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Lessons for QCD from supersymmetry *

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I discuss two topics that have recently emerged in connection with supersymmetry-inspired analyses of non-supersymmetric gauge theories in the limit of large number of colors, $N \rightarrow \infty$. A nonperturbative orbifold conjecture due to Strassler predicts an infinite number of degeneracies in the hadronic spectra of $SU(N) \times SU(N)$ Yang-Mills theory with a bifundamental quark. Analysis of pure Yang-Mills suggest the existence of infinitely many (quasi)stable vacua. Domain walls separating these vacua are very peculiar; in particular, they support hadronic excitations with masses proportional to N .

Everybody knows that breakthrough developments often come from unexpected directions. In the last few years supersymmetry (SUSY) proved to be a powerful guiding principle [1] providing deep insights in gauge dynamics at strong coupling. I am grateful to the organizers of this conference who gave me the opportunity to report in this audience on recent results coming from supersymmetry which may eventually lead to a deep breakthrough in our understanding of QCD. I will dwell on two captivating topics: (i) supersymmetric degeneracies in the spectra of certain non-supersymmetric QCD-type theories; and (ii) proliferation of (quasi)stable vacua in multicolor Yang-Mills theories and mysterious heavy excitations on the domain walls interpolating between these vacua.

1. REMNANTS OF SUSY IN "ALMOST" QCD

Supersymmetric gluodynamics is a distant cousin of QCD. Through orbifoldization (I will explain momentarily what it is) one gets a daughter theory which is much closer to QCD than the parent SUSY gluodynamics. In particular, the

daughter theory has no supersymmetry. Nevertheless, a large number of supersymmetry-based relations survive.

1.1. The parent theory

One starts from supersymmetric gluodynamics, the theory of gluons and gluinos,

$$\mathcal{L} = -\frac{1}{4g_P^2} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{i}{g_P^2} \lambda^{a\alpha} \mathcal{D}_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\alpha}\beta}, \quad (1)$$

where the subscript P stands for the parent theory, λ is the Majorana fermion in the adjoint representation, and I use spinorial notation in the fermion sector. The gauge group is assumed to be $SU(2N)$. This will imply a Z_2 orbifoldization, the simplest possibility. Later on we will tend $N \rightarrow \infty$, following the 't Hooft law, $g_P^2 \propto N^{-1}$. The distinction between QCD and SUSY gluodynamics lies in the fermion sector — the QCD quarks are the Dirac fermions in the fundamental representation of the gauge group.

The theory (1) is supersymmetric. The conserved spin-3/2 current J_β^μ has the form (in the spinorial notation)

$$J_{\beta\alpha\dot{\alpha}} \equiv (\sigma_\mu)_{\alpha\dot{\alpha}} J_\beta^\mu = \frac{2i}{g_P^2} G_{\alpha\beta}^a \bar{\lambda}_{\dot{\alpha}}^a. \quad (2)$$

Moreover, the theory is believed to be confining, with a mass gap. The corresponding spectrum consists of composite (color-singlet) hadrons which enter in degenerate supermultiplets. Note

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that there is no baryon charge in the theory at hand. The only $U(1)$ symmetry of the classical Lagrangian (1),

$$\lambda \rightarrow e^{i\alpha} \lambda, \quad \bar{\lambda} \rightarrow e^{-i\alpha} \bar{\lambda},$$

is broken at the quantum level by the chiral anomaly. A discrete Z_{4N} subgroup, $\lambda \rightarrow e^{\pi i/N} \lambda$, is non-anomalous. It is known to be dynamically broken down to Z_2 , so that the order parameter, the gluino condensate $\langle \lambda \lambda \rangle$, takes $2N$ values,

$$\langle \lambda^a \lambda^a \rangle = N \Lambda^3 \exp \frac{\pi i k}{N}, \quad k = 0, 1, \dots, 2N - 1, \quad (3)$$

labeling $2N$ distinct vacua of the theory (1), see Fig. 1. Here $\langle \dots \rangle$ means averaging over the given vacuum state, and Λ is a dynamical scale.

Figure 1. The gluino condensate $\langle \lambda \lambda \rangle$ is the order parameter labeling distinct vacua in supersymmetric gluodynamics. For $SU(N)$ gauge group one has N discrete degenerate vacua.

