Chern-Simons Couplings for Dielectric F-Strings in Matrix String Theory

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Abstract: We compute the non-abelian couplings in the Chern-Simons action for a set of coinciding fundamental strings in both the type IIA and type IIB Matrix string theories. Starting from Matrix theory in a weakly curved background, we construct the linear couplings of closed string fields to type IIA Matrix strings. Further dualities give a type IIB Matrix string theory and a type IIA theory of Matrix strings with winding.¹

1 Introduction

It is well-known that a collection of D-branes under the influence of a background R-R field strength can undergo an “expansion” into a single higher-dimensional D-brane. This is the so-called dielectric effect, a first analysis of which was performed in [1], at the level of the abelian theory relevant to the description of the single expanded D-brane. It was some years later that the complementary² description from the point of view of the lower-dimensional multiple branes was provided [2]. From this perspective, the expansion takes place because the embedding coordinates of the multiple branes are matrix-valued, and give rise to new non-abelian couplings in the combined Born-Infeld-Chern-Simons action [3, 2].

There is much evidence that the dielectric effect should also exist for fundamental strings, both from the abelian analysis [1] and from the supergravity perspective [4]. However, one would further like to provide the complementary description from the point of view of the fundamental strings. Since, from the strings’ perspective, the dielectric effect should be due to matrix-valued coordinates, one is led to a consideration of Matrix string theory [5]. Moreover, matrix string theory is equivalent to type IIA superstring theory in the light-cone gauge, together with extra degrees of freedom representing D-brane states. Therefore it is the appropriate framework in which to study systems of multiple fundamental strings expanding into higher-dimensional D-branes (see also [6]).

¹Talk given by Bert Janssen.
²See [2] for a discussion on the ranges of validities of the two descriptions.
couplings of closed string fields to type IIA Matrix strings. Further dualities give a type IIB Matrix string theory and a type IIA theory of Matrix strings with winding. From these actions we identify the couplings that are responsible for the dielectric effect.

This letter is a summary of the results published in [7].

2 Matrix Theory and D0-branes

Our starting point is the Chern-Simons action for D0-branes with linear couplings to the R-R background fields, as given in [3]:

\[ S_{\text{linear}} = \frac{1}{R} \int dt \text{STr} \left\{ C^{(1)}(I_0^\mu + C^{(3)}(I_0^{\mu\rho} I_2^{\rho\nu}) + \frac{1}{60} C^{(5)}(I_4^{\mu_1...\mu_5}) + \frac{1}{336} C^{(7)}(I_6^{\mu_1...\mu_7}) \right\}, \]

where STTr denotes the symmetrised trace and \( R \) the radius of the eleventh dimension. These couplings can be derived from the Matrix theory couplings to linear background fields, as shown in [8]. The indices \( \mu, \nu = 0, \ldots, 9 \), the currents \( I_p \) couple to the R-R \((p+1)\)-form potentials \( C^{(p+1)} \) and the potentials \( C^{(5)} \) and \( C^{(7)} \) have been rescaled relative to [3]. The currents appearing in (1) are given in terms of the dimensional reduction of the Born-Infeld field strength,

\[ F_{0i} = -F^{0i} = \partial_t X^i \equiv \dot{X}^i, \quad F_{ij} = F^{ij} = \frac{i}{\beta} [X^i, X^j], \]

where \( i, j = 1, \ldots, 9 \) and \( \beta = 2\pi l_s^3/R \). We will not give the explicit expression but refer to [3]. It can be shown [2, 7] that (1) agrees with the linear order expansion of the multiple D0-brane theory [3, 2]

\[ S = T_0 \int dt \text{STr} \left\{ e^{iX^i} \frac{1}{\lambda} \left( \sum C^{(n)} \right) \right\}, \]

with \( \lambda = 2\pi \alpha' \). Here the interior multiplication is defined as \( (iX^i \Sigma)_{\mu_1,...,\mu_p} = X^i \Sigma_{\mu_1,...,\mu_p} \). This type of contraction of the embedding scalars with the R-R potentials are the origin of the dielectric effect, as explained in [2].

