Theoretical Approaches to HERA Physics

C. Merino
Department of Particle Physics, University of Santiago de Compostela
15706 Santiago de Compostela, Galiza, Spain
e-mail: Merino@fpaxp1.usc.es

Abstract

A review is presented of the different theoretical models proposed to approach consistently the interplay between soft and hard physics, that can now be studied experimentally at HERA for the first time.

1 HERA Physics

The range of the physics scanned by HERA is very broad, from hadroproduction to jets, charm, quark fragmentation and instantons, even though in this contribution we will only focus on low $x$.

The setting up of HERA, the first electron-proton collider, at DESY during the nineties of the last century made possible the experimental study of deep inelastic scattering (DIS) processes and, more generally, of quantum chromodynamics (QCD), under kinematical conditions where the interplay between soft and hard physics should play an important role [1].

The most important measurement in DIS is that of the cross-section of the process:

$$ep \to eX,$$

as a function of any pair of independent Lorentz invariants built from the kinematic variables $q$, the four-momentum transfer mediated by the virtual photon, and $P$, the four-momentum of the incoming proton, e.g. $(Q^2, x)$:

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2Pq},$$

(1)

The large center of mass energy $s$ provided by the $ep$ collider allows the detectors of the two international experimental collaborations working at HERA, H1 [2] and ZEUS [3], the study of the kinematic regimes both at large $Q^2$ and very small $x$. This newly accessible region of very small $x$ is specially interesting from the theoretical point of view. Since

$$x = \frac{Q^2}{Q^2 + s + m_p^2},$$

(2)

at fixed $Q^2$ the limit $x \ll 1$ corresponds to the limit of large $s$, $s \gg Q^2$ (Regge limit), where the Regge Field Theory, which was used to describe the hadronic processes before QCD was accepted as the general field theory accounting for the strong interaction, should be valid. Thus, HERA makes possible the study of DIS where both the perturbative QCD limit, $Q^2 \gg \Lambda^2$, and the Regge limit should apply, i.e. where the hard-perturbative and soft-nonperturbative physics should interplay and shed light on the fundamental question of confinement. On the other hand, very small $x$ means parton densities in which the proton momentum fraction is being shared by many partons (mainly gluons), i.e. very high density parton densities, and thus this region of very small $x$ connects the study of DIS with the physics of the heavy-ion collisions, so important in the next future with the operative start of the Large Hadron Collider (LHC) at CERN.

Theoretically, the DIS differential cross-section can be expressed in terms of two independent structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left[ \left(1 - y + \frac{y^2}{2}\right) F_2 - \frac{y^2}{2} F_L \right],$$

(3)

where $\alpha$ is the electromagnetic coupling constant, $y$ is the fraction of the electron energy transferred to the proton in the proton rest frame ($0 < y < 1$), and $F_L(x, Q^2)$ is the longitudinal structure function:

$$F_L = F_2 - 2xF_1.$$

(4)
The extraction of $F_2$ from the cross-section measurement (3) implies an assumption on $F_L$, since there is no direct $F_L$ measurements in the HERA regime. Once this assumption is made, we can compare the experimental $F_2$ with the theoretical one, that can be written in terms of the quark and antiquark densities, $q_i$ and $\bar{q}_i$, and the quark charges, $e_{q_i}$. However, it has not been possible to derive the hadronic structure from first principles including the interactions of quarks and gluons as given by QCD, and the same happens when calculating the $ep$ cross-section.

In fact, only three things are rigorously calculable in QCD [4], due to the fact that they involve just one large scale: the process $e^+ e^- \rightarrow \text{hadrons}$, $F_2$ in DIS and the form factor in exclusive processes [5]. In DIS, the equations that include the perturbative effects and give the relation between the parton distribution function taken at two different scales are called evolution equations, and they can be obtained rigorously by extracting the perturbative part of $\sigma_{ep}$ in the Operator Product Expansion (OPE) frame and summing up the large logarithmic perturbative corrections in the Renormalization Group (RG) equations [6] for the regime with a large scale:

$$Q^2 \rightarrow \infty, \text{ with } \frac{Q^2}{2\nu} = x \sim 1 \ (\alpha_s \ln x \ll 1).$$