All hadronic states are arranged in supermultiplets. The simplest is the so-called chiral supermultiplet which includes two (massive) spin zero mesons (with the opposite parities), and a Majorana fermion (with the Majorana mass). The interpolating operators producing the corresponding hadrons from the vacuum are G^2 , $G\bar{G}$ and

$G\lambda$. The vector supermultiplet consists of a spin-1 massive vector particle, a 0^+ scalar and a Dirac spinor. All particles from one supermultiplet have degenerate masses. Two-point functions are degenerate too (modulo obvious kinematical spin factors). For instance,

$$\begin{aligned} \langle G^2(x), G^2(0) \rangle &= \langle G\bar{G}(x), G\bar{G}(0) \rangle = \\ \langle G\lambda(x), G\lambda(0) \rangle. \end{aligned} \quad (4)$$

Unlike conventional Yang-Mills theory, both the meson and baryon masses in SUSY gluodynamics are expected to scale as N^0 .

1.2. Orbifoldization

The technique of orbifoldization was developed in the context of string/brane theory and is based on imposing certain nontrivial boundary conditions. I hasten to say that we do not need to know the original procedure. The only thing we need to know, is that orbifolding the parent theory (1) one obtains a self-consistent daughter field theory by judiciously discarding a number of fields in the Lagrangian (1). This is illustrated in Fig. 2. The square presents the color contents of the fields of the daughter theory. The gauge fields which are *retained* in the daughter theory belong to two blocks on the main diagonal. Thus, the daughter theory has gauge symmetry $SU(N) \times SU(N)$ (I do not distinguish between $SU(N)$ and $U(N)$. At large N this distinction is irrelevant.) To emphasize the fact that the gauge group is a direct product I will use “barred” indices for one $SU(N)$, e.g. $A_{\mu}^{\bar{i}\bar{j}}$, and “unbarred” for another, e.g. $G_{\mu\nu}^{ij}$, where $i, j = 1, 2, \dots, N$ while $\bar{i}, \bar{j} = N + 1, \dots, 2N$.

The fermion fields to be retained belong to two off-diagonal blocks in Fig. 2. Thus, each fermion carries one barred and one unbarred color index. In other words, they are fundamentals with respect to the first $SU(N)$ and antifundamentals with respect to the second $SU(N)$. Such fields are called bifundamental. Thus, in the daughter theory we deal with two Weyl bifundamentals. From Fig. 2 it is quite clear that one can combine them to form one four-component (Dirac) bifundamental field,

$$\{\lambda_{\alpha}^{i\bar{j}}, (\lambda_{\alpha}^{\bar{i}j})^{\dagger}\} \rightarrow \Psi^{\bar{i}j}.$$

Figure 2. Color decomposition of fields appearing in the Z_2 orbifold theory.

The Lagrangian of the daughter theory is

$$\mathcal{L} = -\frac{1}{4g_D^2} G_{\mu\nu}^{ij} G_{\mu\nu}^{ij} - \frac{1}{4g_D^2} \bar{G}_{\mu\nu}^{\bar{i}\bar{j}} \bar{G}_{\mu\nu}^{\bar{i}\bar{j}} + \bar{\Psi}^{\bar{i}\bar{j}} i \not{D} \Psi^{\bar{i}\bar{j}}, \quad (5)$$

where

$$D_\mu = \partial_\mu - i A_\mu^a T^a - i \bar{A}_\mu^{\bar{a}} \bar{T}^{\bar{a}}. \quad (6)$$

For the future purposes we will need to rescale the gauge coupling of the daughter theory,

$$g_D^2 = 2g_P^2. \quad (7)$$

The daughter theory is *not* supersymmetric. The color-singlet supercurrent (2) cannot be formed now since the color indices of the gauge fields and the fermion fields cannot be contracted. In this way it seems to be much closer to genuine QCD than SUSY gluodynamics. In fact, exactly this theory is obtained from the three-color three-flavor QCD by gauging the flavor group. (Note, that flavor $SU(2)$ is gauged anyway in the electroweak theory).

The absence of supersymmetry is not the only feature of (5) which distinguishes this theory from

SUSY gluodynamics. The daughter theory possesses a global $U(1)$, with the conserved fermion number current $J_\mu = \bar{\Psi} \gamma_\mu \Psi$. Applying this vector current to vacuum, one produces a vector meson which has no partners in the parent theory. They say that the operator J_μ has no projection onto the parent theory. One can construct many other operators with the similar properties. There are many hadronic states in the daughter theory which have no analogs in the parent one. For instance, for odd N one expects baryons with the quark content $\Psi \Psi \dots \Psi \varepsilon \bar{\varepsilon}$ and masses scaling as N . For even N the daughter theory has no baryons at all.