3 Multiple D-strings

To construct the Matrix string theory of [5], one compactifies Matrix theory on a circle in, say, the \( x^9 \) direction and performs a T-duality transformation. Taking now \( i, j = 1, \ldots, 8 \), and denoting the dual coordinate by \( \hat{x} \), the world volume fields transform as [9]

\[ F^{0i} = \frac{i}{\beta} [X^9, X^i] \longrightarrow \frac{1}{2\pi R_9} \int d\hat{x} \frac{\lambda}{\beta} D_\hat{x} X^i, \quad F^{09} = -\dot{X}^9 \longrightarrow -\frac{1}{2\pi R_9} \int d\hat{x} \lambda \dot{A}_\hat{x}, \]

where \( \hat{R}_9 = \alpha'/R_9 \) is the radius of the dual circle. Turning to the linear action (1), we must consider T-duality applied to both the world volume and background fields. As far as the currents are concerned, we just have to consider the transformation of the Born-Infeld field strength (2) under T-duality, as given by (4) above. This is a simple re-writing, for the results we refer to [3].

\[^{3}\text{Note that there should also be couplings to } C^{(9)}, \text{ but these are not determined by the analysis of [3].}\]
where \(a, b = 0, \ldots, 8\). A simple application of these rules gives

\[
S_{\text{linear}} = \frac{1}{2\pi R R_9} \int dtd\sigma \, \text{STr} \left\{ C^{(0)} I_0^9 + C^{(2)}_{ab} I_0^{ab} + 3C^{(4)}_{abc} I_2^{abc} + C^{(4)}_{abcd} I_2^{abcd} + \frac{1}{12} C^{(4)}_{a_1 a_4} I_4^{a_1 a_4} + \frac{1}{60} C^{(6)}_{a_1 a_5} I_4^{a_1 a_5} + \frac{1}{48} C^{(6)}_{a_1 a_6} I_6^{a_1 a_6} + \frac{1}{336} C^{(8)}_{a_1 a_7} I_8^{a_1 a_7} \right\}.
\]

By construction, the multiple D0-brane action (3) is covariant under T-duality, so the T-dual of the D0-brane action (1) should be equivalent to the linearised version of the D-string action. With the Is as in [3], it is easy to see that the R-R terms can indeed be written as [2]

\[
S = T_1 \int \text{STr} \left\{ P \left[ e^{i(\hat{q}_x \hat{q}_x)/\lambda} \left( \sum C^{(n)} \right) \right] \wedge e^{\lambda F} \right\},
\]

where \(F_{09} = \hat{A}_z\).\(^4\)

4 Matrix string theory in type IIA

Having constructed the \((1 + 1)\)-dimensional theory for multiple D-strings, we are now in a position to perform the so-called 9-11 flip, which acts on the background fields as

\[
C^{(0)} \rightarrow -C^{(1)}, \quad C^{(2)}_{ab} \rightarrow b_{ab}, \quad C^{(2)}_{a_9} \rightarrow -h_{a_9},
\]

\[
C^{(4)}_{a_1 a_4} \rightarrow C^{(5)}_{a_1 a_4}, \quad C^{(4)}_{abc} \rightarrow C^{(3)}_{abc}, \quad C^{(6)}_{a_1 a_6} \rightarrow N^{(7)}_{a_1 a_6},
\]

\[
C^{(8)}_{a_1 a_8} \rightarrow \tilde{b}_{a_1 a_8}, \quad \tilde{C}^{(9)}_{a_1 a_9} \rightarrow -C^{(9)}_{a_1 a_9}, \quad C^{(7)}_{a_1 a_7} \rightarrow -C^{(7)}_{a_1 a_7},
\]

and will give us the type IIA Matrix string theory action in a linear background. Here, \(b_{\mu \nu}, \tilde{b}_{\mu_1 \ldots \mu_5}\) and \(N^{(7)}_{\mu_1 \ldots \mu_7}\) are the NS-NS 2-form, its Hodge dual and the field that couples minimally to the type IIA Kaluza-Klein monopole, respectively. We take the view here that the currents are invariant under the 9-11 flip, since in the flat space case it does not change the worldvolume fields [5].