(5)

An equivalent formulation to the one based on OPE but expressed in terms of parton language is the one of the DGLAP equations [7]. The physical meaning of this method is very clear since it deals with parton densities and fragmentation functions, which are basic quantities for describing short-distance reactions [8]. From (5), we see that the DGLAP equations can be written by neglecting the terms in $\ln (Q^2/s)$ at the leading log approximation in $\ln (Q^2/Q_0^2)$. This leads to consider all ladder diagrams in which the transverse momenta are strongly ordered:

$$Q^2 \ll \cdots k_{T_i}^2 \ll k_{T_{i+1}}^2 \ll \cdots Q^2.$$  

(6)

This is expected to be good enough when $Q^2$ is large but $x$ is not too small. In this approximation the DGLAP evolution equation for the parton density $q_i$ corresponding to the quark of flavor $i$ is (a similar equation can be written for the evolution of the gluon density, $g$):

$$\frac{dq_i(x, Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[ q_i(z, Q^2) P_{qq} \left( \frac{x}{z} \right) + g(z, Q^2) P_{qg} \left( \frac{x}{z} \right) \right].$$

(7)
The $P\left(\frac{z}{x}\right)$ functions in (7) are the calculable splitting functions giving the probabilities for the parton branchings.

When $x$ is small but $Q^2$ is not very large the DGLAP equations can no longer be used. It is for this new regime:

$$\Lambda^2 \ll Q^2 \ll s,$$

where no theoretically rigorous evolution equation is available, that a new evolution equation analytically solvable at fixed $\alpha_s$ has been proposed, the BFKL evolution equation [9], in which we can, from (8), consider the leading log approximation in $\ln(1/x)$, which translates in resumming the ladder diagrams with strongly ordered $x_i$:

$$x_0 \ll \cdots x_i \ll x_{i+1} \ll \cdots x,$$

and no ordering on $k_T$. The BFKL equation is an evolution equation in $x$ of the gluon density $g(x, k^2_T)$, dominant at small $x$:

$$\frac{\partial g(x, k^2_T)}{\partial \ln(1/x)} = \frac{3\alpha_s}{\pi} \int_0^\infty \frac{dk^2_{T'} k^2_T}{k^2_{T'}} \left[ \frac{g(x, k^2_{T'}) - g(x, k^2_T)}{|k^2_{T'} - k^2_T|} + \frac{g(x, k^2_T)}{\sqrt{4k^2_{T'} - k^2_T}} \right].$$

(10)

Thus $g(x, k^2_T)$ can be calculated for any small $x$ once it is known at some starting value $x_0$, for all $k^2_T$.

As already mentioned, the BFKL equation can be solved analytically for fixed $\alpha_s$, with the $x$ and $k_T$ behaviors of the solution being:

$$g(x, k^2_T) \propto \left(\frac{x}{x_0}\right)^{\lambda} \sqrt{\frac{k^2_T}{k^2_{T'}}} \frac{1}{\ln 1/x},$$

(11)

with

$$\lambda^{LO} = \left(n_c \alpha_s / \pi\right) 4 \ln 2 \sim 0.5$$

(12)

for $n_c = 3$ colors, being the famous BFKL Pomeron intercept. At this point one has to note that different problems appear when trying to solve the BFKL equation at the next-to-leading order (NLO) [10], among them the impossibility of treating all the terms in the new solution as Regge terms. Still, the NLO estimates of the BFKL Pomeron intercept seem to lead to a value:

$$\lambda^{NLO} \sim 0.25 \div 0.3.$$

(13)
Also attempts have been made to include the running of $\alpha_s(Q^2)$ in the calculation.

