Magnetic permeability of near-critical 3d Abelian Higgs model and duality

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The Abelian Higgs model is conjecturally dual to a scalar field theory with a global $U(1)$ symmetry. We show that this duality, together with the scaling and universality hypotheses, implies a scaling law for the magnetic permeability $\chi_m$ near the line of second order phase transition: $\chi_m \sim t'^\nu$, where $t$ is the deviation from the critical line and $\nu \approx 0.67$ is a critical exponent of the $O(2)$ universality class. We also show that exactly on the critical lines, the dependence of magnetic induction on external magnetic field is quadratic, with a proportionality coefficient depending only on the gauge coupling. These predictions provide a way for testing the duality conjecture on the lattice in the Coulomb phase and at the phase transition.

Phase transitions, critical exponents, gauge theories, duality

Introduction

Understanding phase transitions in gauge theories is important for the physics of the early Universe RubakovShadn. In many other cases (e.g., in the electroweak theory with a small ratio of the Higgs mass to the $W$ mass, the phase transition can be treated reliably using perturbative techniques ArnoldEspinosa. In some cases, e.g., electroweak theory with a small ratio of the Higgs mass to the $W$ mass, the phase transition can be treated reliably using perturbative techniques

ArnoldEspinosa. In many other cases (e.g., in the electroweak theory with $m_H/m_W \approx 1$ or in QCD) perturbative calculations are unreliable and one has to resort to numerical simulations and other non-perturbative methods to learn about the nature of the phase transitions.

It is thus instructive to investigate simpler models where the phase transitions can be studied in detail. The Abelian Higgs model (AHM, also called the Ginzburg-Landau model), which describes the metal-superconductor transition, is an example of a simple theory with a rather nontrivial phase diagram. This model has two distinct phases: the Higgs phase, where the gauge boson (photon) is massive, and the Coulomb phase with a massless photon. The two phases must be separated by a phase transition. The temperature-induced phase transition is first order deep in the type I regime ($m_H \ll m_W$), as shown by perturbative calculations ArnoldEspinosa, but becomes second order as one goes to the type II regime ($m_H m_W$). The latter has been demonstrated by direct numerical simulations BartholomewKKLP,MHS.

That the phase transition in the AHM can be second order is somewhat surprising, given that it is always first order in $4 - \epsilon$ dimensions with small $\epsilon$ HLM. This fact, as has been argued, might have connection with a duality picture, according to which the three-dimensional AHM allows a dual description as a theory of a complex scalar field. The role of the elementary scalar in the dual theory is played by the Abrikosov-Nielsen-Olesen (ANO) vortex of the AHM BMK,DasguptaHalperin,Kleinert,KovnerRosenstein. Although the exact form of the dual Lagrangian is not known, quantitative predictions of the duality picture are possible near the second order phase transition, where only the symmetries of the dual Lagrangian are important. If the duality is valid, certain quantities in the AHM must behave singularly near the phase transition with the critical exponents of the $O(2)$ universality class (i.e., of the $XY$ model). In this way one can test the duality picture on the lattice. Numerical tests of this sort have been carried out in the Higgs phase, where, according to duality, the tension of an ANO vortex is equal to the mass of the dual scalar, and hence approaches zero as $t'^\nu$, where $t$ is the distance to the critical line and $\nu \approx 0.67$ is a critical exponent of the $XY$ model. Lattice results are still inconclusive, some appear to be inconsistent with this prediction KKLPMHS.

In this paper, we suggest some other tests of the duality hypothesis. In addition to measuring of the vortex tension, we propose to consider the AHM in its Coulomb phase and at the phase transition. What one should measure in the Coulomb phase is the magnetic permeability $\chi_m$, which goes to zero as one approaches the critical line where the Meissner effect (i.e., the Higgs mechanism) starts taking place and the system is perfectly diamagnetic. We shall show that $\chi_m$ is proportional to the square of the decay constant of the Goldstone boson in the dual theory. This mapping is precise (provided duality is valid) and involves only quantities which are not renormalized. Using scaling and universality hypotheses, we then show that the critical behavior for $\chi_m$ is $\chi_m \sim t'^\nu$. Exactly on the critical line, the magnetic permeability vanishes and the dependence of magnetic induction $B$ on external magnetic field $H$ is nonlinear. We shall demonstrate, by using simple scaling arguments, that this dependence is quadratic: $B \sim H^2$, with a proportionality coefficient
depending only on the gauge coupling $e$.

The paper is organized as follows. In section sec:review we review the duality picture. Section sec:arg is devoted to the study of the magnetic permeability of the Coulomb phase. The main line of logic in this section consists of three steps. In the first step (section sec:chimchi) one relates the magnetic permeability of the AHM with the susceptibility of the dual vacuum to the $U(1)$ chemical potential. The second step (section sec:chif) establishes, in the dual theory, the connection between the susceptibility with the decay constant of the Goldstone boson. The third step (section sec:ft) determines the critical behavior of the decay constant. Each step involves fairly well-known arguments, but we believe their synthesis is new. In section sec:BH2 we discuss the response of the AHM to an external magnetic field exactly on the critical line. Section sec:concl contains concluding remarks.

Review of the duality picture sec:review

Although we are mostly interested in the phase transition driven by temperature, thanks to dimensional reduction we can describe the static long-distance physics by an Euclidean three-dimensional AHM theory. Changing the temperature in the (3+1)d theory corresponds varying the parameter of the 3d dimensionally reduced theory. We shall thus start directly from the AHM in three spatial dimensions. It is sometimes useful, especially in our discussion of duality, to turn one spatial dimension into a temporal dimension; we then have a (2+1)d AHM, where the ANO vortex is a particle. This particle is the elementary scalar in the dual theory. The duality was first argued by using a formal representation of the partition function of the AHM in terms of loops BMK,DasguptaHalperin,Kleinert. Subsequently it has been given an operator form in 2+1 dimensions KovnerRosenstein.

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Abelian Higgs model complex scalar theory

The duality maps between the Abelian Higgs model and the dual theory of a complex scalar. Some of the mappings are explained further in the paper.table:duality

We summarize the correspondence between the AHM and the dual complex scalar theory in table table:duality. The Higgs phase is dual to the phase with unbroken $U(1)$ symmetry (the Mott insulator phase), and the Coulomb phase is dual to the phase where the $U(1)$ global symmetry is broken (the superfluid phase) by the condensation of the dual scalar field (“vortex condensation”). The dual of the massless photon in the AHM Coulomb phase is the superfluid Goldstone boson. This is possible because photon has only one polarization in 2+1 dimensions. The vortices in the AHM and the scalar theory, when exist, correspond to a particle in the other theory.

For the purpose of this paper, the most important equation of the duality picture comes from the identification of the magnetic field in the AHM with the $U(1)$ current in the dual theory, equation $e2\pi\epsilon^{\mu\nu\lambda}F_{\nu\lambda}=j_\mu$.duality