The Zee model, which employs the standard Higgs scalar ($\phi$) with its duplicate ($\phi'$) and a singly charged scalar ($h^+$), can utilize two global symmetries associated with the conservation of the numbers of $\phi$ and $\phi'$, $N_\phi + N_{\phi'}$, where $N_\phi + N_{\phi'}$ coincides with the hypercharge while $N_\phi - N_{\phi'}$ ($\equiv X$) is a new conserved charge, which is identical to $L_e - L_\mu - L_\tau$ for the left-handed leptons. Charged leptons turn out to have $e-\mu$ and $e-\tau$ mixing masses, which are found to be crucial for the large solar neutrino mixing. In an extended version of the Zee model with an extra triplet Higgs scalar ($s$), neutrino oscillations are described by three steps: 1) the maximal atmospheric mixing is induced by democratic mass terms supplied by $s$ with $X=2$ that can initiate the type II seesaw mechanism for the smallness of these masses; 2) the maximal solar neutrino mixing is triggered by the creation of radiative masses by $h^+$ with $X=0$; 3) the large solar neutrino mixing is finally induced by a $\nu_\mu-\nu_\tau$ mixing arising from the rotation of the radiative mass terms as a result of the diagonalization that converts $e-\mu$ and $e-\tau$ mixing masses into the electron mass.

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leptons. However, such a constraint on the interactions of \( \phi' \) should be determined by a certain underlying symmetry. We introduce the invariance imposed by two global symmetries associated with the numbers of \( \phi \) and \( \phi' \), \( N_\phi \) and \( N_{\phi'} \). It will be shown that \( N_\phi + N_{\phi'} \) coincides with the hypercharge. The orthogonal combination, \( N_\phi - N_{\phi'} \), turns out to be a new conserved quantum number, which is identical to \( L_e - L_\mu - L_\tau \) for the left-handed leptons. As a result, \( \phi' \) is allowed to couple to \( \mu \) and \( \tau \) and, at the same time, \( \phi \) is constrained to have couplings of \( e-\mu \) and \( e-\tau \) [15]. Since off-diagonal mass terms for charged leptons are generated by \( \phi \), neutrino mixings are affected by the diagonalization of these charged lepton mass terms, which will be used to explain \( \sin^2 2\theta_{12} \neq 1 \).

In order to realize \( \sin^2 2\theta_{12} \neq 1 \), the simplest resolution is to include flavor-diagonal mass terms. It is because the main source of \( \sin^2 2\theta_{12} \approx 1 \) in the \( \nu \tau e \) model comes from the constraint on neutrino masses \( m_{1,2,3} \) of \( m_1 + m_2 + m_3 = 0 \) specific to flavor-off-diagonal mass terms. We employ an additional \( SU(2)_L \)-triplet Higgs scalar \( s \) [17]

\[
s = \begin{pmatrix} s^+ & s^{++} \\ s^0 & -s^+ \end{pmatrix}
\]

(1)
to supply flavor-diagonal mass terms. Tree level neutrino masses are generated by the vacuum expectation value of \( s \), \( \langle 0 | s^0 | 0 \rangle \), via interactions of \( \overline{\nu}_L s \nu_L \), where the subscript \( c \) denotes the charge conjugation including the \( G \)-parity of \( SU(2)_L \). The smallness of the neutrino masses is ascribed to that of \( \langle 0 | s^0 | 0 \rangle \), which is given by \( \sim \mu \langle 0 | \phi | 0 \rangle/m_s^2 \) with \( \langle 0 | \phi | 0 \rangle \) produced by the combined effects of \( \mu \phi^c s \phi^c \) and \( m_s^2 \text{Tr}(s^\dagger s) \), where \( \mu \) and \( m_s \) are mass parameters. The type II seesaw mechanism [18] can be used to ensure tiny neutrino masses by the dynamical requirement of \( \langle 0 | \phi | 0 \rangle \ll m_s \) with \( \mu \sim m_s \).

In the next section, we present a possible neutrino mass matrix to be diagonalized by two mixing angles, \( \theta_{12} \) for the \( \nu_e-\nu_\mu \) mixing and \( \theta_{23} \) for the \( \nu_m-\nu_\tau \) mixing, from which one can find which elements make \( \sin^2 2\theta_{12} \) less than unity. In Sec.III, we explain the role of two conserved charges of \( N_\phi, N_{\phi'} \) and show the Yukawa and Higgs interactions, which provide radiative neutrino masses for \( \nu_e-\nu_\mu \) and \( \nu_e-\nu_\tau \) and charged lepton masses for \( e-\mu \) and \( e-\tau \). The diagonalization of the charged lepton masses is performed in Sec.IV and also shown is sufficient suppression of possible flavor-changing interactions, which are generated by \( \phi \) and \( \phi' \) since charged leptons simultaneously couple to these Higgs scalars. Solar neutrino oscillations are found to exhibit the large mixing of \( \sin^2 2\theta_{12} \sim 0.8 \) as a result of the rotation. The last section is devoted to summary and discussions.

