Frequency Hopping in Quantum Interferometry:
Efficient Up-Down Conversion for Qubits and Ebits

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Abstract

A novel single-photon Mach-Zehnder interferometer terminated at two different frequencies realizes the nonlinear frequency conversion of optical quantum superposition states. The information-preserving character of the relevant unitary transformation has been experimentally demonstrated for input qubits and ebits. Besides its own intrinsic fundamental interest, the new scheme will find important applications in modern quantum information technology.

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Interferometry of quantum particles is rooted at the core of modern physics as it provides a unique tool of investigation and a direct demonstration of fundamental properties of nature as complementarity, nonlocality and quantum nonseparability [1]. In modern times all methods and protocols of quantum information and quantum computation involve interferometry through the very definition of the conceptual cornerstones of this science, viz. the qubit and the ebit [2]. In this framework it is well known that the 1st-order interference of particles, e.g. optical photons in the present work, is a utterly fragile property that can be easily spoiled by de-coherence when the overall system exhibits a certain degree of complexity. More fundamentally it is a common notion that the interfering particles cannot be substantially disturbed by collisions let alone by the hardest possible collisions, i.e. the ones implying the annihilation or the creation of the particles themselves.

In the present paper we demonstrate experimentally that the last seemingly obvious condition is not a necessary one. In facts, there exits a class of information-preserving unitary transformations of parametric type allowing a nonlinear (NL) frequency conversion of all quantum interferometric dynamical structures via a QED particle annihilation or/and creation process. Conceptually the overall process may be considered as the dynamical "reversal" of the quantum injected NL parametric particle amplification/squeezing process that has been realized recently (QIOPA) [3]. Furthermore the frequency conversion process may be noiseless and may be easily realized with a "quantum efficiency" close to its maximum value, $QE = 1$. These are indeed very useful properties that will be of large technological interest in the domain of quantum information and computation.

Refer to Figure 1 showing the schematic diagram of a new kind of single-photon Mach-Zehnder (MZ) 1st-order interferometer (IF). The optical structure consists of an input 50/50 beam splitter (BS) coupled to two photon wavevector (wv) modes $k_j$ ($j = 1, 2$) at the wavelength ($wl$) $\lambda$, in our case lying in the infrared (IR) spectral region. Before standard mode recombination by the output BS$_2$, this simple mode structure is interrupted by a device $U$ providing a unitary nonlinear (NL) transformation $U$ on the input single-photon superposition state, i.e. the qubit defined in a 2-dimensional Hilbert $k$-space:
\[ |\Phi\rangle = \alpha |1\rangle_1 |0\rangle_2 + \beta |0\rangle_1 |1\rangle_2 \] (1)

where the state labels $1, 2$ refer to the IF modes $k_1$ and $k_2$ respectively. Assume now for simplicity and with no lack of generality: $\beta = \alpha \exp(i\Phi)$ and $\alpha \equiv 2^{-1/2}$. Let us consider the frequency up-conversion process. The NL frequency-conversion unitary evolution operator,

\[ U \equiv \exp[\tilde{g} \sum_{j=1,2} (\hat{a}_j\hat{a}_j^\dagger) + \text{h.c.}] \] (2)

provides the QED annihilation of the input qubit (1) defined by the momenta $\hbar k_j$, phase $\Phi$ and wl $\lambda$, and the simultaneous QED creation of a new qubit $|\Psi\rangle$ defined by the momenta $\hbar \vec{k}_j$ ($j = 1, 2$), phase $\Psi$ and wl $\lambda = (\lambda^{-1} + \lambda_p^{-1})^{-1}$ in our case lying in the ultraviolet (UV) spectral region. The 2nd-order tensor parameter $\tilde{g}$ is proportional to the interaction time $t$, to the 2nd-order susceptibility $d^{(2)}$ of the NL medium and to the ”pump” field $E_p$, with wl $\lambda_p$, wv $k_p$, phase $\Theta$. The pump field is assumed to be a single plane-wave coherent ”classical field” undepleted by the interaction. The wl’s and wv’s $\vec{k}_j$, $k_j$, $k_p$ are mutually connected by the energy conservation $\lambda^{-1} = (\lambda^{-1} + \lambda_p^{-1})$ and by the phase-matching condition (PMC) for the 3-wave interaction: $k_j + \vec{k}_j + k_p = 0$, leading in our simple plane case to an equation with two solutions: $j = 1, 2$. Similarly, the qubit phases at different wl’s are also connected by: $\Phi - \Psi + \Theta = 0$. Assume $\Theta = 0$ for convenience. We see that by introduction of the device $U$ and of an additional output $BS_3$ the standard MZ-IF is transformed into a new kind of interferometer terminated at two different wavelengths $\lambda$ and $\lambda$. Correspondingly two sets of interference fringes can be retrieved upon changes $\Delta\Phi = 2^{3/2}\pi X/\lambda$ of the mutual phase of the modes $k_j$ via displacements $X$ of an optical mirror $M$ activated by a piezo-transducer. Note that since in the present experiment PMC couples deterministically each mode $k_j$ to a corresponding $\vec{k}_j$, no additional quantum interference effects arise in the overall NL coupling process [4].

