Jet Tomography of Au+Au Reactions Including Multi-gluon Fluctuations

M. Gyulassy¹, P. Levai², and I. Vitev¹

1 Department of Physics, Columbia University, 538 W. 120th Street, New York, NY 10027, USA
2 KFKI Research Institute for Particle and Nuclear Physics, PO Box 49, Budapest 1525, Hungary

Jet tomography is the analysis of the attenuation pattern of high transverse momentum hadrons to determine certain line integral transforms of the density profile of the QCD matter produced in ultra-relativistic nuclear collisions. In this letter, we calculate the distortion of jet tomography due to multi-gluon fluctuations within the GLV radiative energy loss formalism. We find that fluctuations of the average gluon number, \( \langle N^0 \rangle \sim 3 \) for RHIC initial conditions, reduce the attenuation of pions by approximately a factor \( Z \approx 0.4 - 0.5 \). Therefore the plasma density inferred from jet tomography without fluctuations must be enhanced by a factor \( 1/Z \sim 2 \).

PACS numbers: 12.38.Mh, 24.85.+p, 25.75.-q

I. INTRODUCTION

The discovery of a factor of \( \approx 3 \) suppression of moderate \( p_T \lesssim 4 \) GeV \( \pi^0 \)'s in central \( Au + Au \) reactions by PHENIX [1] and large transverse asymmetries in non-central collisions for \( p_T \lesssim 5 \) GeV by STAR [2] have inspired several attempts [3-7] to begin jet tomographic analysis of the matter density produced in ultra-relativistic nuclear reactions. Jet tomography is the QCD analog of conventional X-ray or positron tomography in that it exploits the attenuation of high energy jets produced in a variable density medium [8]. The main source of attenuation of jets in QCD is induced gluon radiation due to multiple interactions in the medium. While the present data at moderate \( p_T \) are not conclusive, the tomographic analysis suggests that densities up to 100 times nuclear densities may have already been achieved. Soon very high statistics data out to \( p_T \sim 10 - 20 \) GeV will be obtained, and it is important to refine the theory of jet tomography to take into account many sources of distortion of the attenuation pattern. In this letter we provide details of the calculation of the distortions due to gluon number fluctuations and compute the correction factors to the deduced densities. Such density renormalization factors \( Z \sim 0.4 - 0.5 \) have already been applied to calculations of particle spectra at RHIC [9,10].

Jet quenching probes the gluon density of the medium, characterized through the opacity parameter \( L/\lambda_g = \int \sigma_g(\tau) p_g(\tau) \, d\tau \). For the moderate opacities expected in nuclear collisions in the \( \sqrt{s} \sim 200 \) AGeV range, it is convenient to calculate the induced radiation as a power series expansion in the opacity. In Ref. [11] we derived an analytic expression for the medium induced gluon radiation spectrum, \( \rho(x) = dN_g/dx = \sum_{n=1}^{\infty} \rho^{(n)}(x) \), in such a series form, where \( x \) is the light cone momentum fraction carried by the radiated gluon (see also [12] and [13] for the relation of this approach to the asymptotic approach of Ref. [14]).

The result for the \( n^{th} \) order in opacity contribution to \( \rho(x) \) for a jet of energy \( E \) and color Casimir \( C_R \) that is produced inside a medium with opacity \( L/\lambda_g \) can be expressed from [11] in the following form

\[
\rho^{(n)}(x, E) = \frac{C_R \alpha_s}{\pi^2} \frac{1}{x^2 (n+1)!} \left( \frac{L}{\lambda_g} \right)^n \theta \left( x - \frac{\mu}{2E} \right) \int_{K_{\min}}^{K_{\max}} \frac{d^2 k}{\mu^2} \int_1^n \prod_{i=1}^{n} \left[ \frac{d^2 q_i}{2\pi^2} \frac{d^2 \sigma^{eff}(q_i)}{d^2 q_i} \right] \times \sum_{m=1}^{n} A_{n,m} F_{n,m} .
\]

The distribution of transverse momentum impulses is given by an effective dipole-like differential cross section that has an elastic component assumed to be given by a color screened Yukawa form [15] and a \( \delta \)-function component in the forward (jet) direction \( \frac{d\sigma^{eff}(q_i)}{d^2 q_i} = \frac{\mu^2}{\pi} \frac{1}{(q_i^2 + \mu^2)^2} - \delta^2(q_i) \). We take \( \mu \simeq gT \sim 0.5 \) GeV for RHIC initial conditions.