## 1. Neutrino Mass Texture

Before discussing how neutrino masses and oscillations really come out, we first examine which flavor neutrino mass terms affects the deviation of \( \sin^2 2\theta_{12} \) from unity. The mass matrix given by \( M^\nu \) of the form of [15,19]

\[
M^\nu = \begin{pmatrix} a & b & c(= -t_{23}b) \\ b & d & e \\ c & e & f(= d + (t_{23}^{-1} - t_{23})e) \end{pmatrix}
\]

(2)
can be diagonalized by \( U_{MNS} \) defined by

\[
U_{MNS} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\cos \theta_{23} \sin \theta_{12} & \cos \theta_{23} \cos \theta_{12} & \sin \theta_{23} \\ \sin \theta_{23} \sin \theta_{12} & -\sin \theta_{23} \cos \theta_{12} & \cos \theta_{23} \end{pmatrix},
\]

(3)
where \( t_{23} = \sin \theta_{23}/\cos \theta_{23} \), which transforms \( |\nu_{mass}\rangle = (\nu_1, \nu_2, \nu_3)^T \) with masses of \( (m_1, m_2, m_3) \) into \( |\nu_{weak}\rangle = (\nu_e, \nu_\mu, \nu_\tau)^T \): \( |\nu_{weak}\rangle = U_{MNS} |\nu_{mass}\rangle \). The masses and \( \theta_{12} \) are calculated to be [20]:

\[
m_1 = a - \frac{1}{2} \sqrt{b^2 + c^2} \left( x + \eta \sqrt{x^2 + 8} \right), \quad m_2 = (\eta \rightarrow -\eta \text{ in } m_1),
\]

\[
m_3 = d + t_{23}^2 \left( d - a + x \right) \sqrt{b^2 + c^2}/2,
\]

\[
\sin^2 2\theta_{12} = \frac{8}{8 + x^2} \text{ with } x = \frac{a - d + t_{23}e}{\sqrt{(b^2 + c^2)/2}},
\]

(4)

(5)
where \( |m_1| < |m_2| \) is always maintained by adjusting the sign of \( \eta \) (= ±1). The relation of Eq.(5) shows that the significant deviation of \( \sin^2 2\theta_{12} \) from unity is only possible if \( |x| = O(1) \), namely, \( (a - d + t_{23}e)^2 = O(b^2 + c^2) \). In our subsequent discussions, the solution of \( a = 0 \) and \( (d - t_{23}e)^2 = O(b^2 + c^2) \) is realized.
Let us briefly examine Eqs. (4) and (5) in the Zee model. Since the Zee mass matrix is parameterized by $a = d = 0$, leading to $m_1 + m_2 + m_3 = 0$, we find that

$$\Delta m^2_{\text{atm}} = m_3^2 - m_2^2 = \left[ x^2 - \left( 4 + |x| \sqrt{x^2 + 8} \right) \right] \frac{b^2 + c^2}{4},$$

$$\Delta m^2_{\odot} = m_2^2 - m_1^2 = |x| \sqrt{x^2 + 8} \frac{k^2 + c^2}{2}. \quad (6)$$

where $|t_{23}| = 1$ is used for $\Delta m^2_{\text{atm}}$. It is obvious that the requirement of $|x| = \mathcal{O}(1)$ for the large solar neutrino mixing gives $\Delta m^2_{\odot} = \mathcal{O}(\Delta m^2_{\text{atm}})$, which contradicts with the observed result. More explicitly, the mixing angle $\theta_{12}$ is computed to be: [21]

$$\sin^2 2\theta_{12} = \frac{4|m_1 m_2|}{(|m_1| + |m_2|)^2}. \quad (7)$$

Since $(m_2 - m_1)^2 \geq m_3^2$ is derived, $\sin^2 2\theta_{12} \approx 1$ is expected because of $|\Delta m^2_{\text{atm}}| \gg |\Delta m^2_{\odot}|$ implying $|m_1| \approx |m_2|$. In fact, one can readily find that

$$m_1^2 = \frac{(2M - \Delta m^2_{\text{atm}} - 2\Delta m^2_{\odot})}{3}, \quad m_2^2 = \frac{(2M - \Delta m^2_{\text{atm}} + \Delta m^2_{\odot})}{3},$$

$$m_3^2 = \frac{(2M + 2\Delta m^2_{\text{atm}} + \Delta m^2_{\odot})}{3}, \quad (8)$$

where $M = \sqrt{(\Delta m^2_{\text{atm}})^2 + \Delta m^2_{\text{atm}} \Delta m^2_{\odot} + (\Delta m^2_{\odot})^2}$ reduced to $(m_1^2 + m_2^2 + m_3^2)/2$ as expected, which can be derived by the use of $m_1 + m_2 + m_3 = 0$. The approximation of $|\Delta m^2_{\text{atm}}| \gg |\Delta m^2_{\odot}|$, finally, gives $\sin^2 2\theta_{12}$ expressed as: [14]

$$\sin^2 2\theta_{12} \approx 1 - \frac{1}{16} \left( \frac{\Delta m^2_{\odot}}{\Delta m^2_{\text{atm}}} \right)^2. \quad (9)$$

This result is entirely based on the constraint of $m_1 + m_2 + m_3 = 0$, which enables us to relate $m_{1,2,3}^2$ to $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\odot}$. This constraint is violated by the inclusion of diagonal masses supplied by $s$.