Let us now venture in a more detailed account of the experiment shown in Figure 2. A cw single-mode, linearly polarized diode laser, Mod. RLT8810MG, operating at the IR wl $\lambda = 876.1 \text{nm}$ with output power of $2mW$, was highly attenuated by a set of neutral density
filters (ND) to the single photon level. The achievement of this relevant field’s property was tested by two different Hanbury-Brown-Twiss (HBT) methods A and B. The first method (A) consisted of the well known linear photodetection correlation technique at the output of a 50/50 BS. The exact single-photon condition implying perfect anti-correlation is expressed by the zero value of the degree of 2nd-order coherence determined either at wavelength $\lambda$ or at wavelength $\lambda'$: $g^{(2)}(0) = 0$ [5]. To carry out this experiment for the wavelength $\lambda$ (or, independently for wavelength $\lambda'$) the IF phases were set by the mirror M to the value: $\Phi = \Psi = \pi/2$. This condition, carefully tested in the multiphoton regime, implied equal photodetection rates at the output of the detectors $D_A$ and $D_B$ for wavelength $\lambda$ (or $D_A$ and $D_B$ for wavelength $\lambda'$). In the single-photon regime, i.e. after beam attenuation, we obtained for the test at wavelength $\lambda$ a number of detected coincidences $N_C = 0$ with a number of singles $N_A = 1015$ and $N_B = 1223$, over a statistical sample of $10^5$ events. This sets the limit value: $g^{(2)}(0) < 8.05 \cdot 10^{-2}$. For the test at wavelength $\lambda'$ the corresponding figures were: $N_C = 0$, $N_A = 810$, $N_B = 830$, $N = 10^5$ leading to: $g^{(2)}(0) < 14.8 \cdot 10^{-2}$.

By a second nonlinear method (B), never adopted previously to our knowledge, a new 2nd-order quantum correlation function was investigated involving both IR and UV wavelengths $\lambda$ and $\lambda'$ and the density operator $\rho$ of the overall field emerging from the NL crystal:

$$g^{NL(2)}_j(0) = \frac{\text{Tr}(\rho E^{(-)}_{j\lambda} E^{(+)}_{j\lambda'})}{\text{Tr}(\rho E^{(-)}_{j\lambda} E^{(+)}_{j\lambda'})} \frac{\text{Tr}(\rho E^{(-)}_{j\lambda'} E^{(+)}_{j\lambda})}{\text{Tr}(\rho E^{(-)}_{j\lambda'} E^{(+)}_{j\lambda})}$$  \hspace{1cm} (3)

In this case the nonlinear anticorrelation condition $g^{NL(2)}_j(0) = 0$ implied that any up-converted single photon on mode $\overline{k}_j$ was associated with the vacuum field on $k_j$ and vice versa. In particular it also implied that Fock states $|n\rangle$ with $n > 1$ were absent on mode $k_j$ before the interaction, thus providing a further verification of the expression (1). This peculiar nonlinear HBT experiment applied to each mode pair $(k_j, \overline{k}_j)$ $j = 1, 2$ implied the determination of counting correlations between the output signals generated by the pairs of IR and UV detectors $D_j$ and $\overline{D}_j$, coupled respectively to the modes $k_j$ and $\overline{k}_j$ after their mutual NL interaction. For modes $(k_1, \overline{k}_1)$ the following results were obtained: coincidences: $N_C = 0$, singles: $N_D = 2636$, $N_{\overline{D}} = 713$, number of trials: $N = 10^5$. This led
to the upper limit: $g_{1}^{NL(2)}(0) < 5.3 \cdot 10^{-2}$.

All detectors operating at the IR wl $\lambda$ were equal Si avalanche single-photon SPCM-200PQ diodes with Quantum-Efficiency $qe \approx 30\%$ while the two detectors operating at the UV wl $\lambda$ were photomultipliers: Philips-56DUVP ($qe \approx 23\%$) and Hamamatsu-R943-02 ($qe \approx 21\%$). A computer interfaced Stanford Research 400 counter was adopted for counting and averaging the detected signals.