One important advantage of the opacity formalism is that the effects of a finite temperature QCD plasmon frequency cutoff \( \omega_{pl} \sim gT/\sqrt{3} \) of the gluon radiation can be taken into account approximately via the kinematical cut-off both at small \( x \ll x_0 = \frac{\mu}{2E} \) and at small \( k \ll \mu = K_{\min} \). We ignore the \( \sqrt{3} \) difference between the plasma frequency and the screening mass since \( g \sim 2 \) and approximate both by \( \sim 0.5 \) GeV. The upper bound \( K_{\max}(x) = 4E^2 \min(x^2, (1-x)^2) - \mu^2 \) in \( k^2 \) results from requiring the quenched jet as well as the radiated gluons have positive forward momenta. The fraction \( x_c/x \equiv \mu^2 L/(2xE) \) is a measure of the thickness of the medium to the gluon formation length.

The radiation amplitudes are here denoted by \( A_{n,m} = 2 (k - Q_m) \cdot (C_{n,m+1} - C_{n,m}) \), where \( Q_n \equiv q_1 + \cdots + q_n \), with \( Q_0 \equiv 0 \), and \( C_{n,m} = \frac{1}{n} \nabla_k \log(k - Q_m + Q_{m-1})^2 \). Destructive interference suppresses radiation with formation times greater than the thickness, \( L \), of the medium. In addition to one power of \( x_c/x \) in Eq.(1), higher order contributions are further suppressed by a formation factor from Eq. (116) of [11].
\[ F_{n,m} \equiv \text{Im} \prod_{j=1}^{m} \left( 1 + i \frac{x_n^2 (k - Q_n + Q_{j-1})^2}{\mu^2 (n+1)} \right)^{-1}. \]  

(2)

This simple analytic form arises for an exponential distribution, \( \propto \exp \left( - (z_k - z_{k-1}) \frac{L}{n+1} \right) \), between adjacent scattering centers, \( z_k \), in a plasma with mean thickness \( L \) at \( n^{th} \) order in opacity.

Gluon reabsorption from the medium reduces the radiation density \( \rho(x) \) at low \( x \) as shown in [16]. However, this effect is only important for jets with \( E \) less than a few GeV. Here we focus on higher energy jets. We consider only the static plasma geometry in Eq. (2) to simplify the numerical evaluation of \( \rho^{(n)}(x) \) at higher orders. In Refs. [3,6,8] we showed that the main effect of 3+1D (Bjorken+transverse) expansion in the opacity expansion is to reduce the mean radiative energy loss, \( \Delta E = Z_{3+1} \cdot \Delta E_{\text{stat}} \), relative to the static approximation by a renormalization factor \( Z_{3+1} = 2 \tau_0 / L \), where \( \tau_0 \) is the formation time of the matter. The effective static opacity \( L/\lambda = 5 \) that we use to fit the PHENIX data is relatively small because it value reflects the rapid dilution effects due to expansion.

The numerically computed mean number of radiated gluons and the mean energy loss up to third order, 

\[ \langle \Delta E \rangle = \int dx \left( \frac{x}{1} \right) \left( \rho^{(1)}(x,E) + \rho^{(2)}(x,E) + \rho^{(3)}(x,E) \right), \]  

are shown in Figs. 1 and 2. The curves are calculated using \( \mu = 0.5 \) GeV, \( \lambda_g = 1 \) fm, and \( C_R = 3 \) and running \( \alpha_s \).