We start with the “ideal” solution [22] with $t_{23} = \pm 1 \ (\equiv \sigma)$ given by

$$M^\nu_{\text{ideal}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & d & \sigma d \\ 0 & \sigma d & d \end{pmatrix}, \quad (10)$$

which provides $m_1 = m_2 = 0$ and $m_3 = 2d$ and the maximal atmospheric mixing. The deviation from this solution that yields $b (= b_{\text{rad}}) \neq 0$ and $c (= c_{\text{rad}}) \neq 0$ as $\nu_e \nu_\mu$- and $\nu_e \nu_\tau$-terms is caused by radiative effects, leading to $M^\nu_1$:

$$M^\nu_1 = \begin{pmatrix} b_{\text{rad}} & c_{\text{rad}} \\ d & \sigma d \end{pmatrix}, \quad (11)$$

which provides the bimaximal neutrino mixing for $b_{\text{rad}} = -\sigma c_{\text{rad}}$. The diagonalization of the charged lepton masses generates an extra $\nu_\mu \nu_\tau$-term (= $d_{\text{rot}}$) through the rotation of $b_{\text{rad}}$ and $c_{\text{rad}}$, which gives $(d - t_{23}c)^2 = \mathcal{O}(b^2 + c^2)$, leading to $M^\nu_2$:

$$M^\nu_2 = \begin{pmatrix} 0 & b_{\text{rad}} & c_{\text{rad}} \\ b_{\text{rad}} & d & \sigma d \\ c_{\text{rad}} & \sigma d + d_{\text{rot}} & d \end{pmatrix}, \quad (12)$$

which finally provides the large solar neutrino mixing.

**II. MODEL**

To realize interactions that yield the “ideal” solution of Eq. (10) and radiative neutrino masses in the specific entries of $\nu_e \nu_\mu$ and $\nu_e \nu_\tau$, as in Eq. (11), we require all interactions to be invariant under the transformations of two symmetries, $U(1)_{\phi}$ and $U(1)_{\phi'}$ associated with $N_\phi$ and $N_{\phi'}$, where $(N_\phi, N_{\phi'}) = (1, 0)$ for $\phi; = (0, 1)$ for $\phi'; = (0, -1)$ for $\psi_L'; =
Higgs interactions are listed. However, the ordinary Higgs interactions are described by usual Hermitian terms composed of \( \phi^\dagger \phi \) for possible flavor-changing interactions, which cause rare decays such as \( \tau \to \mu \mu \mu, \mu e e, \mu \gamma \) and \( \mu \to e e e, e \gamma \). It will be shown that these interactions are well suppressed.

The Yukawa interactions for leptons are, now, given by

\[
\mathcal{L}_Y = \sum_{i=\mu, \tau} \left( f_{0, \tau}^\dagger \psi_L^\dagger \phi^\dagger \ell_R^\tau + f_{e, \tau}^\dagger \psi_L^\dagger \phi^\dagger \ell_R^\tau + \sum_{(i,j)} (\psi_L^\dagger \phi^\dagger \psi_L^\tau h^+) \right) + \sum_{i,j=\mu, \tau} \left( f_{ij}^\dagger \psi_L^\dagger \phi^\dagger \ell_R^\tau + \frac{1}{2} f_{(ij)} (\psi_L^\dagger \phi^\dagger \psi_L^\tau) \epsilon_{\mu, \tau} \right) + (h.c.),
\]

where \( f \)'s stand for coupling constants and the subscripts of \([ij]\) and \((ij)\), respectively, denote the symmetrization and antisymmetrization with respect to \( i \) and \( j \). The masses of \( \mu \) and \( \tau \) are taken to be diagonal for simplicity. To meet the "ideal" solution of Eq.(10), we set \( f_{(\mu \mu)} = f_{(\tau \tau)} = \sigma f_{(\mu \tau)} \). Although this solution may require other dynamics or a certain symmetry restriction such as the one from a permutation symmetry of \( S_2 \) for the \( \mu \) and \( \tau \) [20,23], we do not further pursue such appropriate physical reasons. Instead, we examine how the large solar neutrino mixing is implemented [24] in our radiative mechanism by adopting this "ideal" solution.