Inverse is also true. For instance, the operator of the supercurrent produces a spin-3/2 hadron from the vacuum state of SUSY gluodynamics, which has no counterpart in the daughter theory. Therefore, in confronting these theories it makes sense to compare only those sectors which have projections onto one another.

Let us note in passing that the axial $U(1)$ current is anomalous, as in the parent theory,

$$\partial^\mu (\bar{\Psi} \gamma_\mu \gamma_5 \Psi) = \frac{N}{16\pi^2} G \tilde{G} + \frac{N}{16\pi^2} \bar{G} \tilde{\bar{G}}. \quad (8)$$

Above I have discussed the Z_2 orbifoldization allowing one to proceed from $SU(2N)$ parent theory to $SU(N)^2$ daughter theory. It is easy to extend the set up to Z_k orbifoldization leading to the transition $SU(kN) \rightarrow SU(N)^k$ where k is arbitrary.

1.3. Planar equivalence

In 1998 a striking relationship was discovered [2,3] between the parent and the daughter theories. It was pointed out that the Green's functions for operators which exist in both theories have identical planar graphs provided the rescaling (7) is implemented. In other words, perturbatively these Green's functions in the parent (supersymmetric) and daughter (non-supersymmetric) theories coincide order by order in the large N limit. In the general case of Z_k orbifoldization the rescaling of the coupling constants is $g_D^2 = k g_P^2$.

Rather than presenting a general proof (it can be found in the original publications) I will illustrate this statement (see Fig. 3) by a simple ex-

ample of the correlation function of two axial currents, $(2/g^2)\bar{\lambda}_\alpha^{ij}\lambda_\alpha^{ji}$. For the computational purposes it is convenient to keep the fermion fields in the daughter theory in the same form λ_α^{ji} , just constraining the color indices. In the parent theory i, j run from 1 to $2N$. In the daughter theory either $i \in [1, N]$, $j \in [N+1, 2N]$ or *vice versa*, $i \in [N+1, 2N]$, $j \in [1, N]$. In the first case we deal with $\lambda^{i\bar{j}}$ in the second $\bar{\lambda}^{i\bar{j}}$.

Figure 3. Counting N factors in the correlation function of the axial currents. The 't Hooft diagrams are depicted in dashed lines.

The easiest way to check the match is analyzing relevant graphs in the 't Hooft representation — each gluon or fermion line carrying two indices is represented by a double line, see Fig. 3. In this way we readily get that the weight of the graph 3a is $4N^2$ (parent), $2N^2$ (daughter), while that of 3b is $8N^3g_P^2$ (parent), $2N^3g_D^2$ (daughter). To get the latter estimate one notes that if the first index of λ — the one forming the outside loop — is unbarred, the second index (corresponding to two inside loops in Fig. 3b) is barred, and *vice versa*. The parent-to-daughter ratio is 2 for both graphs, 3a and 3b, provided $g_D^2 = 2g_P^2$. Proceeding along these lines it is quite easy to see that the parent-to-daughter ratio stays the same for any planar graph.

This statement goes under the name of *perturbative orbifold equivalence*. The next logical step, which has been undertaken by Strassler [4]

is the extension of the planar graph equivalence $\rightarrow$ *nonperturbative orbifold (NPO) conjecture*. According to this conjecture, the Green's functions for the color-singlet operators which appear in both theories are identical at $N \rightarrow \infty$, including all nonperturbative effects. Needless to say that it is the nonperturbative dynamics which is our prime concern in gauge theories at strong coupling. Nonperturbative effects are at heart of key phenomena such as color confinement, chiral symmetry breaking, and so on.

I hasten to add that there are no direct or indirect proofs of Strassler's nonperturbative orbifold conjecture. Since the Yang-Mills theories at strong coupling are not analytically solved, the best strategy could be as follows: (i) extract consequences for the orbifold (non-supersymmetric) theory from the nonperturbative orbifold conjecture (they are quite striking, as we will see shortly); (ii) test these consequences on lattices².