The leading first order gluon energy loss \( \Delta E^{(1)} \), is found to roughly follow the leading log expression \( \Delta E^{(1)} / E \approx \frac{3 \alpha_s \mu L^2}{4 \lambda_g} \log \frac{1}{x} \) from [6,13]. The second order contribution reduces the first order result by a factor \( \sim 2 \) for \( E \lesssim 10 \) GeV. By 40 GeV the second order correction is only 10%. For \( E > 40 \) both the second and third order correction are negligible, but also for \( E \approx 5 - 20 \) GeV the third order contribution largely cancels the second order one. The summed first through third orders shows that the induced fractional energy loss varies from 0.5 at \( E \approx 5 \) GeV down to \( \sim 0.3 \) at 20 GeV. Jet tomography at RHIC is dominated by quark fragmentation for pions with \( p_T > 5 \) GeV with \( \Delta E \) reduced by a factor \( C_F/C_A = 4/9 \).

A similar pattern is seen in the energy dependence of the average number of gluons radiated in Fig. 2. The second and third order corrections largely cancel and the final average gluon number is only 2–3 for the kinematic range accessible at RHIC. Even with sensitivity \( \sim 2 \) on the kinematic cut-offs, \( \langle N^g \rangle \) is small. In fact as a function of energy the gluon number saturates for any given opacity \( L/\lambda_g \). The gluon radiative distributions are strongly peaked at...
small $x$ (which is consistent with the small $x$ approximations used) and naturally need a lower cut-off generated by a characteristic jet energy independent mass scale $m$, i.e. $x_{\text{min}} \sim m/E$. For any gluon radiative distribution that has a form $\rho(x, E) \approx 1/x_c f(x/x_c)$ (with $x_c$ = typical energy scale/E) the mean number of gluons due to induced radiation $\langle N^g \rangle$ is approximately jet energy independent. As the energy of the jet and the radiative energy loss increase $\langle N^g \rangle$ remains small but the radiated gluons become harder.

We note that both $\Delta E$ and $\langle N^g \rangle$ are much smaller than the recent estimates in [14] because our effective transport coefficient $\mu^2/\lambda_g \approx 0.25 \text{ GeV}^2/\text{fm}$, which is constrained by our fit below (see Fig. 4) to the PHENIX pion attenuation data [1], is 4 times smaller than the one considered for illustration in [14]. As noted before, expansion greatly reduces estimates based on the transport properties of the high density initial conditions. In addition, the finite plasmon frequency cut-off, $\sim \mu$, is the medium regulator of soft gluon number divergences. We emphasize that it is the finite medium dependent cut-off of the soft spectrum together with the relatively small transport coefficient and opacity of the plasma produced at RHIC energies that allows the opacity expansion to converge so rapidly even under the extreme conditions produced at RHIC.

![Gluon jets](image)

**FIG. 2.** Average number of gluons, $\langle N^g(E) \rangle$, with $\omega > \mu$ is plotted versus $E$ for opacity $L/\lambda_g = 5$. A set of curves corresponding to the curves in Fig. 1 is presented.

### II. FLUCTUATION SPECTRUM OF RADIATIVE ENERGY LOSS

In the approximation that the fluctuations of the gluon number are uncorrelated, the spectrum of the total radiative energy loss fraction, $\epsilon = \sum_i \omega_i/E$, can be expressed via a Poisson expansion $P(\epsilon, E) = \sum_{n=0}^{\infty} P_n(\epsilon, E)$ with $P_1(\epsilon, E) = e^{-\langle N^g \rangle} \rho(\epsilon, E)$ and

$$P_{n+1}(\epsilon, E) = \frac{1}{n+1} \int_{x_0}^{1-x_0} dx_n \rho(x_n, E) P_n(\epsilon - x_n, E)$$

$$= e^{-\langle N^g(\epsilon) \rangle} \frac{1}{(n+1)!} \int dx_1 \cdots dx_n \rho(x_1, E) \cdots \rho(x_n, E) \rho(\epsilon - x_1 - \cdots - x_n, E). \quad (4)$$