Higgs interactions are described by usual Hermitian terms composed of \( \phi^\dagger \phi \) (\( \phi = \phi, \phi' \), \( h^+, s \)) and by non-Hermitian terms in

\[
V_0 = \mu_0 \phi^\dagger \phi' h^{+\dagger} + \lambda \phi^\dagger s \phi' h^{+\dagger} + \mu \phi^\dagger s \phi^\dagger h'^\dagger + (h.c.),
\]

where \( \mu_0 \) and \( \mu \) represent mass scales and \( \lambda \) stands for a Higgs coupling. Other possible couplings are forbidden by the conservation of \( N_{\phi} - N_{\phi'} \). This situation can be read off from TABLE II, where \( L, N_{\phi}, N_{\phi'} \) and \( N_{\phi} - N_{\phi'} \) for possible Higgs interactions are listed. However, \( N_{\phi} \) and \( N_{\phi'} \) are to be spontaneously broken and a Nambu-Goldstone boson associated with \( N_{\phi} + N_{\phi'} \) is absorbed by the gauge bosons of \( SU(2)_L \times U(1)_Y \), but the one associated with \( N_{\phi} - N_{\phi'} \) remains massless. To avoid the appearance of the massless Nambu-Goldstone boson is achieved by introducing a soft breaking term found in TABLE II.

The use of the conservation of \( N_{\phi} - N_{\phi'} \) organizes mass terms of neutrinos and charged leptons such that \( e-i \) terms and \( i-j \) terms \((i,j=\mu, \tau)\) have different origins, which are translated into the appearance of a generalized \( L' \) symmetry, which is identical to \( L' \) for \( \psi_{L,\mu,\tau} \). It should be noted that another advantage of using \( U(1)_{\phi-\phi'} \) lies in the suppression mechanism for a divergent term of \( \nu_{\tau}/\nu_{\tau} \) at the two loop level, which would require a tree level mass term of \( \nu_{\tau}/\nu_{\tau} \) as a counter term. This is caused by the interaction of \( (h^{+\dagger}h^{+}) \) \((\ell_L^\dagger \ell_L^\dagger) \) as depicted in FIG.1(a). Although this type of the diagram is forbidden by \( U(1)_{\phi-\phi'} \), it is in fact allowed by the formation of \( (0|\phi^0|0) \neq 0 \) and \( (0|\phi'^0|0) \neq 0 \), which yields FIG.1(b) through the interactions of \( \phi'^\dagger \phi h'^\dagger + \phi'^\dagger s \phi^\dagger \). However, this diagram leads to the finite convergent term.

III. NEUTRINO OSCILLATIONS

A. Charged leptons

Since charged lepton masses include off-diagonal terms, the form of the neutrino mass matrix is affected by the diagonalization process of charged lepton masses so as to maintain diagonal weak currents. The Yukawa couplings of Eq.(13) provide the charged lepton mass matrix, \( M_0 \), parameterized by

\[
M_0 = \begin{pmatrix}
0 & \delta m_{\mu}\tau & \delta m_{\mu}\tau \\
\delta m_{\mu}\mu & m_\mu^0 & 0 \\
\delta m_{\mu}\tau & 0 & m_\tau^0
\end{pmatrix},
\]

arising from \( \langle 0|M_0^\dagger(\phi, \phi')|0 \rangle \) with \( M_0(\phi, \phi') \) defined by

\[
M_0(\phi, \phi') = \begin{pmatrix}
0 & f_{\mu}\phi & f_{\tau}\phi \\
f_{\mu}\phi & 0 & f_{\mu}\phi' \\
f_{\tau}\phi & f_{\tau}\phi' & 0
\end{pmatrix}.
\]
where \( f^\phi_i = f^\phi_{ei} (=f^\phi_i) \) for \( i = \mu, \tau \) have been assumed. The masses of \( m_i \) and \( \delta m \) are given by

\[
\delta m_{ei} = f^\phi_i \nu_{ei}, \quad m_0 = f^\phi_i \nu_{e'}. \tag{17}
\]

for \( \nu_{ei} (\nu_{e'j}) = \langle 0|0|0 \rangle (|0|0|0) \rangle \) and should at least satisfy \( |\delta m_{ei}| \ll m_0 \rangle \) to meet the hierarchy of \( m_\tau \ll m_\mu \ll m_\tau \). To realize \( m_\tau \ll m_\mu \ll m_\tau \) requires new physics beyond the standard physics; however, we do not intend to discuss how the hierarchy is physically explained but we simply use the parameterization based on the “hierarchical” one [26] to examine its influence on neutrino oscillations [27].