There is another idea which immediately comes to one's mind; it is less comprehensive but can still provide useful hints. Nonperturbative effects exist even at short distances when the running coupling constant is small. Therefore, one can confront them in the parent and daughter theories, under controllable conditions, without waiting for the full analytic solution to emerge. For instance, one can consider both theories on small tori, and calculate instanton or toron effects. If they diverge, this might indicate an inconsistency of the nonperturbative orbifold conjecture. On the other hand, their coincidence could be an argument — although, admittedly inconclusive — in favor of this hypothesis.

1.4. Consequences of the NPO conjecture

Thus, a relationship is established between two theories in the large N limit — one is supersymmetric, the other is not; both are gauge theories at strong coupling with no analytic solution. Why is it so exciting? The point is that quite a lot is known about SUSY gluodynamics. Supersymmetry is a powerful tool of allowing one to get interesting results.

Let us start from perturbation theory. The ex-

²Work on supersymmetry on the lattice is already in progress, see e.g. [5].

act β function of SUSY gluodynamics was established [6] almost twenty years ago,

$$\beta(\alpha_P) \equiv \frac{d\alpha_P}{d\ln\mu} = -\frac{1}{2\pi} \frac{6N\alpha_P^2}{1 - (N\alpha_P)/\pi}, \quad (9)$$

(for $SU(2N)$). This formula is exact to all orders in α_P and nonperturbatively. Due to the planar correspondence it implies that the full β function in the daughter theory is also known (in the 't Hooft limit),

$$\beta(\alpha_D) = -\frac{1}{2\pi} \frac{3N\alpha_D^2}{1 - (N\alpha_D)/(2\pi)}, \quad (10)$$

where I have taken into account the relation (7) between the coupling constants in the parent and orbifold theories. The β function (10) is exactly the same as in $SU(N)$ SUSY gluodynamics.

Let us pass to nonperturbative aspects. In the parent theory the gluino condensate was calculated exactly [7], see Eq. (3). The NPO conjecture then implies that the bifundamental fermions of the orbifold theory condense too, and the fermion condensate takes $2N$ distinct values,

$$\langle \bar{\Psi} \Psi \rangle \propto N \Lambda_D^3 \exp \frac{\pi i k}{N}. \quad (11)$$

Although it looks similar to the quark condensate of QCD, one should remember that in QCD the quark condensate is the order parameter for a continuous (axial) symmetry, while here we deal with a discrete subgroup, so that there are no Goldstone bosons associated with its spontaneous breaking.

The results above are moderately surprising. Much more counterintuitive is the relationship between spectra. For the daughter theory one should consider the spectrum of hadrons invariant under Z_2 corresponding to the interchange of the two gauge groups, $SU(N) \times SU(N)$. By gauge invariance all such hadrons with less than N constituents will be bosons. It is a folklore statement that the lightest baryon in the daughter theory will have mass $\propto N$, while that of the lightest bosons scales as N^0 . Thus, supersymmetry seems to be maximally broken.

Such spectrum is in sharp contrast to that of the parent theory where supersymmetry ensures

the boson-fermion degeneracy. Moreover, supersymmetry guarantees an infinite number of degeneracies in the bosonic sector. For instance, the scalar and pseudoscalar glueballs produced by G^2 and $G\tilde{G}$ must be degenerate between themselves and degenerate with the spin-1/2 glueballino. The latter is gone in the daughter theory. However, the masses of the scalar and pseudoscalar glueballs must continue to be equal (up to $1/N^2$ corrections), with no apparent symmetry reason! All bosons which used to belong to one and the same supermultiplet in the parent theory are expected to be degenerate in the daughter one. These degeneracies will occur throughout the spectrum, along entire Regge trajectories.

Needless to say that such a remarkable spectral property goes against our intuition. The first desire is to say that this cannot be true because this can never happen. One can suggest several explanations. (i) The NPO conjecture may turn out to be wrong. (ii) The baryon states in the daughter theory may defy common wisdom and may appear as solitons, with masses of order of N^0 ; (iii) The gauge symmetry of the daughter theory $SU(N) \times SU(N)$ may be spontaneously broken down to a diagonal $SU(N)$. In the cases (ii) and (iii) supersymmetry is effectively restored. For instance, if $SU(N) \times SU(N)$ is spontaneously broken down to a diagonal $SU(N)$, one can easily recover a gauge invariant (under the diagonal $SU(N)$) supercurrent.

