Orbitally excited $D$ and $B$ mesons in the approach of the QCD string with quarks at the ends

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Abstract

In this letter we discuss the masses and the splittings of $1^{2S+1}P_J$ states in the spectrum of $D$ and $B$ mesons, as they appear in the approach of the QCD string with quarks at the ends. We find good agreement of our predictions with those of other QCD-motivated models as well as with the lattice and experimental data, including recent experimental results. We discuss the ordering pattern for $P$ levels in $D$- and $B$-mesonic spectrum.

Data on the spectroscopy of heavy-light mesons coming from various experimental collaborations are challenge for theorists, and these are $D$ and $B$ mesons to play an important role in checks of the validity and accuracy of the models.

In this letter we address questions concerning the masses of orbitally excited $D$ and $B$ mesons in the method of the QCD string with quarks at the ends, paying special attention to the ordering of the $P$ levels. The choice of $D$ and $B$ mesons is not accidental and is stipulated by the recent data on the masses and decays of the above mentioned heavy-light mesons coming from various experimental collaborations. Despite of the fact that there is no agreement between them and some resonances are not yet confirmed, still we find it interesting to compare these experimental data, as well as those provided by other models and lattice simulations, with the predictions of our approach. First, let us remind the reader the basic ideas of the latter.

Starting from the gauge-invariant wave function of the $q\bar{q}$ meson,

$$\Psi_{q\bar{q}}(x, y|A) = \bar{\Psi}_q(x) \Phi(x, y) \Psi_{\bar{q}}(y),$$

(1)

with $\Phi$ being the parallel transporter, we write the Green’s function of the meson,

$$G_{q\bar{q}} = \langle \bar{\Psi}_{q\bar{q}}^+(\bar{x}, \bar{y}|A) \Psi_{q\bar{q}}(x, y|A) \rangle_{q\bar{q}A}.$$

(2)
and perform the integration over the quark and the gluonic fields. For the latter case we make use of the minimal area law asymptotic for the Wilson loop bounded by the quark and the antiquark trajectories (see, e.g., [1]),

\[ \langle Tr P \exp \left( ig \oint C dz A_\mu \right) \rangle_A \sim \exp \left( -\sigma S_{\text{min}} \right), \]

where \( \sigma \) is the string tension in the fundamental representation of the SU(3) colour group, and the area \( S_{\text{min}} \) can be approximated by means of the straight-line anzatz [2],

\[ S_{\text{min}} = \int_0^T dt \int_0^1 d\beta \sqrt{(\dot{w}^r)^2 - \dot{w}^2 w'^2}, \]

with \( w_\mu(t, \beta) = \beta x_1^\mu(t) + (1 - \beta) x_2^\mu(t), \)

with \( x_{1,2} \) being the coordinates of the quark and the antiquark. Now, applying the Feynman-Schwinger representation to the single-quark propagators and introducing the einbein fields \( \mu_{1,2} \) to simplify the relativistic kinematics [3], we, finally, arrive at the following expression for the Hamiltonian of the meson [4, 5]:

\[ H = H_0 + V_{\text{str}} + V_{\text{sd}}, \]

\[ H_0 = \sum_{i=1}^2 \left( \frac{\vec{p}_i^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \sigma r - \frac{\kappa}{r} - C_0, \]

\[ V_{\text{str}} \approx -\frac{\sigma(\mu_1^2 + \mu_2^2 - \mu_1 \mu_2)}{6\mu_1^2 \mu_2^2} \frac{\vec{L}^2}{r} + \frac{\sigma^2(\mu_1 + \mu_2)(4\mu_1^2 - 7\mu_1 \mu_2 + 4\mu_2^2)}{72\mu_1^2 \mu_2^2} \vec{L}^2, \]