It is straightforward to reach \( U_\ell \) that transforms \( M_{\ell}^0 \) into \( M^\ell = U_\ell^\dagger M_{\ell}^0 U_\ell = \text{diag}(-m_e, m_\mu, m_\tau) \):

\[
U_\ell = \begin{pmatrix}
1 - \frac{r_{ee}^2 + r_{\tau\tau}^2}{2} & r_{ee} & r_{e\tau} \\
r_{ee} & 1 - \frac{r_{ee}^2}{2} & r_{e\tau} m_0 r_{m\mu} \\
r_{e\tau} & r_{e\tau} - \frac{m_0}{m_\tau - m_\mu} & 1 - \frac{r_{e\tau}^2}{2}
\end{pmatrix}, \tag{18}
\]

up to the second order of \( \delta m^\ell \)'s contained in \( r_{ee, e\tau} \):

\[
r_{ee} = \frac{\delta m_{ee}^\ell}{m_0^\mu}, \quad r_{e\tau} = \frac{\delta m_{e\tau}^\ell}{m_0^\mu} \tag{19}
\]

and the diagonal masses are calculated to be

\[
m_e = m_\mu^0 r_{ee}^2 + m_\tau^0 r_{e\tau}^2, \quad m_\mu = m_\mu^0 (1 + r_{ee}^2), \quad m_\tau = m_\tau^0 (1 + r_{e\tau}^2), \tag{20}
\]

which give the upper bounds on \( r_{ee, e\tau} \):

\[
|r_{ee}| \lesssim \sqrt{m_e/m_\mu}, \quad |r_{e\tau}| \lesssim \sqrt{m_e/m_\tau}, \tag{21}
\]

respectively, from \( m_e \geq r_{ee}^2 m_\mu^0 \approx r_{ee}^2 m_\mu \) and \( m_e \geq r_{e\tau}^2 m_\tau^0 \approx r_{e\tau}^2 m_\tau \).

Even after the rotation of the mass term given by \( U_\ell^\dagger (0|M_\ell^0(\phi, \phi')|0)U_\ell \), our Yukawa interactions corresponding to \( U_\ell^\dagger M_{\phi, \phi'}^0(U_\ell = M^\ell (\phi, \phi')) \) contain flavor-off-diagonal couplings. We find that \( M^\ell (\phi, \phi') \) is calculated to be:

\[
M^\ell (\phi, \phi') = U_\ell^\dagger M_{\phi, \phi'}^0 U_\ell \\
= \begin{pmatrix}
- (2\alpha_\phi - \alpha_\phi') m_e & (\alpha_\phi' - \alpha_\phi') r_{e\mu} m_0 \\
(\alpha_\phi' - \alpha_\phi) r_{e\mu} m_0 & (\alpha_\phi' + (2\alpha_\phi - \alpha_\phi') r_{e\mu}) m_0 \\
(\alpha_\phi' - \alpha_\phi) r_{e\tau} m_0 & (\alpha_\phi - \alpha_\phi') r_{e\tau} m_0 + (\alpha_\phi' + (2\alpha_\phi - \alpha_\phi') r_{e\tau}) m_0
\end{pmatrix}, \tag{22}
\]

where \( \alpha_\phi = \phi/v_\phi \) and \( \alpha_\phi' = \phi'/v_\phi' \), which induces flavor-changing interactions for \( \tau \) and \( \mu \). Of course, Eq.(22) with the identification of \( \alpha_\phi' \) with \( \alpha_\phi \), corresponding to the case of the standard model, only contains diagonal Higgs couplings giving rise to diag.\( (-m_e, m_\mu, m_\tau) \). The flavor-changing interactions are roughly controlled by the suppression factor of order \( r_{ee} r_{e\tau} (m_\mu/v_{\text{weak}})(m_\tau/v_{\text{weak}}) (v_{\text{weak}}/m)^2 (\sim \xi_1) \) for \( \tau \rightarrow \mu \mu e, e\gamma \) and \( r_{e\mu} (m_e/v_{\text{weak}})(m_\mu/v_{\text{weak}})(v_{\text{weak}}/m)^2 (\sim \xi_2) \) for \( \tau \rightarrow \mu \mu e, e\gamma \) and \( r_{ee} (m_e/v_{\text{weak}})(m_\mu/v_{\text{weak}})(v_{\text{weak}}/m)^2 (\sim \xi_3) \) for \( \mu \rightarrow eee, e\gamma \), where \( m \) is a mass of the mediating Higgs scalar and \( v_\phi \sim v_\phi' \sim v_{\text{weak}} = (2\sqrt{2}G_F)^{-1/2}=174 \text{ GeV} \). We find that to suppress these interactions to the phenomenologically consistent level requires a rough estimate of \( |\xi_{1,2,3}| \lesssim 10^{-5} \) [20], which can be fulfilled because of \( m \approx v_{\text{weak}} \) and Eq.(21) for \( r_{ee, e\tau} \). It should be noted that there are no such Higgs interactions for quarks that only couple to \( \phi \). The similar flavor-changing interactions caused by \( h^\ell \) [25] are sufficiently suppressed because of the smallness of the \( \ell^\ell \)-couplings to leptons to be estimated in Eq.(35).

