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Pseudospin symmetry and spin symmetry in the relativistic mean-field with deformed potential

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Abstract

We compared the pseudospin pair-levels and the spin pair-levels in the relativistic mean-field calculation. So long as the pseudospin symmetry is well satisfied, the good agreement in the absolute values of *unphysical* amplitudes (with *unnatural* ℓ_z in the centrifugal term compared with the asymptotic quantum number) of pseudospin pair-levels is found both for the large and small components. On the other hand, the good agreement in the absolute values of *physical* (with *natural* ℓ_z) amplitudes is found both for the large and small components in the spin pair-levels when good spin symmetry is satisfied. The relation between the deformed wave function and the spherical wave function are discussed at the spherical limit.

21.10.Pc, 21.10.Hw, 21.60.Jz, 21.30.-x, 24.10.Jv, 27.70.+q

I. INTRODUCTION

One of the advantages in the relativistic treatment is that it clarifies the origin of the spin-orbit interaction, which is also related with the pseudospin concept. Recently, there appeared many works with the purpose to find the root of the pseudospin concept to the relativistic mean-field equation [3–9]. We found the general and realistic conditions for the pseudospin approximation both for the spherical potential [6–8], and the deformed potential [7,9]. It is found that the condition found in the large component of Dirac wave function works much better than that in the small component wave function for the deformed case in contrast to the spherical case [9].

The pseudospin concept in the deformed nuclei is as follows: The single-particle levels with $j_z = \Omega = \ell_z + 1/2$, and $j'_z = \Omega + 1 = \ell_z + 2 - 1/2$ lie very close in energy, and these two levels are labeled as $j_z = \tilde{\ell}_z - 1/2$ and $j'_z = \tilde{\ell}_z + 1/2$ with $\tilde{\ell}_z = \ell + 1$. If we use the asymptotic quantum number $[N, n_z, \ell_z]\Omega$ with $j_z = \Omega$ to identify the deformed state, which is usually used in the non-relativistic calculation, the pseudospin pair levels become $[N, n_z, \ell_z]\Omega$ and $[N, n_z, \ell_z + 2](\Omega + 1)$, or equivalently $[N, n_z, \tilde{\ell}_z](\tilde{\ell}_z - 1/2)$ and $[N, n_z, \tilde{\ell}_z](\tilde{\ell}_z + 1/2)$. On the other hand, the spin pair levels are labelled by $[N, n_z, \ell_z]\Omega$ and $[N, n_z, \ell_z]\Omega + 1$, or equivalently $[N, n_z, \ell_z](\ell_z - 1/2)$ and $[N, n_z, \ell_z](\ell_z + 1/2)$, as they are splitted only by the spin-orbit interaction.

In this paper, we compare the pseudospin pair levels with $\tilde{\ell}_z \pm 1/2$, and the spin pair levels with $\ell_z \pm 1/2$ by using the relativistic mean-field program [10] for ^{154}Sm . In Sec. II our theory is reviewed, and in Sec. III the numerical analysis is carried over ^{154}Sm . The conclusion is given in Sec. IV.

II. THEORY

The eigenfunction $\psi(\vec{r})$ for the Dirac equation with an axially symmetrically deformed potential has two components, i.e., upper (large) component g and lower (small) component

f . We use the cylindrical coordinates (ρ, φ, z) , and then each of g and f in $\psi(\vec{r})$ has two components,

$$\begin{aligned} g &= \frac{1}{\sqrt{2\pi}} \begin{pmatrix} ig_{+,k}^{\Omega} e^{i(\Omega-1/2)\varphi} \\ ig_{-,k}^{\Omega} e^{i(\Omega+1/2)\varphi} \end{pmatrix}, \\ f &= \frac{1}{\sqrt{2\pi}} \begin{pmatrix} f_{+,k}^{\Omega} e^{i(\Omega-1/2)\varphi} \\ f_{-,k}^{\Omega} e^{i(\Omega+1/2)\varphi} \end{pmatrix}. \end{aligned} \quad (2.1)$$

