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Affine Malcev Algebra and $N = 8$ KdV*

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In this talk I report the results of two recent papers concerning the realization of the $N = 8$ supersymmetry from the division algebra of the octonions. At first I discuss a Sugawara realization for the “Non-associative $N = 8$ SCA” in terms of a superaffinization of the algebra of octonions. Next, I discuss the fact that the $N = 8$ SCA provides a generalized Poisson brackets structure for an $N = 8$ super-KdV.

Key-word: Supersymmetry.

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1 Introduction

In this talk I report some results obtained in two recent papers. In the first one [1], the superaffinization of the octonionic algebra has been introduced extending a procedure originally introduced in [2] in the purely bosonic case. In [1] it has been further proven that such superaffinization is related (via a suitable Sugawara limiting procedure) to the so-called “Non-associative $N = 8$ Superconformal algebra” first introduced in [3]. This algebra is constructed with the help of the octonionic structure constants and does not fulfill the Jacobi identities (due to the non-associativity of the octonions). This is the reason why it presents a central extension, overcoming a no-go theorem which states that no central extensions are allowed for *ordinary* superconformal algebras with $N > 4$. The presence of the central extension makes it tempting to consider the $N = 8$ SCA as a generalized Poisson brackets structure which induces a generalized KdV dynamics with large N supersymmetries. This is the content of [4], in which the first example of an $N = 8$ KdV has been constructed. Previous examples of supersymmetric extensions of KdV were given in [5] for the $N = 2$ case and in [6] in the $N = 4$ case.

The existence of both the $N = 8$ KdV and the $N = 8$ Sugawara realization relating the affine superfields with the superconformal ones, induces an $N = 8$ dynamics on the affine superfields which generalizes both the NLS and the mKdV systems of equations.

2 The $N = 8$ superaffine Malcev algebra

It consists of a superalgebra satisfying the Malcev property. It is constructed in terms of manifest $N = 1$ superfields $\Psi_a(X)$ and the totally antisymmetric octonionic structure constants C_{abc} (Greek indices run from 1 to 7, while Latin indices from 0 to 7), whose only non-vanishing components are given by

$$C_{123} = C_{147} = C_{165} = C_{246} = C_{257} = C_{354} = C_{367} = 1. \quad (1)$$

The algebra is explicitly given by

$$\{\Psi_a(X), \Psi_b(Y)\} = 2C_{abc}\Psi_c(Y)\delta(X, Y) + k \cdot \Pi(\tau_a \cdot \tau_b)D_Y\delta(X, Y), \quad (2)$$

where k is an affine central charge and $\Pi(\tau_a \cdot \tau_b)$ denotes the projection over the identity **1** in the composition law of the octonions τ_a 's.

The existence under a suitable limit of a Sugawara reconstruction formula for the fields entering the $N = 8$ SCA from the affine fields $\Psi_a(X)$, guarantees in particular that (2) is $N = 8$ supersymmetric. While the Sugawara formula will not be reported here, the “Non-associative $N = 8$ SCA” is explicitly given by

$$\begin{aligned} \{T(x), T(y)\} &= -\frac{1}{2}\partial_y^3\delta(x-y) + 2T(y)\partial_y\delta(x-y) + T'(y)\delta(x-y), \\ \{T(x), Q(y)\} &= \frac{3}{2}Q(y)\partial_y\delta(x-y) + Q'(y)\delta(x-y), \\ \{T(x), Q_\alpha(y)\} &= \frac{3}{2}Q_\alpha(y)\partial_y\delta(x-y) + Q'_\alpha(y)\delta(x-y), \end{aligned}$$

$$\begin{aligned}
\{T(x), J_\alpha(y)\} &= J_\alpha(y)\partial_y\delta(x-y) + J'_\alpha(y)\delta(x-y), \\
\{Q(x), Q(y)\} &= -\frac{1}{2}\partial_y^2\delta(x-y) + \frac{1}{2}T(y)\delta(x-y), \\
\{Q(x), Q_\alpha(y)\} &= -J_\alpha(y)\partial_y\delta(x-y) - \frac{1}{2}J'_\alpha(y)\delta(x-y), \\
\{Q(x), J_\alpha(y)\} &= -\frac{1}{2}Q_\alpha(y)\delta(x-y), \\
\{Q_\alpha(x), Q_\beta(y)\} &= -\frac{1}{2}\delta_{\alpha\beta}\partial_y^2\delta(x-y) + C_{\alpha\beta\gamma}J_\gamma(y)\partial_y\delta(x-y) + \\
&\quad + \frac{1}{2}(\delta_{\alpha\beta}T(y) + C_{\alpha\beta\gamma}J'_\gamma(y))\delta(x-y), \\
\{Q_\alpha(x), J_\beta(y)\} &= \frac{1}{2}(\delta_{\alpha\beta}Q(y) - C_{\alpha\beta\gamma}Q_\gamma(y))\delta(x-y), \\
\{J_\alpha(x), J_\beta(y)\} &= \frac{1}{2}\delta_{\alpha\beta}\partial_y\delta(x-y) - C_{\alpha\beta\gamma}J_\gamma(y)\delta(x-y).
\end{aligned} \tag{3}$$

3 The $N = 8$ KdV

The $N = 8$ SCA can be regarded as a generalized Poisson brackets structure. By looking for the most general hamiltonian with the right dimension and $N = 8$ -invariant, we arrive at a unique solution (up to the normalizing factor), namely

$$H = \int dx (-T^2 - Q'Q - Q'_\alpha Q_\alpha + J''_\alpha J_\alpha), \tag{4}$$

The equations of motion derived from (4) with the $N = 8$ SCA Poisson brackets correspond to the $N = 8$ generalization of KdV. They are explicitly given by

$$\begin{aligned}
\dot{T} &= -T''' - 12T'T - 6Q''_\alpha Q_\alpha + 4J''_\alpha J_\alpha, \\
\dot{Q} &= -Q''' - 6T'Q - 6TQ' - 4Q''_\alpha J_\alpha + 2Q_\alpha J''_\alpha - 2Q'_\alpha J'_\alpha, \\
\dot{Q}_\alpha &= -Q_\alpha''' - 2QJ''_\alpha - 6TQ'_\alpha - 6T'Q_\alpha + 2Q'J'_\alpha + 4Q''J_\alpha - \\
&\quad - 2C_{\alpha\beta\gamma}(Q_\beta J''_\gamma - Q'_\beta J'_\gamma - 2Q''_\beta J_\gamma), \\
\dot{J}_\alpha &= -J_\alpha''' - 4T'J_\alpha - 4TJ'_\alpha + 2QQ'_\alpha + 2Q'Q_\alpha - C_{\alpha\beta\gamma}(4J_\beta J''_\gamma + 2Q_\beta Q'_\gamma).
\end{aligned} \tag{5}$$

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