Performing the transformations (8) on the linear action (6), one finds

\[
S = \frac{1}{2\pi} \int d\tau d\sigma \, \frac{R'}{R^2} \text{STr} \left\{ -C^{(1)}_{0 9} - h_{a_9} I_0^a + 3b_{a_5} I_2^{a_5} + C^{(3)}_{abc} I_2^{abc} + \frac{1}{12} C^{(5)}_{a_1 a_4} I_4^{a_1 a_4} + \frac{1}{60} \tilde{b}_{a_1 a_5} I_4^{a_1 a_5} + \frac{1}{48} N^{(7)}_{a_1 a_6} I_6^{a_1 a_6} - \frac{1}{336} C^{(7)}_{a_1 a_7} I_7^{a_1 a_7} \right\}.
\]

Here, we have defined the dimensionless world sheet coordinates

\[
\sigma = \frac{1}{R_9} \hat{x}^9, \quad \tau = \frac{R}{\alpha'} t.
\]

Writing the currents \(I\) in terms of the dimensionless quantities \(\tau\) and \(\sigma\), this is the action describing Matrix string theory in a weakly curved background [10, 7].

In particular, if we set the Born-Infeld field to zero, we find for the R-R three-form couplings:

\[
S_{C^{(3)}} = \frac{1}{4\pi g_s} \int d\tau d\sigma \, \text{STr} \left\{ \frac{\sqrt{\alpha'}}{R} \sqrt{2} C^{(3)}_+ + \frac{\sqrt{\alpha'}}{R} \sqrt{2} C^{(3)}_- + \hat{X}^i C^{(3)}_i \right\},
\]

\(^4\)We should note that only half of the terms necessary to form the pullback of \(C^{(8)}\) are present in the linear action (6). The missing terms come from the missing \(C^{(9)}\) coupling in the D0-brane action (1).
and made use of light-cone coordinates. This coupling has been given before in [10]. We showed however in [7] that extra $C^{(3)}$ terms arise if one takes into account the couplings coming from the Born-Infeld part of the action. A dielectric solution involving only the couplings in (11) was given in [10] which we have interpreted in [7] in terms of gravitational waves expanding into a transverse D2-brane.

It can be seen from (9) that as regards the R-R 5-form potential, only the terms of the form $C^{(5)}_{a_1\ldots a_9}$ contribute. Setting the Born-Infeld vector to zero, the remaining 5-form R-R field couplings can be written as:

$$S_{C^{(5)}} = \frac{i}{4\pi g_s} \frac{R}{\sqrt{\alpha'}} \int d\tau d\sigma \text{STr} \left\{ \sqrt{\alpha'} C^{(5)}_{\mu
u} + X^i C^{(5)}_{i+} \right\},$$

where we have defined

$$C^{(5)}_{\mu\nu} = [X^k, X^j] DX^i C^{(5)}_{ijkl\mu\nu}. \quad (14)$$

Similarly, the $C^{(7)}$ couplings only have contributions involving terms of the form $C^{(7)}_{a_1\ldots a_7}$. These can be written as

$$S_{C^{(7)}} = \frac{i}{96\pi g_s} \frac{R^2}{\alpha'} \int d\tau d\sigma \text{STr} \left\{ \sqrt{\alpha'} C^{(7)}_{\mu
u} + X^i C^{(7)}_{i+} \right\},$$

with

$$C^{(7)}_{\mu
u} = [X^k, X^j][X^l, X^m] DX^i C^{(7)}_{ijkl\mu\nu}. \quad (16)$$

Let us also consider for the sake of completeness the couplings to $\tilde{b}^{(6)}$ and $N^{(7)}$. As for $C^{(5)}$, only the terms with a 9-component appear in the action. The $\tilde{b}^{(6)}$ couplings can be written as:

$$S_{\tilde{b}^{(6)}} = \frac{1}{16\pi g_s} \frac{R}{\sqrt{\alpha'}} \int d\tau d\sigma \text{STr} \left\{ \sqrt{\alpha'} \tilde{b}^{(6)}_{++} + X^i \tilde{b}^{(6)}_{+i} - X^i \tilde{b}^{(6)}_{-i} \right\},$$

where

$$\tilde{b}^{(6)}_{\mu\nu} = [X^i, X^k][X^j, X^l] \tilde{b}^{(6)}_{ijkl\mu\nu}, \quad (18)$$

and the $N^{(7)}$ couplings are given by

$$S_{N^{(7)}} = -\frac{1}{16\pi g_s} \frac{R^2}{\alpha'} \int d\tau d\sigma \text{STr} \left\{ \sqrt{\alpha'} N^{(7)}_{++} - X^i N^{(7)}_{+i} + X^i N^{(7)}_{-i} \right\},$$

where

$$N^{(7)}_{\mu\nu} = [X^m, X^i][X^k, X^j] DX^i N^{(7)}_{ijkl\mu\nu}. \quad (20)$$

Note that the couplings of the different fields occur at a different order of the expansion parameter $R/\sqrt{\alpha'}$. One should look at these couplings when studying fundamental strings expanding into D4-, D6- or NS5-branes, or Kaluza-Klein monopoles.