Here $g_{\pm,k}^{\Omega}$ and $f_{\pm,k}^{\Omega}$ are functions of ρ and z , k denotes the other quantum numbers except for Ω . The $\pm$ sign in $g_{\pm,k}^{\Omega}$ and $f_{\pm,k}^{\Omega}$ corresponds to the spin wave function with $s_z = \pm 1/2$. Inserting this equation into the Dirac equation, we get the coupled equations. Then, we found the equations for $g_{\pm,k}^{\Omega}$ and the equations for $f_{\pm,k}^{\Omega}$ have symmetric form to each other [7,9].

$$\begin{aligned} -V_1 V_2 g_{\pm,k}^{\Omega} &= [\partial_{\rho}^2 - \frac{(\Omega \mp 1/2)^2}{\rho^2} + \frac{\partial_{\rho}}{\rho} + \partial_z^2] g_{\pm,k}^{\Omega} \\ &\quad - \frac{\partial_{\rho} V_2}{V_2} [(\partial_{\rho} \mp \frac{\Omega \mp 1/2}{\rho}) g_{\pm,k}^{\Omega} \mp \partial_z g_{\mp,k}^{\Omega}] \\ &\quad - \frac{\partial_z V_2}{V_2} [(\pm \partial_{\rho} + \frac{\Omega \pm 1/2}{\rho}) g_{\mp,k}^{\Omega} + \partial_z g_{\pm,k}^{\Omega}], \end{aligned} \quad (2.2)$$

$$\begin{aligned} -V_1 V_2 f_{\pm,k}^{\Omega} &= [\partial_{\rho}^2 - \frac{(\Omega \mp 1/2)^2}{\rho^2} + \frac{\partial_{\rho}}{\rho} + \partial_z^2] f_{\pm,k}^{\Omega} \\ &\quad - \frac{\partial_{\rho} V_1}{V_1} [(\partial_{\rho} \mp \frac{\Omega \mp 1/2}{\rho}) f_{\pm,k}^{\Omega} \mp \partial_z f_{\mp,k}^{\Omega}] \\ &\quad - \frac{\partial_z V_1}{V_1} [(\pm \partial_{\rho} + \frac{\Omega \pm 1/2}{\rho}) f_{\mp,k}^{\Omega} + \partial_z f_{\pm,k}^{\Omega}]. \end{aligned} \quad (2.3)$$

Here $\partial_{\rho}^2 = \partial^2/\partial\rho^2$, $\partial_{\rho} = \partial/\partial\rho$, $\partial_z^2 = \partial^2/\partial z^2$ and $\partial_z = \partial/\partial z$. The eigenvalue of Dirac equation ϵ is included in two kinds of potentials V_1 and V_2 ,

$$V_1 = M - \epsilon_{k,\Omega} - V_S - V_V, \quad V_2 = \epsilon_{k,\Omega} + M + V_S - V_V, \quad (2.4)$$

with vector potential V_V , scalar potential V_S and mass M . As is seen in Eqs. (2.2) and (2.3), $g_{\pm,k}^{\Omega}$ and $f_{\pm,k}^{\Omega}$ have symmetric forms by exchanging V_2 by V_1 . It is also seen from Eqs. (2.2) and (2.3) that there exists a symmetry relation among $g_{\pm,k}^{\Omega}$ ($f_{\pm,k}^{\Omega}$) and $g_{\pm,k}^{-\Omega}$ ($f_{\pm,k}^{-\Omega}$) in the axially symmetrically deformed case.

$$g_{+,k}^\Omega = +A_{k,\Omega}g_{-,k}^{-\Omega}, \quad g_{+,k}^\Omega = -A_{k,\Omega}g_{-,k}^{-\Omega}, \quad f_{+,k}^\Omega = -A_{k,\Omega}f_{-,k}^{-\Omega}, \quad f_{-,k}^\Omega = +A_{k,\Omega}f_{+,k}^{-\Omega}. \quad (2.5)$$

where $|A_{k,\Omega}|^2 = 1$. It indicates $\pm\Omega$ components of the eigenfunction are not independent. So long as Ω is limited to the positive value, four amplitudes $f_{\pm,k}^\Omega$ and $g_{\pm,k}^\Omega$ are independent. From now on, we limit our discussion to the positive value of Ω and use both $f_{\pm,k}^\Omega$ and $g_{\pm,k}^\Omega$.