There are serious problems with these explanations, however. In the case (ii) there is no known example of how baryonic states with mass scaling as N^0 can appear in the theory where all local gauge invariant interpolating operators with the baryonic quantum numbers are built of N constituents. In the case (iii) one cannot build a local gauge invariant order parameter which could indicate the breaking $SU(N) \times SU(N) \rightarrow SU(N)_{\text{diag}}$. From the standpoint of the gauge symmetry the appropriate operator would be λ^{ij} . Unfortunately, it is not Lorentz-scalar and cannot condense. The situation remains a mystery so that an input from the lattice community would be most welcome.

1.5. Gauge theories on small tori

As I have already mentioned, one can try to test the NPO conjecture at weak coupling, considering certain known nonperturbative mechanisms. For instance, one can assume that both, the parent and daughter theories, are formulated on four-dimensional tori of size $L \ll \Lambda^{-1}$. Imposing twisted boundary conditions due to 't Hooft allows one to consider torons. In fact, essentially all necessary computations had been done long ago [8], and we only need to cast them in the form appropriate for the given problem [9]. In SUSY gluodynamics the minimal toron has two fermion zero modes (supersymmetric modes). They give rise to the gluino condensate $\langle \lambda \lambda \rangle \propto \exp(-4\pi^2/(Ng_P^2))$. At the same time, in the daughter theory with bifundamentals, the minimal toron configuration is that with a “double twist” — gauge fields from both $SU(N)$'s are twisted. The action is $16\pi^2/(Ng_D^2)$. Equation (8) then implies that there are four fermion zero modes, and the fermion condensate that emerges has the form

$$\langle \bar{\Psi} \Psi \bar{\Psi} \Psi \rangle \propto \exp\left(-\frac{16\pi^2}{Ng_D^2}\right). \quad (12)$$

The matching is not obvious, and the issue calls for further analysis.

2. STABLE VACUA IN MULTICOLOR YANG-MILLS THEORY, D WALLS AND HEAVY STATES

In 1998 Witten analyzed the vacuum structure in a theory belonging to the same universality class as pure Yang-Mills, using a D-brane picture [10]. He confirmed his earlier expectations that in the limit $N \rightarrow \infty$ there are infinitely many degenerate and stable vacua in (non-supersymmetric) gluodynamics. These vacua can coexist being separated by a domain wall/brane whose tension is $\propto N$ while the thickness is $\propto N^{-1}$. Color flux tubes generated by probe quarks in the bulk can end on these domain walls. Sure enough, one can eliminate probe quarks and imagine a minimal flux tube with both ends attached to the wall.

I will not delve into the D-brane picture; rather, I will try to illustrate what is going on from the

field theory side. To this end let us return to SUSY gluodynamics and have a closer look at Fig. 1. The $SU(N)$ theory has N degenerate vacua, labeled by the value of the gluino condensate, $\langle \lambda \lambda \rangle_k = N\Lambda^3 \exp(2\pi i k/N)$. As in any theory with discrete degenerate vacua, one can imagine a situation with two coexisting vacua separated by a domain wall. This wall is rather unusual.

First, its tension T is exactly calculable [11],

$$T = \frac{N}{8\pi^2} |\langle \lambda \lambda \rangle_{k+1} - \langle \lambda \lambda \rangle_k| \propto N\Lambda^3. \quad (13)$$

Linear dependence on N is unexpected. At the same time, the volume energy density outside the wall, in the bulk, vanishes due to supersymmetry, while inside the wall it is proportional to $N^2\Lambda^4$. The factor N^2 is quite standard, it reflects N^2 degrees of freedom, gluons and gluinos.

Combining these two scaling laws one arrives at the conclusion [12] that in the large N limit the wall thickness must scale as $N^{-1}\Lambda^{-1}$, the second stunning feature of the domain wall under consideration. Why is it so surprising?

Supersymmetric gluodynamics is believed to be a conventional confining theory, with meson and baryon masses scaling as N^0 . If a wall is in any way “built” of the colorless degrees of freedom, its thickness must scale as N^0 too. This is what one would expect *a priori*. Indeed, the exponential decay law of the wall “tails” (far away from the wall in the perpendicular direction) is determined by the masses of the lightest excitations. The fact that in the case at hand we deal with a totally different dynamical pattern implies very peculiar underlying gauge dynamics.