5 Multiple IIB F-strings

To describe fundamental strings in the type IIB theory, we perform another T-duality in the $x^9$ direction, as in (5). As before, we assume that the world volume fields do not change. The linear action (9) becomes

$$S_{\text{linear}}^{\text{IIB}} = \frac{1}{2\pi} \int d\tau d\sigma \frac{R}{R^2} \text{STr} \left\{ b_{ab} I_0^a - C^{(0)} I_0^9 + C^{(4)}_{abc} I_2^{ab} + 3b_{ab} I_2^{ab0} \right\}.$$
which should describe Matrix strings in the IIB theory. Note that precisely the same
action is obtained if one applies the S-duality rules to the D1-brane action (6), although
it is not clear \textit{a priori} how such an S-duality should be done directly, since we are dealing
with non-abelian fields. The action above can be rewritten in a more convenient form,
filling in the expressions for the currents:

$$S = \frac{1}{2\pi} \int d\tau d\sigma \text{STr} \left\{ P \left[ \frac{\alpha'}{R} \gamma^{(2)} + \frac{g_s \alpha'}{R} C^{(0)} \wedge F \right. + \frac{i}{g_s} \frac{\alpha'}{\alpha} (iXiX) \tilde{C}^{(4)} + i (iXiX) b^{(2)} \wedge F \left. - \frac{1}{2g_s^2} (iXiX)^2 \tilde{b}^{(6)} \right] - \frac{R}{2g_s^2 \alpha} (iXiX)^2 C^{(4)} \wedge F + \frac{iR}{6g_s^2 \sqrt{\alpha'}} (iXiX)^3 \tilde{C}^{(8)} - \frac{iR^2}{6g_s^2 \alpha'} (iXiX)^3 \tilde{b}^{(6)} \wedge F \right\}. \quad (22)$$

Again we note that only half of the terms necessary to form the pullback of $C^{(8)}$ are
present in the linear action. Some of the above couplings have been given before in [11].
In [7] it was shown that the $C^{(4)}$ coupling gives rise to a solution of F-strings expanding
into a D3-brane.

6 Multiple F-strings with winding in type IIA

Type IIA F-strings with winding number can be obtained from IIB strings by performing
a T-duality transformation in a direction transverse to the IIB strings. Calling $z$ the T-
duality direction and $a = (0, i)$, where now $i = 1, \ldots, 7$, the linear action that is obtained
from (21) is given by:

$$S^{\text{IIB}}_{\text{linear}} = \frac{1}{2\pi} \int d\tau d\sigma' \text{STr} \left\{ \tilde{b}_{d2} I_0^a + h_{2g} I_0^z - C^{(1)}_z I_0^9 + C^{(5)}_{abc} I_2^{abc} - 3C_{abh} I_2^{dbz} + 3b_{ab} I_2^{dbz} \right. \left. - 6h_{a2} I_2^{a2} + \frac{1}{60} N^{(7)}_{a1 \ldots a5} I_4^{a1 \ldots a5} + \frac{1}{12} \tilde{b}_{a1 \ldots a42} I_4^{a1 \ldots a42} + \frac{1}{12} C^{(5)}_{a1 \ldots a4z} I_4^{a1 \ldots a4z} + \frac{1}{3} C^{(3)}_{abc} I_4^{abc} \right. \left. - \frac{1}{336} C^{(9)}_{a1 \ldots a7z} I_6^{a1 \ldots a7z} + \frac{1}{48} C^{(7)}_{a1 \ldots a6z} I_6^{a1 \ldots a6z} + \frac{1}{48} N^{(7)}_{a1 \ldots a6z} I_6^{a1 \ldots a6z} + \frac{1}{8} \tilde{b}_{a1 \ldots a5z} I_6^{a1 \ldots a5z} \right\}. \quad (23)$$

The direction $z$ in which the T-duality is performed appears as an isometry direction in
the transverse space of the strings. We denote the corresponding Killing vector as $k^\mu = \delta^\mu_z$, $k_\mu = \eta_{2\mu} + h_{2\mu}$. In a manner similar to the Kaluza-Klein monopole, the non-abelian strings
do not see this special direction, the embedding scalar $X^z$ is not a degree of freedom
of the strings, but is transformed under T-duality into a world volume scalar $\omega$.