Let us assume the case where the following conditions are satisfied,

$$\partial V_2 / \partial \rho = 0, \quad \partial V_2 / \partial z = 0. \quad (2.6)$$

In this special region, the coupled equation (2.2) becomes,

$$\begin{aligned} -V_1 V_2 g_{-,k}^\Omega &= [\partial_\rho^2 - \frac{(\Omega + 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] g_{-,k}^\Omega \\ -V_1' V_2' g_{+,k'}^{\Omega+1} &= [\partial_\rho^2 - \frac{(\Omega + 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] g_{+,k'}^{\Omega+1}, \end{aligned} \quad (2.7)$$

where V_1 and V_2 have $\epsilon_{k,\Omega}$, while V_1' and V_2' have $\epsilon_{k',\Omega+1}$ as is defined in Eq. (2.4). This is nothing but the pseudospin doublets, if k corresponds to $[N, n_z, \ell_z]$, k' to $[N, n_z, \ell_z + 2]$, and $\epsilon_{k,\Omega} \sim \epsilon_{k',\Omega+1}$ or $V_1 = V_1'$ and $V_2 = V_2'$. They obey the same equation with $\tilde{\ell}_z = \Omega + 1/2$ in the centrifugal term. However, they have *unnatural* ℓ_z , because the true value of ℓ_z for the former level must be $\ell_z = \Omega - 1/2$ as $j_z = \Omega = \ell_z + 1/2$, and the true value of ℓ'_z for the latter level is $\ell'_z = \Omega + 3/2$ as $j'_z = \Omega + 1 = \ell_z + 2 - 1/2$. We name them as *unphysical* amplitudes [9]. Then, we can say that the pseudospin doublet is found in the *unphysical* amplitudes of upper components.

If both of k and k' are the same with $[N, n_z, \ell_z]$ and $\epsilon_{k,\Omega}$ nearly equals to $\epsilon_{k,\Omega+1}$, Eq. (2.7) also corresponds to the spin doublets, as $\Omega = \ell_z - 1/2$ and $\Omega + 1 = \ell_z + 1/2$. In this case, spin doublets have *natural* $\ell_z = \Omega + 1/2$ in the centrifugal term. We name them as *physical* amplitudes [9]. Then, we can say that the spin doublet is found in *physical* amplitudes of upper components. We must pay attention to the fact that the condition (2.6) is effective in both the pseudospin doublets and the spin doublets.

It is found that the region where $\partial_\rho V_1 = 0$ overlaps with the region where $\partial_\rho V_2 = 0$, and the region where $\partial_z V_1 = 0$ equals to the region where $\partial_z V_2 = 0$ [9]. However, we have not yet

confirmed that there exists the region where $\partial_\rho V_2 = 0$ equals to the region where $\partial_z V_2 = 0$. In deriving Eq. (2.7), we have assumed both $\partial_\rho V_2 = 0$ and $\partial_z V_2 = 0$. To make it sure, we show the behaviours of the relative ratio of the derivatives of $V_V + V_S$ ($V_V - V_S$) to $V_V + V_S$ ($V_V - V_S$) in Fig. 1. The details of the calculation are explained in Sec. III. It is seen the region where $\partial_\rho(V_V + V_S)/(V_V + V_S)$ almost agrees with those where $\partial_z(V_V + V_S)/(V_V + V_S)$, $\partial_\rho(V_V - V_S)/(V_V - V_S)$ and $\partial_z(V_V - V_S)/(V_V - V_S)$ in the realistic calculation. As the region where $\partial_\rho V_2 = 0$ and $\partial_z V_2 = 0$ agrees with the region where $\partial_\rho V_1 = 0$ and $\partial_z V_1 = 0$, we can apply the same discussion to the lower components as that to the upper components. If the following condition is satisfied,

$$\partial_\rho V_1 = 0, \quad \partial_z V_1 = 0, \quad (2.8)$$

then

$$\begin{aligned} -V_1 V_2 f_{-,k}^\Omega &= [\partial_\rho^2 - \frac{(\Omega + 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] f_{-,k}^\Omega \\ -V_1' V_2' f_{+,k'}^{\Omega+1} &= [\partial_\rho^2 - \frac{(\Omega + 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] f_{+,k'}^{\Omega+1}. \end{aligned} \quad (2.9)$$