**B. Neutrinos**

To estimate effects on the neutrino mass matrix through the rotation due to \( U_\ell \), we shift the original base into the one with the diagonalized charged lepton masses, which forces us to rotate the original neutrinos \( (|\nu|) \) into \( |\nu_{\text{weak}}| \) in
the weak base by $|\nu_{\text{weak}}\rangle = U_f^T |\nu\rangle$. The radiative neutrino masses to be denoted by $\delta m_{ij}^\nu$, are generated by interactions corresponding to FIG.2. For the sake of simplicity, we set $v_\phi = v_\nu^{s\nu}$ and $m_\phi = m_\nu^{s\nu}$, 1 where $m_\phi, m_\nu^{s\nu}$ denote the masses of $\phi^+$ and $\phi^{s\nu}$, and calculate $\delta m_{ij}^\nu$ to be:

$$
\delta m_{ij}^\nu = K_{\phi'} \left( U_f^T f M_0^f M_0^f U_f \right)_{ij} - (i \leftrightarrow j),
$$

(23)

for $|\nu_{\text{weak}}\rangle$, where $f_{ij} = f_{[ij]}$ and $K_{\phi'}$ is the one-loop factor:

$$
K_{\phi'} = \frac{\mu_0}{16\pi^2} \ln \frac{m_h^2}{m_{\phi'}^2},
$$

(24)

where $m_h$ is a mass of the $h^+$ scalar. We find that

$$
\delta m_{ij}^\nu = K_{\phi'} f_{[ij]} \left( m_{\phi'}^2 - m_{\nu}^2 \right),
$$

(25)

where the couplings of $F_{[ij]}$ are defined by $F_{[ij]}= (U_f^T f U_f)_{ij}$, which result in

$$
F^{e\mu}_{[\mu\mu]} = f^{e\mu}_{[\mu\mu]}, \quad F^{e\tau}_{[\tau\tau]} = f^{e\tau}_{[\tau\tau]}, \quad F^{\mu\tau}_{[\mu\tau]} = r_{\mu\tau} f^{\mu\tau}_{[\mu\tau]} - r_{\tau\mu} f^{\mu\tau}_{[\mu\tau]}.
$$

(26)

The tree level masses, $m_{ij}^\nu$, are given by the type II seesaw mechanism to be:

$$
m_{ij}^\nu = f_{[ij]} v_s \approx f_{[ij]} \mu^2 v_s^2
$$

(27)

where $f_{[\mu\mu]} = f_{[\tau\tau]} = \sigma f_{[\mu\tau]} (\equiv f_s)$ and $v_s = (0|s^0|0)$.

Collecting these results, we find that our mass matrix of Eq.(2) has the following mass parameters:

$$
a = 0, \quad b = \delta m_{e\mu}^\nu = K_{\phi'} f^{e\mu}_{[\mu\mu]} m_{\mu}^2, \quad c = \delta m_{e\tau}^\nu = K_{\phi'} f^{e\tau}_{[\tau\tau]} m_{\tau}^2,
$$

$$
d = m^\nu, \quad e = \sigma m^\nu + \delta m_{e\tau}^\nu = \sigma m^\nu + K_{\phi'} f^{\mu\tau}_{[\mu\tau]} m_{\tau}^2, \quad f = m^\nu,
$$

(28)

where $m^\nu = f_s v_s$ and $m_{\mu}^2 \ll m_{\nu}^2$ has been used to estimate Eq.(25). The two-loop convergent contribution shown in FIG.1(b) to the $\nu_e\nu_e$ mass for the $a$-term is well suppressed by the presence of $m_s$ contained in the propagator of $s$ and does not jeopardize $a=0$. The maximal atmospheric neutrino mixing defined by $|t_{23}| = 1$ calls for the “inverse” hierarchy of $f^{e\mu}_{[\mu\mu]}$ and $f^{e\tau}_{[\tau\tau]}$ [28] expressed as

$$
f^{e\mu}_{[\mu\mu]} m_{\mu}^2 = -\sigma f^{e\tau}_{[\tau\tau]} m_{\tau}^2.
$$

(29)

For the solar neutrino oscillations, the radiative $\nu_\mu - \nu_\tau$ mixing term of $F_{[\mu\tau]}$ arises via $r_{\mu\tau}$ as a result of the rotation due to the $e-\mu$ and $e-\tau$ mixings. It is this term that gives a deviation of $\sin^2 2\theta_{12}$ from unity. The masses of neutrinos satisfy the “normal” mass hierarchy of $|m_1| < |m_2| < m_3$ determined by Eq.(4) to be:

$$
m_1 = -\eta \sqrt{8 + x^2} - |x| \delta m_{\nu_{\text{rad}}}^\nu, \quad m_1 = \eta \sqrt{8 + x^2} + |x| \delta m_{\nu_{\text{rad}}}^\nu,
$$

$$
m_3 = 2m^\nu + \eta \xi \delta m_{\nu_{\text{rad}}}^\nu
$$

(30)

with $\delta m_{\nu_{\text{rad}}}^\nu = \sqrt{\left( \delta m_{\nu_{\text{rad}}}^\nu \right)^2 / 2}$, where the sign of $\eta$ has been adjusted to yield $\eta x = -|x|$ for $|m_1| < |m_2|$ and $x$ measures the ratio of the $\nu_\mu - \nu_\tau$ mixing mass over the $\nu_\tau - \nu_\tau$ mixing mass, which is defined by

$$
x = \frac{\delta m_{\nu_{\tau}}^\nu}{\delta m_{\nu_{\tau}}^\nu} = \frac{F_{[\mu\tau]} - r_{\mu\tau} f_{[\mu\tau]}}{|F_{[\tau\tau]}|}.
$$

