Comment on No–Signaling Condition and Quantum Dynamics

In carefully worded paper buz1, the authors tried to derive linearity (i.e. affinity on ‘density matrices’ =: DM) and complete positivity (CP) of general quantum mechanical dynamics g from usual (nonrelativistic) kinematics of quantum mechanics (QM), and from an additional “no–signaling condition” (NS). I shall try to show here that the declared goals of buz1 were not attained there.

The authors consider a given system A in an arbitrary state described by a DM \( \rho_A \), as a subsystem of a composed system \( A&B \) occurring in a pure state \( |\psi_{AB} \rangle \). The subsystems A and B are spacelike separated. Different convex decompositions of the reduced DM \( \rho_A = \sum_k q_k \rho_k \) are obtained by different choices of discrete measurements on B. ‘Measurements’ of \( \lambda I_B \) give trivial decomposition of \( \rho_A \), other non-maximal measurements give decompositions of \( \rho_A \) to density matrices. The pure-state decompositions (corresponding to maximal measurements) are interpreted in buz1 as representing the corresponding different “probabilistic mixtures” (PM) in the sense of (classical) statistical ensembles (of quantal systems), sometimes in literature called Gemenge, or also genuine mixtures in bon.

The time evolution transformation \( g \) (“not necessarily linear”) of A “is a priori defined only on pure states….” Boldface in quotations are my emphases. P.B. g : \( P_\psi \mapsto g(P_\psi) \). An explicit extension of \( g \) to all considered states of \( A \), e.g. to all decompositions \( \{ \rho_k, q_k \} \) of \( \rho_A \), is essential, however, for the forthcoming discussion: Effects of any deterministic (no collapse!) semi-group of time transformations \( g \) are supposed to be uniquely determined in QM by its initial conditions \( [\{ g(P_\psi) \} \) does not have to be a pure state” of \( A \) (!)].

The following observation will also support my criticism:

(*) “…the results of measurement on \( A \) will be completely determined by the reduced DM of the system.” [pp.2-3] buz1.

Decisive for proving linearity of \( g \) is: (***) “…every PM of pure states corresponding to the DM \( \rho_A \) can be prepared via appropriate measurements on \( B \)” (this is supported by calculations of probabilities at \( A \) conditioned by results of measurements on \( B \)); such a process is classified in EPR as the “reduction of the wave packet”, i.e. a use of the projection postulate (having an ontological meaning), what is, however, strongly rejected in [pp. 1 and 2] buz1. The linearity of \( g \) is then implied by:

*1 cmg(\( \rho_A \)) = g(\( \{ \rho_k, q_k \} \)) = \( \sum_j p_j g(P_{v_j}) \),

(\( \dagger \)) \( g(\rho_A) := \rho'_A(\{ P_{v_j}, p_j \}) \) in buz1, if valid for arbitrary (or at least pure) decompositions \( \rho_A = \sum_k q_k \rho_k = \sum_j p_j P_{v_j} ; (\dagger) \) was deduced in buz1 from (**), and from a use of NS.

My criticism is concentrated to two points, i.e., mainly, to (first) criticism of the way of the deduction of the restriction (\( \dagger \)) imposed on \( g(\{ \rho_k, q_k \}) \), leading to linearity of \( g \), and to, less important, (second) criticism of the statement of the implication: \{linearity \& positivity(of each timemeasurement)\} \( \Rightarrow \{ \text{complete positivity of } g \} \)

(first): The necessity of (\( \dagger \)) in buz1 is given by mere “statics” of buz1, without NS, since that kinematics (embracing all ‘state space points’ \( \bullet \) appearing as initial conditions for \( g \), and also its values \( g(\bullet) \)) does not contain in buz1 any means (i.e. corresponding observables) to ascertain locally a distinction between different kinds of interpretation of \( \rho_A \), cf. (*): then the value of \( g(\rho_A) \) should be here the same for \( \rho_A \) considered as an indecomposable quantity describing a quantum state of each single system \( A \) in an ensemble of equally prepared couples \( A&B \), as well as for \( \rho_A \) representing a specific ensemble of subsystems \( A \) each of which being in one of the states \( \rho_k \) taken from the set composing the chosen convex decomposition \( \{ \rho_k, q_k \} \) of \( \rho_A \).

It can be introduced, however, a state space for \( A \) (as it was partly done implicitly in buz1) consisting of all probability measures on density matrices (interpreted as corresponding PM’s, and encompassing different decompositions of the same density matrix as different points) with observables distinguishing them; let us define then \( g(\{ \rho_k, q_k \}) := \sum q_k g(\rho_k) \) for the case of PM \( \{ \rho_k, q_k \} \), and let \( g(\rho) \) be ‘independently’ given for any (not decomposed!) density matrix \( \rho \), cf. [2.1-e] bon. Then the proof of linearity of \( g \) in buz1 (with a use of my emphases. P.B.,
of NS) depends on possibility of an empirical check of \(^{(* *)\text{ (i.e. of existence of physical differences between different decompositions of } \rho_A \text{ at the instant of the measurements on } B\)}\) without a use of results of measurements on \(B\). Its negative result (due to NS) does not imply \((\dagger)\): All the physically indistinguishable “at a distance prepared PM’s” are described by \(\rho_A\) and all of them evolve to \(g(\rho_A) \equiv g(\sum q_k \rho_k) \neq g((\rho_k, q_k))\) for nonlinear \(g\).

(second): Assuming linearity and positivity of each physical time evolution transformation \(g\), authors infer CP of \(g\) by applying these properties to extensions \(A&B\) of the considered system \(A\). Their arguments consist, however, of a rephrasing of the definition of CP and of its physical motivation published in [Sec. 9.2]\text{dav}. My conclusion is that the authors did not succeed in their effort to prove in buz1 effectiveness of new quantummechanical axiom called the “no–signaling condition”, and the declared aims of the paper buz1 were not achieved.

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