In the region where $x$ is small and $Q^2$ is not too large, now accessible at HERA, the old Regge Field Theory should work. The Regge Field Theory [11] is a method based on analyticity, crossing symmetries between the $s$ and $t$ channels and unitarity in the complex angular momenta plane, valid to compute the cross-section of hadron-hadron collisions at high energies (the equivalent in DIS to small $x$), by squaring the sum over the scattering amplitudes due to the mesons that can be exchanged in the process. These mesons can be empirically grouped along the so called Regge trajectories, parametrized as straight lines which relate the mass $m$ and the spin $J$ of the exchanged mesons, $J = \alpha(m^2)$:

$$\alpha(t) = \alpha_0 + \alpha' t,$$

where $\alpha_0$ is the intercept and $\alpha'$ the slope of the trajectory. Thus the elastic cross-section for a hadron-hadron collision will be calculated by summing over all the Regge trajectories whose resonances can be exchanged in the reaction, and the final result can be written as:

$$\frac{d\sigma_{el}}{dt} \propto (\beta(t))^{2s^{\alpha(t)-2}},$$

with $\beta(t)$ an unknown real function. Now, by using the optical theorem which relates the total cross-section to the forward elastic amplitude, we can predict the behavior of the total hadron-hadron cross-section:

$$\sigma_{tot} \propto s^{\alpha_0-1}.$$  \hspace{1cm} (16)

The huge amount of experimental data on hadronic cross-sections for many different processes shows an universal and steady rise at large energies that can only be accounted for by parametrizing all these cross-sections as the sum of two different components:

$$\sigma_{tot} = A s^{\alpha_R(0)-1} + B s^{\alpha_P(0)-1},$$  \hspace{1cm} (17)

with $A$ and $B$ process-dependent constants and the intercepts $\alpha_R(0) \sim 0.5$ and $\alpha_P(0) \sim 1.08$, universal process-independent constants. While the first term in Eq. (17) represents the exchange of the experimentally detected mesonic resonances (secondary Reggeon), the second term represents the exchange of a hypothetical object, the soft Pomeron, which has the vacuum quantum numbers (electrically and
color neutral, isospin 0 and $C$-parity +1), and it is the one responsible, through the value of its intercept larger than 1 (supercritical Pomeron), for the rise of $\sigma^{tot}$ at large energies. This soft Pomeron, as it also happens for the BFKL Pomeron, is suspected to be of gluonic nature. Of course, Equation (17) violates the unitarity constraints imposed by the Froissart bound on the cross-section behavior, so we must interpret the soft Pomeron just as an useful phenomenological tool.

As a matter of fact, DIS processes at small $x$ can be viewed in terms of the Regge Field Theory as virtual photon-proton scattering at high energy, with $F_2$ dominated by the gluon content of the proton:

$$F_2 = -\frac{Q^2}{4\pi^2\alpha_s} \sigma^{tot}_{\gamma p} \rightarrow F_2 \propto \left(\frac{1}{x}\right)^{\alpha(P(0))}, \quad (18)$$

where this should be compared with the solution of the BFKL evolution equation, so we can write:

$$F_2 \propto \left(\frac{1}{x}\right)^{\alpha_{BFKL}}, \quad (19)$$

and we view the exchange of a hard BFKL Pomeron in the Regge language as the sum over graphs with one gluon ladder between the interaction particles in perturbative QCD.

Now we are in conditions to address the problems at the origin of the theoretical models that we pretend to review in this contribution. The first question is how to connect QCD and the Regge Field Theory in a theory which could compute quantitatively any DIS or hadronic process in both the soft and hard regimes. The second problem concerns the perturbative QCD prediction of a strong increase of the parton densities at low $x$, which led to the idea [12] that the density of partons (gluons) becomes so big that at some point these gluons cannot be considered as independent partons any more, but they interact among them. To determine whether this is actually the case, and whether these interactions must be taken into account for a consistent description of DIS processes in this small $x$ regime, is what is called the saturation problem, at present intensively analyzed, both theoretically and experimentally at HERA. To precisely establish [13] the way saturation relates to the idea of unitarity, a better understanding of the fundamental question of confinement is needed.
2 Theoretical Approaches