The input single-photon field with wl $\lambda$ was injected into the input 50/50 beam splitter $BS_1$ of the double MZ-IF with output modes $k_j$, $j = 1, 2$. These modes were mutually $\Phi$ – *dephased* by a piezoelectrically driven mirror (M) and the associated fields were brought by a $f = 3.5cm$ lens (L) into a common focal region, with diameter $\phi \approx 20\mu m$, within a NL LiIO$_3$, $l = 1mm$ thick crystal slab, cut for Type I phase matching. Here a strong NL 3-wave interaction took place between the input field associated with $k_j$, the up-converted field associated with the $\overline{k}_j$ and a single mode high intensity "pump" field associated with the ultrashort pulses emitted with wl $\lambda_p = 795nm$ by a mode-locked 76MHz Coherent MIRA 900 Ti-Sa femtosecond laser. Finally, the output beams $k_j$ emerging from the NL crystal were again superimposed by a 50/50 beam splitter $BS_2$ thus completing the usual MZ-IF scheme at the input wl $\lambda$. In a similar way the two up-converted UV output beams at the UV wl $\lambda = 416.8nm$ were superimposed on an independent 50/50 $BS_3$, thus completing the MZ-IF scheme at the up-converted wl $\lambda$. The difficult task of filtering the very weak beam at the WL $\lambda$ against the very strong UV beam at $\lambda_p/2$ was overcome by spatial discrimination after the NL crystal and by the adoption of two interference filters at $416.8nm$ with bandwidth 10nm.

Note that the up-conversion unitary transformation of the quantum superposition state (or "qubit") at the IR wl $\lambda$ into the "UV qubit" with $\lambda$ is a *noise free* process since energy conservation doesn’t allow any amplification of the input vacuum state. Of course, the inverse transformation process is also possible as an input qubit with $\lambda > \lambda_p$ can be frequency "down-converted" into a corresponding one with $\lambda = \lambda_p(\lambda - \lambda_p)^{-1} > \lambda$. 

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This "optical parametric amplifying" (OPA) process is nevertheless affected by a *squeezed vacuum* noise due to the amplification of the input vacuum state at \( \lambda \) [3].

The expression of the quantum efficiency (QE) of the up-conversion process, defined as the ratio between the average numbers of scattered and input photons on the modes \( \mathbf{k}_j, k_j \) as a function of the peak intensity of the pump pulse \( I_p \) and for perfect collinear interaction, can be obtained by previous evaluation of the field at the output of the nonlinear interaction: \( U^\dagger \hat{a} U \) and \( U^\dagger \hat{\pi} U \) [6]. This leads to the expression,

\[
QE = \sin^2 \left( \frac{\pi}{\varepsilon_0} \sqrt{\frac{I_p}{\lambda n_o n_e (\vartheta_m)}} \varrho^{(2)} l \right).
\]

where \( n_o \) is the "ordinary ray" refraction index at \( \lambda \) while \( n_e (\vartheta_m) \) is the "extraordinary ray" refraction index at \( \lambda \) for the angle \( \vartheta_m \) determined by \( k_p \) and by the optic axis of the crystal (phase matching angle) [7].

The expression of \( QE \) just given in Equation 4 refers to the case of a collinear interaction, a condition which can be approximated by improvement of the spatial superposition of the beams in the NL interaction region. We report in Figure 3 the theoretical value of \( QE \) as function of the pump intensity \( I_p \), calculated for the 1\( \text{mm} \) thick \( \text{LiIO}_3 \) crystal adopted in the experiment. The theoretical value of \( I_p \) corresponding to the limit condition \( QE = 1 \) is found \( I_p = 200\text{GW/cm}^2 \). This figure may be compared with the experimental result \( QE \approx 0.4 \) we obtained in a side experiment by focusing a low repetition rate 100\( \text{fs} \), \( I_p = 200\text{GW/cm}^2 \) pulse on the same crystal: Figure 3 [6]. This discrepancy is attributed to the non perfect realization of the plane wave collinear interaction in the active focal spot of the focusing lens. In the present MZ-IF experiment the peak intensity of each Ti-Sa laser pulse was \( \approx 1\text{GW/cm}^2 \), and the the corresponding measured value of the quantum efficiency was: \( QE \approx 3 \cdot 10^{-3} \).

The persistence of the quantum superposition condition within the \( IR \rightarrow UV \) frequency hopping process is demonstrated in Figure 4 by the two correlated interference fringe patterns showing an equal periodicity upon changes of the mutual dephasing \( \Delta \Phi \) of the IR modes \( k_1, k_2 \). As previously emphasized in a different context [3], this is but one aspect of a very
general *information preserving* transformation of all unitary NL parametric up- (or down-)
conversion transformations. By these ones any input qubit at \( \omega l \lambda \) and expressed by
Equation 1 is generally transformed into another at \( \omega l \lambda' \neq \lambda \) keeping the *same*
complex parameters \( \alpha, \beta \) of the original one, i.e. fully reproducing its *quantum information*
content. In addition and most important, the present work shows that these transformations can be
*noise-free* and can be realized with a quantum efficiency close to its maximum value.