\[ V_{\text{sd}} = \frac{8\pi\kappa}{3\mu_1 \mu_2} (\vec{S}_1 \vec{S}_2) \left| \psi(0) \right|^2 - \frac{\sigma}{2r} \left( \frac{\vec{S}_1 \vec{L}}{\mu_1^2} + \frac{\vec{S}_2 \vec{L}}{\mu_2^2} \right) + \frac{\kappa}{r^3} \left( \frac{1}{2\mu_1} + \frac{1}{\mu_2} \right) \frac{\vec{S}_1 \vec{L}}{\mu_1} + \frac{\kappa}{r^3} \left( \frac{1}{2\mu_2} + \frac{1}{\mu_1} \right) \frac{\vec{S}_2 \vec{L}}{\mu_2} + \frac{\kappa}{\mu_1 \mu_2 r^3} \left( 3(\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) - (\vec{S}_1 \vec{S}_2) \right) + V_{\text{loop}}(\kappa^2), \]

where in (5) we supply the purely nonperturbative interaction, coming from the string-like picture of confinement, by the perturbative Coulomb interaction \( (\kappa = \frac{4}{3} \alpha_s) \), as well as by the constant negative shift, \( C_0 \), due to the light-quark self-energy [6] strongly needed to bring the Regge trajectory intercepts into their experimental values. The term \( V_{\text{str}} \) deserves special attention, since it is originated from the square root in (4) and describes the contribution of the QCD string into the total inertia of the rotating \( q\bar{q} \) system. This contribution is important to establish the correct slope of the mesonic Regge trajectories [7]. We keep the first two terms in its expansion in powers of \( \sigma/\mu^2 \). The term \( V_{\text{sd}} \) contains spin-dependent interaction generated by both, perturbative and nonperturbative, potentials. Finally, the last term, \( V_{\text{loop}}(\kappa^2) \), comes

<table>
<thead>
<tr>
<th>Meson</th>
<th>( \sigma, \text{GeV}^2 )</th>
<th>( \alpha_s )</th>
<th>( C_0, \text{MeV} )</th>
<th>( m_Q, \text{MeV} )</th>
<th>( m_q, \text{MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>0.17</td>
<td>0.4</td>
<td>196</td>
<td>1400</td>
<td>9</td>
</tr>
<tr>
<td>( B )</td>
<td>0.17</td>
<td>0.39</td>
<td>169</td>
<td>4800</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the Hamiltonian (5)-(8).
Table 2: Solutions of the eigenvalue problem for the Hamiltonian (6) and the coefficients from equation (9) for the set of parameters given in Table I. All quantities are given in MeV.

<table>
<thead>
<tr>
<th>Meson</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu$</th>
<th>$M_0$</th>
<th>$\Delta E_{str}$</th>
<th>$E_{st}$</th>
<th>$E_{so_1}$</th>
<th>$E_{so_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1522</td>
<td>597</td>
<td>429</td>
<td>2444</td>
<td>-26</td>
<td>14</td>
<td>15</td>
<td>-13</td>
</tr>
<tr>
<td>$B$</td>
<td>4847</td>
<td>675</td>
<td>593</td>
<td>5780</td>
<td>-26</td>
<td>6</td>
<td>7</td>
<td>-10</td>
</tr>
</tbody>
</table>

Table 3: The matrix elements of the spin-dependent operators between $P$ states given in the form: $\langle \vec{S}_1 \vec{L} \rangle$, $\langle \vec{S}_2 \vec{L} \rangle$, $\langle (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) \rangle$.

\[
\begin{array}{c|c|c|c}
& ^3P_0 & ^1P_1 & ^3P_1 \\
\hline
^3P_0 & -1, -1, -1/4 & 0, 0, -1/4 & 1/\sqrt{2}, -1/\sqrt{2}, 0 \\
^1P_1 & 0/\sqrt{2}, -1/\sqrt{2}, 0 & -1/2, -1/2, 1/4 & \\
^3P_1 & 1/\sqrt{2}, -1/\sqrt{2}, 0 & -1/2, -1/2, 1/4 & 1/2, 1/2, 1/20 \\
\end{array}
\]

from the one-loop corrections to the potential. It is given by equations (3.1) and (3.2) of the paper [8] with the obvious change $m_{1,2} \rightarrow \mu_{1,2}$, and we choose the renormalization scale to be equal to the reduced effective mass $\mu$. Finally, to fix the Hamiltonian (5)-(8), we use the values of the parameters listed in Table I.

Einstein fields $\mu_{1,2}$ are kept as variational parameters and the spectrum is minimized then with respect to them. The extremal values of $\mu$’s play the role of the constituent masses of the quarks and appear dynamically due to the interaction. This feature of the given approach allows one to start from the current mass of the constituent (gluons also can be described in this formalism) and to arrive at its effective constituent mass self-consistently.