Many models have been proposed in the attempt to make the connection between soft and hard physics in a theoretically consistent way during the last years. Given the current lack of both theoretical and experimental tools needed in order to articulate an universal and fundamental approach, all these models are phenomenological to a larger or smaller degree, and they can basically be classified in three main groups. The first class of models explicitly includes saturation, as the dipole models which use the idea that the virtual photon actually splits into a $q$-$\bar{q}$ dipole and it is this dipole which subsequently interacts with the proton. Here, the size of the dipole will determine the soft or hard character of the interaction. The second class of models, that we will call phenomenological parametrizations of structure functions at low $Q^2$, provides a parametrization of the structure function $F_2(x, Q^2)$ describing the experimental data in the (nonperturbative) region where perturbative QCD is supposed to fail, and then uses this parametrization to be plugged in as an initial condition in the perturbative evolution equations to obtain a description of the experiment in the whole kinematic range. The philosophy behind the third group of models (dynamical models for the low $Q^2$ behavior of $F_2$) is just the opposite. Now a QCD-based consistent description of the data in the perturbative region is considered, and then extrapolated down to the region of not large $Q^2$, through the evolution equations and under certain ad-hoc conditions imposed to maintain the consistency of the description in the region where the perturbative treatment is supposed not to work any more.

The first explicit realization of the idea of saturation was presented not for the nucleon, but for the nuclear case [14], when it was shown that the Glauber model without gluon interaction leads to the saturation of the $1/k_T^2$ growth of the quark and gluon distributions at fixed $s$ as $k_T^2$ becomes small.

One dipole model very transparent in its physical interpretation is the GW model [15]. Here the structure function $F_2$ is separated into two terms:

$$F_2 = F^T + F^L,$$  \hspace{1cm} (20)

with, for $x \ll 1$:

$$F^{T,L} = \frac{Q^2}{4\pi^2\alpha} \int d^2\vec{r}dz|\Psi^{T,L}(\vec{r}, z, Q^2)|^2 \hat{\sigma}(x, \vec{r}).$$  \hspace{1cm} (21)
Here $\Psi^{T,L}$ is the known wave function for a transverse (T) or longitudinal (L) polarized virtual photon $\gamma^*$ to split into a $q$-$\bar{q}$ dipole, $\hat{\sigma}$ is the dipole cross-section describing the interaction of the dipole with the proton, and $\vec{r}$ is the transverse separation of the $q$-$\bar{q}$ pair. Unitarity is built in by the phenomenological form of the dipole cross-section:

$$\hat{\sigma} = \sigma_0 (1 - exp(-r^2/4R_0^2(x)))$$

and

$$R_0(x) = (1/Q_0)(x/x_0)\Delta/2,$$

where $R_0$ is the so-called saturation radius, $Q_0 = 1$ GeV, and parameters $\sigma_0$, $x_0$ and $\Delta$ are fitted to all inclusive DIS data with $x < 0.01$. Thus in this model, at small $r$ one has color transparency and a strong growth of $\hat{\sigma}$ with $x$:

$$\hat{\sigma} \sim r^2 x^{-\Delta},$$

while at large $r$ (or $x \to 0$) $\hat{\sigma}$ approaches the black-disk constant value $\sigma_0$ (saturation). The transition to saturation is governed by $R_0(x)$.

There are models including other saturation mechanisms. One of them is the interaction between partons in the parton cascade [16], not taken into account in QCD evolution equations, but that could become important to slow down the growth of parton densities. Here, the parton interactions will create an equilibrium-like system of partons with a definite value for the average transverse momentum, $Q_s(x)$ (saturation scale). A different proposed saturation mechanism is the percolation of strings [17], a second order phase transition which takes place when clusters of overlapping strings, with size of the order of the total transverse area available, appear. This phase transition is used as an indication for the onset of saturation of the density of partons. In this approach, which can be generalized to the nuclear case, a multiple exchange model for $ep$ collisions is needed.

