from 0.62 and 0.19 to 0.28 and 0.3 respectively. We see that the FSI is responsible for changing the shape of \(|f|^2\) from a constant to a rapidly falling one as a function of energy.

Figure 3: The data and the solid lines are as in Fig. 2. The dashed line is the FSI calculation retaining only the pole term in Eq. (3).
In Fig. 3 we study the off-shell effects in the FSI. As seen in section 2, we describe the \( \eta^3\text{He} \) final state interaction through an off-shell T-matrix for \( \eta^3\text{He} \) scattering. Previous estimates of FSI in literature have been made using on-shell amplitudes and hence it is important to check the validity of such an approximation. The \( \eta^3\text{He} \) T-matrix appears in an integral in Eq. (3) which can be split into the principal value and pole term. Retaining only the pole term in Eq. (3) and setting the principal value (which involves the off-shell \( \eta^3\text{He} \) scattering) to zero, we get the dashed line in Fig. 3. The pole term alone is unable to reproduce the shape of the data and in fact reduces the magnitude of the results obtained without FSI. The solid line represents the full calculation. We find the off-shell effects to be important and in fact responsible for producing the energy dependence of \( |f|^2 \).

![Figure 4: Angular distribution of \( \eta \) in the \( pd \rightarrow ^3\text{He} \eta \) reaction at different beam energies. The solid (dashed) curves are the calculations with (without) FSI included.](image)

Next, in Fig. 4, we study the effect of the FSI on the angular distributions at different energies. We use \( a_{\eta N} = (0.62, 0.19) \), since this value gives a good
description of the $|f|^2$ data. As observed in Fig. 2 too, we see that the FSI increases the magnitude of the cross sections with the increase being maximum at threshold (Fig. 4a). The angular dependence of the cross sections without FSI (dashed lines) is isotropic at threshold and deviates by a small amount from the isotropy with increase in beam energy. The small anisotropy at a few MeV above threshold is somewhat amplified by the final state interaction. Thus, as one goes to higher energies (Fig. 4c), though the FSI does not increase the magnitude of the cross section too much, it does changes the angular dependence.

Figure 5: Total cross section for $pd \rightarrow ^3\text{He} \eta$ as a function of beam energy. Solid curve is obtained by numerically integrating the calculated $d\sigma/d\Omega$ with FSI over all angles. The dashed curve is obtained by multiplying $d\sigma/d\Omega$ (with FSI) at $180^0$ with $4\pi$, thus assuming the angular distribution to be isotropic. The data is from Ref. [1].

In Fig. 5 we show the angle integrated total cross section as a function of beam energy. The total cross sections without FSI are shown by the thin solid line. The thick solid line is obtained by numerically integrating the angular distribution with FSI included. The dashed curve indicates the total cross
section calculated with FSI included but assuming the angular distribution to be isotropic (i.e. $\sigma_{\text{tot}} = 4\pi (d\sigma/d\Omega)_{\theta = 180^\circ}$). The data of Ref. [1] seem to indicate a negligible forward-backward asymmetry (which is consistent with zero within 5% at all energies from 0 to 11 MeV above threshold) in the $pd \to ^3\text{He}\,\eta$ reaction. Within the two-step FSI model of the present work however, we find small deviations from isotropy at energies away from threshold. Since the total cross section assuming isotropic angular distribution gives better agreement with data, it seems that the two step model of the present work underestimates the cross section at forward angles. The data on the squared amplitude $|f|^2$ at $180^0$ is however very well produced as seen earlier in Fig. 2.

![Figure 6: Effects of including the binding energy of $^3\text{He}$ in the calculation of FSI between $^3\text{He}$ and $\eta$. The solid and dashed curves are calculations including FSI with the binding energy $\varepsilon = -7.718$ and zero respectively (see Eq. (5)). The effect is shown for three different values of $\eta$-nucleon scattering length $a_{\eta N}$.](image)


Finally, in Fig. 6 we study the effects on the $pd \rightarrow ^3\text{He}\eta$ reaction, of including the FSI which incorporates the binding effects in $^3\text{He}$. Once again we plot the amplitude squared $|f|^2$ calculated at three different values of $a_{\eta N}$. The solid curves are the calculations using $\varepsilon = -7.718$ MeV in Eq. (5) for $T_{\eta^3\text{He}}$. The dashed curves represent the calculation with $\varepsilon = 0$, i.e. no binding effects in the final state interaction. The calculated results without the binding effects are larger than those which include them. The magnitude of the increase, however, depends upon the value of $a_{\eta N}$. It is a factor of 2 when we use the best fit value $a_{\eta N} = (0.62, 0.19)$ of the present work, and negligibly small for $a_{\eta N} = (0.28, 0.19)$.