The action (23) can be written in a covariant way as a gauged sigma model, where gauge
covariant derivatives $D_\alpha X^\mu$ are used to gauge away the embedding scalar corresponding
to the isometry direction [12]:

$$D_\alpha X^\mu = D_\alpha X^\mu - k_\rho D_\alpha X^\rho k^\mu, \quad (24)$$

with $\alpha = \sigma, \tau$. These gauge covariant derivatives reduce to the standard covariant
derivatives $D_\alpha X^\mu$ for $\mu \neq z$ and are zero for $\mu = z$. The pull-backs in the action of the F-strings
with winding are constructed from these gauge covariant derivatives. For example,

$$P \left[ b^{(2)} \right] = b_{\mu\nu} DX^\mu DX^\nu dt dx \quad (25)$$

\footnote{This world volume scalar forms an invariant field strength with $b_{az}$ (see [11] for the details), and can therefore be associated to fundamental strings wrapped around the isometry direction $z$, which themselves end on the Matrix strings.}
S_{\text{linear}} = \frac{1}{2\pi} \int d\tau d\sigma \text{STr} \left\{ \mathcal{P} \left[ -\frac{g_s \sqrt{\alpha'}}{R} i_k C^{(1)} \wedge F + \frac{g_s}{R^2} b^{(2)} \wedge D\omega + i\langle [X, \omega] \rangle k^{(1)} \wedge F \right. \right.
\left. + \frac{i}{g_s R} \left( i\langle [X, \omega] \rangle C^{(3)} + \frac{i}{g_s} (i\langle X \rangle X) C^{(3)} \right) \wedge D\omega + \frac{R}{g_s \sqrt{\alpha'}} (i\langle X \rangle X) (i\langle X, \omega \rangle) C^{(3)} \wedge F - \frac{i}{g_s R} (i\langle X \rangle X) i_k C^{(5)} \right.
\left. + \frac{R}{2g_s \sqrt{\alpha'}} (i\langle X \rangle X)^2 i_k C^{(5)} \wedge F + \frac{1}{g_s} (i\langle X, \omega \rangle) (i\langle X \rangle X) i_k b^{(6)} + \frac{R}{2g_s \sqrt{\alpha'}} (i\langle X \rangle X)^2 i_k b^{(6)} \wedge D\omega \right.
\left. + \frac{R^2}{2g_s \alpha'} (i\langle X, \omega \rangle) (i\langle X \rangle X)^2 i_k b^{(6)} \wedge F - \frac{1}{2g_s^2} (i\langle X \rangle X)^2 i_k N^{(7)} - \frac{iR^2}{6g_s^2 \sqrt{\alpha'}} (i\langle X \rangle X)^3 i_k N^{(7)} \wedge F - \frac{iR^2}{2g_s^2 \alpha'} (i\langle X, \omega \rangle) (i\langle X \rangle X)^3 i_k C^{(7)} + \frac{R}{6g_s^2 \alpha'} (i\langle X \rangle X)^3 i_k C^{(9)} \right\}, \quad (26)

where we have introduced the following types of interior multiplication:
\[(i_k \Sigma)_{\mu_1 \ldots \mu_p} = k^\rho \Sigma_{\rho \mu_1 \ldots \mu_p} = \Sigma_{\rho \mu_1 \ldots \mu_p}, \quad \langle i\langle X, \omega \rangle \Sigma \rangle_{\mu_1 \ldots \mu_p} = [X^i, \omega] \Sigma_{\mu_1 \ldots \mu_p}. \quad (27)\]

Again only half of the terms to form the pullback of $C^{(7)}$ and $C^{(9)}$ are present in the linear action (23). Some of the couplings in (26) have been derived earlier in [11]. It can be shown [7] that the $C^{(5)}$ couplings lead to a solution of F-strings expanding into a D4-brane that is smeared in the $z$ direction and the $C^{(3)}$ couplings to an unstable solution of F-strings expanding into a cylindrical D2-brane.

References