To tie loose end together, it was argued [13] that the colorless excitations of which the domain walls are actually built have masses of order $N\Lambda$. Remember, these are not regular baryons, since in SUSY gluodynamics there are no quarks in the fundamental representation. Rather, these states have to be of a solitonic nature. In the D-brane picture they are likely to be associated with open strings with the end points attached to the branes. If so, the $N\Lambda$ mass states are confined to the surface of the wall, they cannot propagate in the bulk. At the same time, the regular mesons, with mass of order Λ , which can propagate in the

bulk, must be weakly coupled to the wall, with all couplings suppressed by powers of $1/N$. Such walls were called D walls [13].

Now let us try to guess what happens if supersymmetry is broken, gluinos decouple and we are left with pure Yang-Mills. The strategy is as follows. First, we introduce a small gluino mass term, so that SUSY breaking is small and all SUSY-based calculations are under control. In this way we establish a tendency. Then we let the gluino mass term to grow. Eventually it will become larger than Λ ; at this point gluinos will decouple. If there is no phase transition *en route*, then the conclusions obtained at small SUSY breaking parameter will remain qualitatively valid in pure Yang-Mills. Certainly, this is just a poor man's substitute of the full solution (which might come one day from lattices, who knows?).

The gluino mass term is added as follows

$$\Delta\mathcal{L} = (m_g/g^2)\lambda\lambda + \text{h.c.} \quad (14)$$

With $m_g \neq 0$ the vacuum degeneracy is lifted. It is easy to see that to the first order in m_g

$$\begin{aligned} E_{k\text{-th vac}} &= -\text{Re} \frac{m_g}{g^2} \langle \lambda\lambda \rangle_{k\text{-th vac}} = \\ &- \left(2 \cos \frac{2\pi k}{N} \right) \left(\frac{m_g}{g^2} \right) N\Lambda^3. \end{aligned} \quad (15)$$

A schematic plot of the vacuum energy is depicted in Fig. 4. The vacuum states which used to be exactly degenerate in SUSY gluodynamics, become local (nondegenerate) minima. They become quasistable vacua. For $k \sim N^0$ the splittings are $\propto N^0$ while the barrier heights are $\propto N^2$. Therefore, the tunneling must be highly suppressed. In fact, it is not difficult to estimate the tunneling probability, the decay rate of the false vacuum $|\text{Vac}\rangle_1$ into the true vacuum $|\text{Vac}\rangle_0$.

The tunneling occurs through the creation of a bubble of the true vacuum in the false one (Fig. 5). In the thin wall approximation [14]

$$\Gamma_{\text{decay}} = \exp[-CT^4/(\Delta E)^3] = \exp[-CN^4], \quad (16)$$

where ΔE is the difference between the energies of the neighboring minima of the potential in Fig.

Figure 4. A schematic plot of the potential energy in (non-supersymmetric) Yang-Mills at large N . The central minimum is the true stable vacuum, other local minima become stable vacua in the limit $N \rightarrow \infty$.

4, see Eq. (15). Moreover, C is a constant calculable at $m_g \ll \Lambda$. Thus, what remains to be done is to check the applicability of the thin wall approximation. This is illustrated in Fig. 5. The bubble radius scales as N^1 , while the wall thickness $\propto N^{-1}$. The result presented in Eq. (16) is expected to be qualitatively valid even if $m_g > \Lambda$, when gluinos decouple. In this limit the constant C is not calculable, of course; its order of magnitude can be estimated, however.

Thus, the first conclusion we arrive at is the existence of infinitely many stable domain walls in pure Yang-Mills at $N \rightarrow \infty$. Although the consideration above bears a qualitative nature at best (it would be probably more appropriate to speak of an educated guess), being combined with Witten's D-brane argument, it presents a circumstantial evidence that this phenomenon is real. It would be very interesting to try to explore it on lattices. The existence of a large number of (quasi)stable (nearly) degenerate vacua may have other conspicuous implications. On finite size lattices the proliferation of vacua may cause contamination of the hadronic spectra. Moreover, very unusual hadronic excitations must exist on the surface of the walls. In a sense, they should be similar to QCD baryons, but — remember — we are talking of the theory with no quarks in the Lagrangian! Here we again enter uncharted waters. Although the topic is obviously quite difficult, one can expect intriguing findings.

Figure 5. Quasistable vacuum decay through bubble production.

3. CONCLUSIONS

I would like to conclude with two remarks of a general nature.

★ Analytic methods based on supersymmetry (even qualitative and imprecise) may and should serve as a source of inspiration for lattice calculations;

★ Fruitful coexistence and cross-fertilization of various approaches — supersymmetry, lattice calculations, light-cone analyses, etc. — is mutually beneficial.

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