(31)

1 Our later discussions are still valid as far as $|r K_\phi - r^{-1} K_{\phi'}| \leq |K_{\phi'}| / 10$ for $r = v_\phi^{s\nu} / v_\nu$. For instance, $a = (\delta m_{\nu_{\tau}}^\nu)$ is computed to be $2(r K_\phi - r^{-1} K_{\phi'}) (r_{\mu\tau} - r_{\tau\mu}) f^{e\mu}_{[\mu\mu]} m_{\mu}^2$ for $f^{e\mu}_{[\mu\mu]} m_{\mu}^2 = -\sigma f^{e\tau}_{[\tau\tau]} m_{\tau}^2$, which is at most $r_{\mu\tau} b/10$ being harmless for our estimation, and similarly for other elements.
Then, $\Delta m_{\text{atm,}2}^2$ and $\sin^2 2\theta_{12}$ are calculated to be:

$$
\Delta m_{\text{atm,}2}^2 \approx 4m_{\nu}^2 + 4\eta m_{\nu} \delta m_{\text{rad}}^\nu, \quad \Delta m_{\nu}^2 \approx |x|\sqrt{8 + x^2} \delta m_{\text{rad}}^\nu,
$$

$$
\sin^2 2\theta_{12} = \frac{8}{(8 + x^2)}.
$$

(32)

To get an estimate of $x$, we parameterize $r_{e\mu, e\tau}$ as

$$
r_{e\mu} = c_\ell \sqrt{\frac{m_\mu}{m_\tau}}, \quad r_{e\tau} = s_\ell \sqrt{\frac{m_\mu}{m_\tau}}
$$

(33)

to satisfy the constraint of Eq.(20) on the electron mass, where $c_\ell = \cos \theta_\ell$ and $s_\ell = \sin \theta_\ell$. The parameter of $x$ turns out to be given by

$$
|x| = |0.07c_\ell + 4.86\sigma s_\ell|,
$$

(34)

where the hierarchical coupling condition of Eq.(29) has been used. For $|x| = \sqrt{2}$ corresponding to $s_\ell \sim \pm 0.3$, $\sin^2 2\theta_{12} = 0.8$ and $\delta m_{\text{rad}}^\nu = 3.2 \times 10^{-3}$ eV are obtained. The tree level mass of $m_\nu^\nu$ is estimated to be $\sim 0.027$ eV for $\Delta m_{\text{atm}}^2 = 3 \times 10^{-3}$ eV$^2$. The type II seesaw mechanism yields an estimate of the mass parameter: $m_\alpha (= \mu) = 1.2 \times 10^{14} \times (|F_{\nu\mu}|/|e|)$ GeV for $v_\phi (= v_\phi') = v_{\text{weak}}/\sqrt{2}$ to meet $v_\phi^2 + v_\phi'^2 = v_{\text{weak}}^2$ for the weak boson masses, where $e$ is the electromagnetic coupling. From Eq.(29) for the maximal atmospheric neutrino mixing,

$$
f_{[\mu \nu]} \sim 2.9 \times 10^{-5}, \quad f_{[e \tau]} \sim -\sigma \times 10^{-7},
$$

(35)

together with $|F_{[\mu \nu]}| = \sqrt{2}|f_{[e \tau]}|$ are obtained, where $\mu_0 = m_\phi = m_\phi' = v_{\text{weak}}$ and $m_{h^+} = 3v_{\text{weak}}$ are used to compute the loop-factor of $K_{\nu'}$. The masses of $m_{1,2,3}$ are predicted to be:

$$
|m_1| = 2.8 \times 10^{-3} \text{ eV}, \quad |m_2| = 7.3 \times 10^{-3} \text{ eV}, \quad m_3 = 5.5 \times 10^{-2} \text{ eV}.
$$

(36)