For the nuclear case, the first approach [14] presenting the $k_T^2$ saturation curves for the quark and gluon distributions at fixed $s$ has been extended [18] to the case where interaction among gluons is taken into account, obtaining the $x$ dependence of those curves. Also all multiple Pomeron LO exchanges have been included [19] in deriving a small $x$ evolution equation of $F_2$ for a large nucleus from the first nuclear approach without interaction [14], and it is shown that in the double leading log limit this equation reduces to the GLR equation [12]. In a different model [20], the valence quarks of the nucleons of the nucleus are treated as the sources of the small $x$ gluon distribution of the nucleus.
Among the phenomenological parametrizations of structure functions at low $Q^2$, the DL model [21] is a parametrization of $F_2$ that uses two separate (soft and hard) Pomerons:

$$F_2(x, Q^2) = f_0(q^2)x^{-\epsilon_0}(1 - x)^7$$
$$+ f_1(q^2)x^{-\epsilon_1}(1 - x)^7$$
$$+ f_2(q^2)x^{-\epsilon_2}(1 - x)^3,$$

with $\epsilon_0 = 0.4372$ (hard), $\epsilon_1 = 0.0808$ (soft), and $\epsilon_2 = -0.4525$ (valence). The ABY model [22] is another parametrization of $F_2$ using two different components, a hard Pomeron plus a soft Pomeron that in this model is taken as a flat term.

The CKMT model [23] is also a phenomenological parametrization of $F_2$ at low $Q^2$, but it uses only one effective Pomeron. The CKMT model proposes for the nucleon structure functions:

$$F_2(x, Q^2) = F_S(x, Q^2) + F_{NS}(x, Q^2),$$

the following parametrization of its two terms in the region of small and moderate $Q^2$. For the singlet term, corresponding to the Pomeron contribution:

$$F_S(x, Q^2) = A_S x^{-\Delta(Q^2)}(1 - x)^{n(Q^2)+4} \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)},$$

where the $x \to 0$ behavior is determined by an effective intercept of the Pomeron, $\Delta$, which takes into account Pomeron cuts and, therefore (and this is one of the main points of the model), it depends on $Q^2$. This dependence is parametrized as:

$$\Delta(Q^2) = \Delta_0 \left( 1 + \frac{\Delta_1 Q^2}{Q^2 + \Delta_2} \right).$$

Thus, for low values of $Q^2$ (large cuts), $\Delta$ is close to the effective value found from analysis of hadronic total cross-sections ($\Delta \sim 0.08$), while for high values of $Q^2$ (small cuts), $\Delta$ takes the bare Pomeron value, $\Delta \sim 0.2 \div 0.25$. The parametrization for the non-singlet term, which corresponds to the secondary Reggeon ($f, A_2$) contribution, is:

$$F_{NS}(x, Q^2) = B_{NS} x^{1-\alpha_R}(1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R},$$

where the $x \to 0$ behavior is determined by the secondary Reggeon intercept $\alpha_R$, which is in the range $\alpha_R \sim 0.4 \div 0.5$. The valence quark contribution can be separated
into the contribution of the u \((B_u^{NS})\) and d \((B_d^{NS})\) valence quarks, the normalization condition for valence quarks fixes both contributions at one given value of \(Q^2\) \((Q^2_v = 2\, GeV^2\) has been used in the calculations). For both the singlet and the non-singlet terms, the behavior when \(x\to1\) is controlled by \(n(Q^2)\), with \(n(Q^2)\) being

\[
n(Q^2) = \frac{3}{2} \left(1 + \frac{Q^2}{Q^2 + c}\right).
\]  

(29)

Therefore, for \(Q^2 = 0\), the behavior of the valence quark distributions is given by Regge intercepts, \(n(0) = \alpha_R(0) - \alpha_N(0) \sim 3/2\), while the behavior of \(n(Q^2)\) for large \(Q^2\) is taken to coincide with dimensional counting rules. The total cross-section for real \((Q^2 = 0)\) photons can be obtained from the structure function \(F_2\) using the following relation:

\[
\sigma_{\gamma p}^{\text{tot}}(\nu) = \left[\frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2)\right]_{Q^2 = 0}.
\]  

(30)

The proper \(F_2(x, Q^2)\) behavior when \(Q^2\to0\), is given in the model by the last factors in Equations (26) and (28), leading to the following form of the \(\sigma_{\gamma p}^{\text{tot}}(\nu)\) in the CKMT model:

\[
\sigma_{\gamma p}^{\text{tot}}(\nu) = 4\pi^2\alpha \left(A^{S} a^{-1-\Delta_0 (2m\nu)^\Delta_0} + (B_u^{NS} + B_d^{NS}) b^{-\alpha_R (2m\nu)^{\alpha_R-1}}\right).
\]  