The form of this spectrum guarantees that the mean value is as in Fig. 1:

$$\int_0^\infty d\epsilon \ P(\epsilon, E) \epsilon = \frac{\Delta E}{E}. \quad (5)$$
The above distribution differs considerably from that computed in [14] because $\langle N^0(E) \rangle$ is finite and small in our case and because $\rho(x) = 0$ for $x < \mu/(2E)$ due to the plasma frequency cutoff that we impose on the low frequency modes in the medium. Therefore, we have explicitly a finite $n = 0$ (no radiation) contribution $P_0(\epsilon, E) = e^{-(N^0(E))} \delta(\epsilon)$. Assuming negligible kinematic correlations the numerical iteration of the recursion relation in Eq. (4) becomes very fast and is computed to high order ($n \leq 25$). However, by not enforcing that $\sum x_i \leq 1$, there is a “leakage” error into the unphysical $\epsilon > 1$ range. We calculate this “leakage” error $\int_1^\infty d\epsilon P(\epsilon, E)$ and correct the normalization of $P(\epsilon, E)$ in the unphysical range $\epsilon \in [0, 1]$.

The resulting spectrum (without the delta function contribution) is shown in Fig. 3 for three different jet energies. The finite intercept at $\epsilon = 1$ provides a measure of the “leakage” error, which is acceptable in this case. The low frequency plasmon cut-off at $\sim \mu$ is clearly visible. In addition multi-gluon iterations in the probability distribution $P(\epsilon, E)$ exhibit a slight oscillatory pattern in multiples of $x_0$. The high frequency random oscillations provide an indication of the accuracy of our Monte Carlo numerical integrations methods.

The results indicate that $P(\epsilon, E)$ is approximately constant from $x_0 = \mu/(2E)$ up to a scale $\sim x_c = \mu^2L/(2E)$. For $x \gg x_c$, $P(\epsilon, E)$ decreases rather quickly. The (normalized to unity) probability distribution per gluon, $\rho(x, E = 40 \text{ GeV})/\langle N^0 \rangle$ is also shown for comparison. Multi-gluon fluctuations flatten the rapid small $x$ rise of $\rho$ even though Eqs. (3,5) dictate that the first moment of both $\rho(x, E)$ and $P(\epsilon, E)$ are the same.

![FIG. 3. Probability density of total fractional energy loss $\epsilon = \sum \omega_i/E$ for a gluon get with $E=10,20$, and 40 GeV traversing matter with opacity $L/\lambda_q = 5$. The numerical curves include orders $1+2+3$ order in the opacity expansion for $\rho(x)$. The $n = 0$ no radiation delta function contribution, $P_0(\epsilon)$, is not shown above. The low frequency plasma cut-off is at $\epsilon = \mu/(2E)$. The probability density per gluon is also included.]

**III. THE QUENCHING PATTERN OF $\pi^0$**

We apply the energy loss spectrum to calculate the quenched spectrum of hadrons by modifying the mean energy loss pQCD formulas from Refs. [3-7,17]. We concentrate on mid-rapidity hadron production ($y_{cm} = 0$). A jet of flavor $c$ and transverse momentum $p_c$ produced in a hard pQCD scattering $a + b \rightarrow c + d$ is attenuated prior to hadronization by the radiative energy loss to $p_c^* = p_c(1-\epsilon)$. This shifts the hadronic fragmentation fraction $z_c = p_h/p_c$ to $z_c^* = z_c/(1-\epsilon)$.