Apart from the fundamental relevance of these results due to the peculiar paradigmatic
and historical status of single-particle interferometry in the quantum mechanical context,
the present work is expected to have a large impact on modern quantum information tech-
nology. This can be illustrated by the following example. Consider a case in which quantum
information is encoded on a single microwave photon with \( \omega l \lambda \), e.g. within the cavity of
a micromaser. If we want to transfer conveniently this information at a large distance we
need to use an optical fiber exhibiting its low loss behaviour in the IR spectral region, at
\( \omega l \lambda' \). This can be done in a *"information lossless"* manner in a NL waveguide by the up-
conversion \( \lambda \rightarrow \lambda' \). If now this information is to be transferred to a set of trapped atoms in
an optical cavity we may need a further lossless up-conversion into the visible: \( \lambda' \rightarrow \lambda'' \), etc.
This scenario may generally represent an appealing alternative to other linear methods, but
in some cases it may indicate the *only available* solution to sort quantum information out
of a nanostructure quantum device and, most important, to interconnect it efficiently within
a large information network made of heterogeneous components. This may the case of a
NMR quantum gate operating at a radiofrequency \( \omega l [8] \) or of a superconducting quantum
dot gate or a SQUID device operating at still lower electromagnetic (e.m.) frequencies [9].

So far we have been dealing only with conversion of "qubits". The extension of our NL
method to a two photon entangled state, or specifically to elementary entangled information
carriers, i.e. *ebits*, can be easily realized in several ways depending on the nature of the
entanglement. Consider for instance a linear-polarization (\( \pi \)) entangled 2-photon state emit-
ted over two spatial modes \( k_1, k_2 \) by a Spontaneous Parametric Down Conversion (SPDC)
process in a NL crystal: \( |\Phi\rangle = \alpha |\uparrow\rangle_1 |\uparrow\rangle_2 + \beta |\downarrow\rangle_1 |\downarrow\rangle_2 [3,10] \). With reference to a technique
recently adopted by P.Kwiat et al. [10] the modes $k_j$ and the strong coherent pump beam with wavevector $k_p$ could be focused by the common lens $L$ beam into a combination of two equal thin Type I NL plane crystal slabs, e.g. $LiIO_3$, $l = 1mm$ thick, and placed in mutual contact along their plane orthogonal to $k_p$. If these slabs are mutually rotated around the axis parallel to $k_p$ by an angle $\phi = \pi/2$ and the linear polarization $\pi_p$ of the pump beam is also rotated by $\phi = \pi/4$, both nonlocally correlated orthogonal $\pi$-state components of the injected entangled state undergo equal NL transformations given by [2] thus realizing an overall, *information preserving* up- (or down-) frequency conversion of $|\Phi\rangle$.

By recent works [11] [12] a new conceptual and formal perspective has been introduced in quantum information according to which the optical *field’s modes* rather than the photons are taken as the carriers of quantum information and entanglement. Furthermore in that picture any *qubit* is physically implemented by a two-dimensional subspace of Fock states of the e.m. field, specifically the state spanned by the vacuum state and the 1-photon state. According to this perspective the class of information preserving NL transformations of the state given by Equation [1] investigated in the present work should be more correctly referred to *entangled states* and may indeed provide a useful new set of unitary transformations for the Hilbert space evolution of these new information states.

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REFERENCES


[4] It is possible to conceive a NL 3-wave interaction in which the PMC is relaxed, e.g. in a very thin crystal, and the input modes are coupled non-deterministically to the up-converted ones. The Feynman path indistinguishability implied by the additional quantum chance condition may lead to additional interference phenomena that interplay with the dominant behaviour of the nonlinear MZ-IF.


Figure Captions

Figure 1: Schematic diagram of the nonlinear Mach-Zehnder interferometer (MZ-IF) terminated at two different correlated frequencies.

Figure 2: Lay-out of the single-particle MZ-IF experiment.

Figure 3: Up-conversion quantum efficiency $Q E$ as function of the laser pump intensity $I_p$: theoretical (continuous line) and experimental results.

Figure 4: Single photon interference fringes obtained within the same experiment at the different correlated wavelengths $\lambda$ and $\bar{\lambda} = (\lambda^{-1} + \lambda_p^{-1})^{-1}$. The phase period of the fringing patterns has been found is in agreement with the figure of merit (0.7 nm/V) of the piezoelectrical transducer activating the mirror M.