Since the Hamiltonian $H_0$, which plays the role of the zeroth order approximation for the problem, conserves the angular momentum $\vec{L}$, the total spin $\vec{S}$, and the total momentum $\vec{J} = \vec{L} + \vec{S}$ separately, then its eigenstates can be specified as terms, $n^{2S+1}L_J$, with $n$ being the radial quantum number. In the remainder of this letter we shall concentrate on the states with $n = L = 1$. Their masses can be represented as

\[
M(1^{2S+1}P_J) = \langle 1^{2S+1}P_J | H | 1^{2S+1}P_J \rangle = M_0 + \Delta E_{str} + E_{so_1} \langle \vec{S}_1 \vec{L} \rangle + E_{so_2} \langle \vec{S}_2 \vec{L} \rangle + E_{st} \langle 3(\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) - (\vec{S}_1 \vec{S}_2) \rangle,
\]

where $\Delta E_{str}$ is the contribution of the string correction and the term with the spin-spin interaction does not contribute since the wave function at the origin vanishes for orbitally excited states, whereas the corresponding one-loop contribution is negligible. The results of numerical calculations, including the values of the coefficients entering equation (9), are listed in Table II (see [5] for the details of the calculations).

In Table III we give the matrix elements of the spin-tensor and spin-orbit operators between $P$-level states. Since the spin-orbit interaction mixes states with different total spin, then the masses of the physical states with the total momentum $J = 1$ are subject to a matrix equation,

\[
\begin{bmatrix}
\langle 1^1P_1 | H | 1^1P_1 \rangle & \langle 1^1P_1 | H | 1^3P_1 \rangle \\
\langle 1^3P_1 | H | 1^1P_1 \rangle & \langle 1^3P_1 | H | 1^3P_1 \rangle
\end{bmatrix} = 0.
\]
In Tables IV, V we give our predictions for the masses of the $P$-level $D$ and $B$ mesons and compare them with the predictions of other models as well as with the lattice and experimental data coming from various collaborations.

From Tables IV, V one can deduce several conclusions. First, all three mentioned models give good description of the $1P_2$ states, whereas all of them fail to reproduce a very heavy $1P_0$ $B$-mesonic state reported by OPAL [14]. If this experimental value is confirmed, then this will serve as a signal that all theoretical approaches miss something, and this question deserves additional careful study. On the other hand, lattice simulations give the mass $5.754 GeV$ for this state [11], which is also about $100 MeV$ lower than the OPAL value. This stresses once again that the experimental situation strongly needs clarification. A similar state in the spectrum of $D$ mesons is not reported yet by experimental collaborations, though all models and the lattice simulations give a consistent prediction for it to be around $2430 \pm 2440 MeV$.

Another conclusion which one can make from Tables IV, V is that there is no agreement concerning $1P_1$ states. Different models give different splitting patterns (see also the discussion in [11]). To have a better insight into the nature of this splitting let us study the heavy-quark limit, $m_Q = m_1 \rightarrow \infty$, analytically, which is possible in our approach. Only the coefficient $E_{so2}$, in notations of equation (9), survives in this limit, and the expression for it reads

$$E_{so2} = -\frac{\sigma}{2\mu^2} \langle r^{-1} \rangle + \frac{\kappa}{2\mu^2} \langle r^{-3} \rangle + \frac{9\kappa^2}{16\pi\mu^2} \left[ \frac{19}{18} + \gamma_E \right] \langle r^{-3} \rangle + \langle r^{-3} \ln(\mu r) \rangle,$$  \hspace{1cm} (11)

where $\gamma_E = 0.5772$ is the Euler constant and the averaging is performed over the zeroth-order wave function $\psi_{nl}(r)$ corresponding to both states, $P_{1/2}$ and $P_{3/2}$, which are now the true eigenstates of the Hamiltonian $1$. As discussed in [19, 5], solution of the eigenstate problem for the Hamiltonian (6) in this limit is given by solutions to the Schrödinger equation