(31)

The parameters in the model were determined from a joint fit of the \(\sigma_{\gamma p}^{\text{tot}}\) data and NMC data on the proton structure function in the region \(1\, GeV^2 \leq Q^2 \leq 5\, GeV^2\), obtaining a very good description of the available experimental data. The next step in this approach is to introduce the QCD evolution in the partonic distributions of the CKMT model and thus to determine the structure functions at higher values of \(Q^2\). For this, the evolution equation in two loops in the \(\overline{\text{MS}}\) scheme with \(\Lambda = 200\, MeV\) was used. The results obtained by taking into account the QCD evolution in this way are in a very good agreement with the experimental data on \(F_2(x, Q^2)\) at high values of \(Q^2\).

The ALLM parametrization [24] of \(F_2\) also uses \(Q^2\)-dependent powers of \(x\), but it does not introduce QCD evolution.

Among the dynamical models of the low \(Q^2\) behavior of \(F_2\), we should mention the GRV model [25], in which the QCD evolution equations are extended down to the very low \(Q^2\) region \((Q^2 < 1\, GeV^2)\) using dynamical parton densities generated radiatively from valence-like inputs at some resolution scale. In this model both LO and NLO
approximations are used. Another dynamical model is the KP model [26], where the DGLAP evolution equation at NLO is solved by giving analytical parametrizations for the parton distributions. This model takes into account the contributions of higher-twist (renormalon-type) operators of the Wilson OPE, which are important at low $Q^2$. Finally, the BK model [27] considers the contributions both from the parton model with QCD corrections extended to the low $Q^2$ region and from the low mass vector mesons. There are other vector meson dominance (VMD) models [28].

On top of these, one can find models which are in the middle of two of the groups above. Thus, in the NZZ model [29] a color dipole approach is used to solve the BFKL equation with running coupling constant, while another dipole model with both a hard and a soft Pomeron has been proposed [30] where the large dipoles couple to the soft Pomeron and small dipoles couple to the hard Pomeron. This model has been applied to the case of the charm structure function $F^c_2(x, Q^2)$. Also an explicit dipole model with the CKMT pattern of energy behavior (effective $Q^2$-dependent Pomeron intercept) has recently been presented [31]. Other attempts have been made by interpolating between Regge behavior and the high $Q^2$ DGLAP asymptotics [32].

3 Discussion and Conclusions

In this contribution we have presented a basic introduction to HERA physics, mainly focused on the field of low $x$ physics. The low $x$ HERA physics provides an important experimental tool to obtain a better understanding of the interplay between soft and hard physics and the relation between concepts as saturation and unitarity, and to address more consistently the fundamental question of confinement. Models including saturation present a decrease of the effective Pomeron intercept as $x \to 0$, while in other models the effective intercept should increase as energy (or $1/x$) increases.

The experimental evidence of saturation at HERA, specifically the presence of a change in the $Q^2$ dependence of $F_2$ at very small $x$ and moderate $Q^2$, presented in a very graphic way as the turnovers of the logarithmic slopes of $F_2$, in particular of $\partial F_2/\partial \ln Q^2$ at moderate $Q^2$ and small $x$, is not conclusive.

Essential information on the behavior of the structure functions in the region of extremely small $x$, not accessible at HERA, will be available at LHC in a hopefully near future.
Acknowledgments

It is a pleasure to thank and congratulate Stephan Narison for organizing this conference and for his effort to connect, in a two-way profitable relation, our physics community to such a great country as Madagascar and to its scientists. I am also grateful to Marie Razafindrakoto for her success in implementing a friendly organization for the participants before and during the conference, and in dealing with the proceedings. Thanks are due to Fy Rafam’andrianjafy for her help on the week-end tour to the National Park of Andasibe and Manambato, and to all members of the local committee. I wish to thank M.A. Braun and G. Parente for reading and improving the manuscript, and E.G. Ferreiro for her useful comments. Finally, I would like to express my hope that this conference will become only the first of a long and fruitful series to come.

References


M. Derrick et al., ZEUS Collaboration, *Z. Phys. C* 63, 391 (1994);