The invariant distribution of $\pi^0$ reduced by energy loss in central $A + A$ collision is then given by

$$E_h \frac{dN^{AA}_{\pi^0}}{d^3p} = T_{AA}(0) \sum_{abcd} dx_1 dx_2 f_{a/A}(x_1, Q^2) f_{b/A}(x_2, Q^2) \frac{d\sigma_{ab\rightarrow cd}}{dt} \int d\epsilon P(\epsilon, p_c) \int_0^{z_c^*} \frac{dz_c}{z_c} \frac{D_{x^0/c}(z_c^*, Q^2)}{\pi z_c},$$

(6)
where $T_{AA}(0)$ is the Glauber profile density in central collisions. The pion fragmentation function $D_{\pi^0/c}(z,Q^2)$ is taken from BKK [18]. We take the GRV94 LO [19] structure functions for $f_{u/p}(x,Q^2)$ and include isospin dependence ($Z$ protons and $A-Z$ neutrons). Nuclear shadowing, intrinsic $k_T$ broadening and Cronin effect can be taken into account as in [20-23]. The interplay between the soft and hard components of hadron production studied in [7,10] lead to modifications of the spectral shapes in the low $p_T$ region and are neglected in this analysis. The factor $z^*/z_c$ appears because of the in-medium modification of the fragmentation function [17]. Thus, the invariant cross section Eq. (6) depends on the average opacity $L/\lambda_g$ through the effect of $P(\epsilon, p_c)$.

We consider three different approximations to $P(\epsilon, E)$:

1. Use only the mean energy loss with $P(\epsilon, E) \approx \delta(\epsilon - \Delta E(E)/E)$ as in [3-7,17]
2. Use the full fluctuating spectrum, $P(\epsilon, E)$, from Eq. (4)
3. Use a renormalized average energy loss with $P(\epsilon, E, Z) \approx \delta(\epsilon - Z \cdot \Delta E(E)/E)$

![Graph](image.png)

**FIG. 4.** Quenching pattern for $\pi^0$ versus transverse momentum order by order in the opacity expansion. Curves labeled $<\Delta E_{1+\ldots}>$ are calculated using the average energy loss up to the order indicated. The average opacity is taken to be $L/\lambda_g = 5$. The data are from PHENIX [1]. Curves labeled $P_{1+\ldots}(\epsilon)$ average over the fluctuating distributions illustrated in Fig. 3. The dot-dashed curves correspond to using a reduced ($Z$ renormalized) first order average energy loss.

The ratio, $R_{AA}(p_T)$, compares the quenched to the unquenched $\pi^0$ distributions. In the case (1), the convergence of the opacity series using the mean energy shift to first and up to third order appears to be reasonably fast even though the second order correction is still uncomfortably large below $p_T \lesssim 10$ GeV. Improved numerical methods need to be developed to enable summing higher order terms to verify our expectation that the summed results to third order are not significantly changed by higher order due to the additional $\frac{1}{(n+1)}$ and $F_{n,m}$ factors in Eq. (1).

We see from Fig. 4 that with even the modest value of the opacity $L/\lambda_g = 5$, the mean energy loss approximation over predicts the observed quenching by about a factor of two. Including the fluctuations in the Poisson approximation via $P(\epsilon, E)$ leads to less energy loss by approximately a factor of two and brings the attenuation in line with the observed results. This renormalization of the effective energy loss can be inferred from the dot-dashed curves using approximation (3) above with $Z \approx 0.4 - 0.5$. We conclude that the distortion of jet tomography due to gluon number fluctuations in the Poisson approximation can be well approximated by renormalizing the mean energy loss calculations by a factor $Z \sim 0.5$.

While the $p_T$ range of the available data is still too low to draw definitive conclusions, the effective static opacity with gluon fluctuation renormalization above corresponds from the results of [7] to an estimated initial gluon rapidity
density \( dN^g/dy \sim 800 \pm 100 \) and implies that the initial gluon density produced at RHIC may have reached \( \rho_g \approx \frac{(dN^g/dy)}{(\tau_0 \pi R^2)} \sim 20/fm^3 \sim 100 \rho_A \).

ACKNOWLEDGMENTS

This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-FG02-93ER40764 and by the U.S. NSF under INT-0000211 and OTKA No. T032796.