$$\left( -\frac{d^2}{dx^2} + \frac{\lambda}{|x|} - \frac{\lambda^2}{|x|^2} \right) \chi_{\lambda} = a(\lambda) \chi_{\lambda},$$  \hspace{1cm} (12)

with the reduced Coulomb-potential strength $\lambda$ being the solution of the equation (we put the light-quark current mass equal to zero for simplicity)

$$\lambda^2 = \frac{4}{3} \kappa^2 \left( a + 2\lambda \frac{\partial a}{\partial \lambda} \right),$$  \hspace{1cm} (13)

which is $\lambda_0 = 1.215$ for $\alpha_s = 0.39$ and $\lambda_0 = 1.250$ for $\alpha_s = 0.4$. The reduced effective mass $\mu$ takes the value

$$\mu = \frac{1}{2} \sqrt{\sigma} \left( \frac{\lambda_0}{\kappa} \right)^{3/2} \approx 0.7 GeV$$  \hspace{1cm} (14)

and

$$\langle r^N \rangle = (2\mu \sigma)^{-N/3} \int_0^{\infty} x^{N+2} |\chi_{\lambda}(x)|^2 dx.$$

Then the difference of the masses of the two eigenstates corresponding to $j_2 = \frac{1}{2}$ and $j_2 = \frac{3}{2}$ is

$$M_{P_{3/2}} - M_{P_{1/2}} = -\frac{3}{2} E_{so2},$$  \hspace{1cm} (16)

so that the picture of the splitting depends on the sign of the coefficient (11). Numerically this difference equals to $+9 MeV$ for $D$ mesons and $+11 MeV$ for $B$'s.
From Tables IV, V one can see that the predictions of our method for the masses of the $1^P_1$ states are in good agreement with the lattice calculations [11] as well as with the experimental data. Namely, as far as the spectrum of $D$ mesons is concerned, we have good coincidence with the results of CLEO [13] (see Table IV). In the $B$-mesonic spectrum we identify the state $B_1$ with the mass $m(B_1) = 5.71 \pm 0.02 \text{GeV}$, recently claimed by CDF [17], with the lightest member of the $J = 1$ doublet, whereas the heaviest one can be associated with the resonance reported by OPAL [14] (see Table V). Unfortunately, experimental resolution does not enable one to disentangle both $P_1$ states simultaneously, so that at present time one rather has to rely on available lattice simulations. In such a situation other models, as well as improved lattice calculations, are welcome to attack this problem to have well established and clear predictions for experimentalists.

In Fig. 1 we give the splitting pattern for the $1P$ levels for both, $D$ and $B$, mesons for the heavy-quark mass varying from infinity to about 1.3 GeV with the vertical dashed line giving the actual masses of $c$ and $b$ quarks for $D$ and $B$ mesons, respectively. It is also worth mentioning that according to our model the $1P_0$ and $1P_2$ levels change their ordering around the heavy-quark mass $m_Q \approx 7.9 \text{GeV}$ for the $D$-like meson (left plot in Fig.1), and around $m_Q \approx 5.5 \text{GeV}$ for the $B$-like one (right plot in Fig.1).

In conclusion, let us briefly summarize the results reported in this letter. We addressed the question on masses and splitting pattern of the $P$-level $D$ and $B$ mesons in the method of the QCD string with quarks at the ends. We took into account the proper dynamics of the QCD string, encoded in the so-called string correction, and supplied the interquark interaction with the one-loop corrections adapted to the case of the self-consistently generated dynamical masses of the quarks. Using the standard values for the string tension, the strong coupling constant and the current quark masses, we calculated the spectrum of $P$-level $D$ and $B$ mesons and found good agreement of our results with the lattice and experimental data, including those reported recently. Finally, we give our predictions for the splittings between $1^P_1$ states.

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1We follow the standard notations using the total momentum of the light quark, $\vec{J}_2 = \vec{L} + \vec{S}_2$, as the subscript.
Figure 1: Splitting pattern for the $D$-(left plot) and $B$-mesonic (right plot) $P$ levels as a function of the heavy-quark mass. The vertical dashed line corresponds to $m_Q = m_c = 1.4\,\text{GeV}$ for the left plot and $m_Q = m_B = 4.8\,\text{GeV}$ for the right one, respectively.

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