5 Summary

The $pd \rightarrow ^3\text{He}\eta$ reaction has been studied in the present work within a two step model, incorporating the final state interaction (FSI) of the $^3\text{He}$ nucleus and $\eta$ meson in a rigorous way. The peculiar behaviour of the cross section for this $\eta$ producing reaction as compared to a similar pion production re-action $pd \rightarrow ^3\text{He}\pi^0$ is seen to originate due to the interaction between $^3\text{He}$ and $\eta$. The FSI changes the energy dependence of the squared amplitude from a constant (without FSI) to one which falls by a factor of 3.75, from threshold to 11 MeV above threshold. We incorporate the FSI through an off-shell T-matrix for $\eta^3\text{He}$ elastic scattering. This T-matrix is evaluated by numerically solving few body equations which include the nuclear binding effects. Both the off-shell as well binding effects in $\eta^3\text{He}$ scattering are found to be important in the calculation of the $pd \rightarrow ^3\text{He}\eta$ reaction near threshold. Earlier investigations of this reaction involving on-shell and approximate ways of calculating the FSI should hence be treated with caution.

The $\eta$ nucleus interaction has generated a lot of interest in the past few years, particularly due to the possibility of forming $\eta$ mesic nuclei. Since it is difficult to obtain data on elementary $\eta$ nucleon scattering, little is known about the $\eta N$ interaction. The scattering length in $\eta N$ scattering is a much debated quantity and different estimates and limiting values (for the possible formation of an $\eta$ mesic nucleus) of this parameter exist in literature. Within the model of the present work we find that the $\eta N$ scattering length, $a_{\eta N} = (0.62, 0.19)$, which gives the $\eta^3\text{He}$ scattering length, $a_{\eta^3\text{He}} = (-1.18, 4)$, leads to a good reproduction of the $pd \rightarrow ^3\text{He}\eta$ data near threshold.
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Appendix

In what follows, we discuss in detail the constituents of the Born amplitude (Eq. (12)) for the \( pd \rightarrow ^3\text{He}\eta \) reaction. To start with, we write the deuteron wave function in Eq. (12) as,

\[
\sum <pn|d> = \frac{1}{\sqrt{2}} \left\{ \sum_{m_n} <1/2 m_p 1/2 m_n | 1 m_2 > \frac{\phi^d_0(p_1)}{\sqrt{4\pi}} \right. \\
+ \sum_{m_i} <1/2 m_p 1/2 m_n | 1 M_s > <1 M_s 2 m_l | 1 m_2 > Y_{2,m_l}(\hat{p}_1) \phi^d_2(p_1) \right\}
\]

where \( m_n \) and \( m_p \) are the spin projections of the off-shell neutron and proton respectively in the intermediate state and \( m_2 \) is the spin projection of the target deuteron. The factor \( \frac{1}{\sqrt{2}} \) comes from isospin overlap. \( \phi^d_l(p_1) \) is the deuteron wave function in the \( l^{th} \) partial wave and \( \hat{p}_1 \) is the relative momentum of the p-n pair inside the deuteron. Writing

\[
\phi^d_0(p_1) = (2/\pi)^{1/2} \sum_{j=1}^n \frac{C_j}{p_1^2 + m_j^2}
\]

\[
\phi^d_2(p_1) = (2/\pi)^{1/2} \sum_{j=1}^n \frac{D_j}{p_1^2 + m_j^2},
\]

the parameters \( C_j \), \( D_j \) and \( m_j \) for the Paris potential are given in Ref. [20].

The \( ^3\text{He} \) wave function is written as,

\[
\sum <^3\text{He} | pd >= \sum_{m_i' m_p'} <1 m_2' 1/2 m_p' | 1/2 m_3 > \frac{\chi_0(p_2)}{\sqrt{4\pi}} + \quad (A.4)
\]

\[
\sum_{m_i'} <1 m_2' 1/2 m_p' | 3/2 m > <2 m_i' 3/2 m | 1/2 m_3 > \chi_2(p_2) Y_{2,m_l}(\hat{p}_2)
\]
where $m'_2$ and $m'_p$ are the spin projections of the off-shell deuteron and proton respectively. The spin projection of $^3$He is $m_3$ as shown in Fig. 1. $\chi_l(p_2)$ is the helium wave function in the $l^{th}$ partial wave and $\vec{p}_2$ is the relative momentum of the p-d pair inside $^3$He.

$$\chi_l(p_2) = \sum_{i=1}^{n} \frac{a_i}{p_2^2 + m_i^2}$$  \hspace{1cm} (A.5)

The parameters $a_i$ and $m_i$ given in Ref. [21] are chosen corresponding to p-d clustering in $^3$He. The normalization of the wave function is such that,

$$\int p_2^2 dp_2 \{\chi_0(p_2)^2 + \chi_2(p_2)^2\} = 1.5$$  \hspace{1cm} (A.6)

The four momentum $k_\pi = \{E_\pi, \vec{k}_\pi\}$ appearing in the pion propagator in Eq. (12) is written using energy and momentum conservation at the $\pi^+ n \rightarrow \eta p$ and $pp \rightarrow \pi^+ d$ vertices respectively. Thus,

$$E_\pi = E_\eta + \frac{1}{3} E_{He} - \frac{1}{2} E_d$$  \hspace{1cm} (A.7)

and

$$\vec{k}_\pi = \frac{\vec{k}_p}{2} + \frac{2}{3}\vec{k}_\eta + \vec{p}_1 + \vec{p}_2$$  \hspace{1cm} (A.8)

References