IV. SUMMARY

Summarizing our discussions, we have demonstrated that the use of $U(1)_2$ and $U(1)_{\phi'}$ successfully opens a window for both $\sin^2 2\theta_{23} = 1$ for atmospheric neutrino oscillations and the LMA solution with $\sin^2 2\theta_{12} \sim 0.8$ for solar neutrino oscillations. The mass mixings in the $e-\mu$ and $e-\tau$ entries in the neutrino mass matrix, i.e. the $\nu_e-\nu_\mu$ and $\nu_e-\nu_\tau$ mixing masses of $\delta m_{\nu}^\mu$ and $\delta m_{\nu}^\tau$, determine the atmospheric neutrino mixing: $\tan \theta_{23} = -\delta m_{\nu}^\nu/\delta m_{\nu}^\mu$ and supply solar neutrino masses proportional to $\delta m_{\text{rad}}^\nu (= \sqrt{(\delta m_{\nu}^\mu)^2 + (\delta m_{\nu}^\tau)^2}/2)$. On the other hand, mass mixings in the charged lepton mass matrix, i.e. the $e-\mu$ and $e-\tau$ mixing masses of $\delta m_{\nu}^\mu$ and $\delta m_{\nu}^\tau$, are rotated into the electron mass given by $\delta m_{\nu}^\mu/m_\mu + \delta m_{\nu}^\tau/m_\tau$ and induce the $\delta m_{\nu}^\nu$-mixing mass as effects of the rotation, which yields the deviation of $\sin^2 2\theta_{12}$ from unity. This induced $\nu_e-\nu_\mu$ mixing determines $\sin^2 2\theta_{12} = 8/(8 + x^2)$ for $x = \delta m_{\nu}^\nu/\delta m_{\nu}^\mu$, leading to $\sin^2 2\theta_{12} = 0.8$ for $|x| = \sqrt{2}$, which corresponds to the mixing angle of $s_\ell \sim \pm 0.3$ that measures the $\tau$-contribution in the electron mass as in Eq.(33). To induce the observed neutrino oscillations is thus based on the radiative mechanism and the rotation process to get the electron mass, which can be illustrated by

$$
\begin{pmatrix}
0 & 0 & 0 \\
0 & m_\nu & \sigma m_\nu \\
0 & \sigma m_\nu & m_\nu
\end{pmatrix}
\xrightarrow{\text{radiative}}
\begin{pmatrix}
0 & \delta m_{\nu}^\mu & \delta m_{\nu}^\nu \\
\delta m_{\nu}^\mu & m_\nu & \sigma m_\nu \\
\delta m_{\nu}^\nu & \sigma m_\nu & m_\nu
\end{pmatrix}
\xrightarrow{\text{rotation}}
\begin{pmatrix}
0 & \delta m_{\nu}^\nu & \delta m_{\nu}^\nu \\
\delta m_{\nu}^\nu & m_\nu & \sigma m_\nu + \delta m_{\nu}^\mu \\
\delta m_{\nu}^\mu & \sigma m_\nu + \delta m_{\nu}^\mu & m_\nu
\end{pmatrix}.
$$

Once the democratic mass structure for $\nu_\mu$ and $\nu_\tau$ as the $m^\nu$-terms is realized, we can generate radiative neutrino masses whose texture is compatible with the large solar neutrino mixing.

Acknowledgements
The authors are grateful to Y. Koide for valuable discussions. One of the authors (M.Y.) also thanks to the organizers and participants in Summer Institute 2001 at FujiYoshida, Yamanashi, Japan, for useful comments. The work of M.Y. is supported by the Grants-in-Aid for Scientific Research on Priority Areas A, "Neutrino Oscillations and Their Origin," (No 12047223) from the Ministry of Education, Culture, Sports, Science, and Technology, Japan.


### TABLES

**TABLE I.** The lepton number ($L$), $N_\phi, N_{\phi'}$ and $N_\phi \pm N_{\phi'}$ for leptons and Higgs scalars, where $N_\phi + N_{\phi'}$ is nothing but the hypercharge.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\psi_L^\mu$</th>
<th>$\psi_L^\tau$</th>
<th>$\epsilon_R$</th>
<th>$\mu_R, \tau_R$</th>
<th>$\phi$</th>
<th>$\phi'$</th>
<th>$h^+$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$N_\phi$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$N_{\phi'}$</td>
<td>-1</td>
<td>-1</td>
<td>-2</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$N_\phi + N_{\phi'}$</td>
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<td>-1</td>
<td>-2</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$N_\phi - N_{\phi'}$</td>
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<td>-2</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>2</td>
</tr>
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</table>

**TABLE II.** $L$, $N_\phi, N_{\phi'}$ and $N_\phi - N_{\phi'}$ for Higgs interactions, where $N_\phi + N_{\phi'} = 0$.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\phi'\phi h^+\dagger$</th>
<th>$\phi'\phi$</th>
<th>$(h^+h^+)^\dagger\det s$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$N_\phi$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$N_{\phi'}$</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$N_\phi - N_{\phi'}$</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
<th>$\phi'\phi^c h^+\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>$N_\phi$</td>
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<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$N_{\phi'}$</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$N_\phi - N_{\phi'}$</td>
<td>4</td>
<td>-2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\langle | \phi \rangle & \quad \langle | \phi' \rangle \\
\langle | \phi' \rangle & \quad \langle | \phi'' \rangle \\
\langle | \phi'' \rangle & \quad \langle | \phi''' \rangle
\end{align*}
\]

\[
\mu \tau = \mu \tau
\]

\[
\langle | \phi' \rangle | \phi' \rangle
\]