If k corresponds to $[N, n_z, \ell_z]$, and k' to $[N, n_z, \ell_z + 2]$, $f_{-,k}^\Omega$ and $f_{+,k'}^{\Omega+1}$ for the pseudospin doublets ($\epsilon_{k,\Omega} \sim \epsilon_{k',\Omega+1}$), obey the same equation with $\tilde{\ell}_z$, but *unnatural* ℓ_z in the centrifugal term. In the other word, the pseudospin doublet is found in the *unphysical* amplitudes of lower components. If both of k and k' are defined as $[N, n_z, \ell_z]$, $f_{-,k}^\Omega$ and $f_{+,k}^{\Omega+1}$ for the spin doublets ($\epsilon_{k,\Omega} \sim \epsilon_{k,\Omega+1}$), have *natural* $\ell_z = \Omega + 1/2$ in the centrifugal term. In the other word, the spin doublets are found in *physical* amplitudes of lower components. Again, the condition (2.8) which is equivalent to (2.6) works both for the pseudospin doublets and spin doublets.

Now we consider of the other amplitudes for the states with (k, Ω) and $(k', \Omega + 1)$, i.e., $g_{+,k}^\Omega$ ($f_{+,k}^\Omega$) and $g_{-,k'}^{\Omega+1}$ ($f_{-,k'}^{\Omega+1}$). Under the same conditions of (2.6) or (2.8), they become

$$\begin{aligned} -V_1 V_2 g_{+,k}^\Omega &= [\partial_\rho^2 - \frac{(\Omega - 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] g_{+,k}^\Omega \\ -V_1' V_2' g_{-,k'}^{\Omega+1} &= [\partial_\rho^2 - \frac{(\Omega + 3/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] g_{-,k'}^{\Omega+1} \end{aligned}$$

$$\begin{aligned}
-V_1 V_2 f_{+,k}^\Omega &= [\partial_\rho^2 - \frac{(\Omega - 1/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] f_{-,k}^\Omega \\
-V_1' V_2' f_{-,k'}^{\Omega+1} &= [\partial_\rho^2 - \frac{(\Omega + 3/2)^2}{\rho^2} + \frac{\partial_\rho}{\rho} + \partial_z^2] f_{-,k'}^{\Omega+1}.
\end{aligned} \tag{2.10}$$

These amplitudes have *natural* ℓ_z and $\ell_z + 2$ for the pseudospin doublets of $k = [N, n_z, \ell_z]$ and $k' = [N, n_z, \ell_z + 2]$. In the other word, they belong to *physical* amplitudes. They obey different equations except for large ρ where the contribution from the centrifugal term becomes negligible. The pseudospin doublets with $g_{+,k}^\Omega$ ($f_{+,k}^\Omega$) of $k = [N, n_z, \ell_z]$ and $g_{-,k'}^{\Omega+1}$ ($f_{-,k'}^{\Omega+1}$) of $k' = [N, n_z, \ell_z + 2]$ obey the different equation but with *natural* ℓ_z . If we choose $k = k' = [N, n_z, \ell_z]$ in Eq. (2.10), these amplitudes have *unnatural* ℓ_z . For the spin doublets with $k = [N, n_z, \ell_z]$, $g_{+,k}^\Omega$ ($f_{+,k}^\Omega$) and $g_{-,k}^{\Omega+1}$ ($f_{-,k}^{\Omega+1}$) obey the different equation with *unnatural* ℓ_z .

From Fig. 2 to Fig. 5, we have compared these *physical* and *unphysical* amplitudes for both pseudospin doublets and spin doublets. In all the figures, the *unphysical* amplitudes are shown by dashed or dot-dashed lines, while the *physical* amplitudes are shown by solid or dotted lines. Both the dashed and the solid lines correspond to the level with Ω , and both the dot-dashed and dotted lines correspond to the level with $(\Omega + 1)$.

In Fig. 2, the absolute values of the *unphysical* amplitudes (dashed and dot-dashed lines) almost agree both in upper and lower components, while those of the *physical* amplitudes (solid and dotted lines) do not. This is already pointed out in our last paper [9]. As we commented in Eq. (2.10), the difference in the centrifugal term becomes negligible at larger ρ . It is seen in Fig. 3 which shows the behaviour of the pseudospin doublets as a function of z at $\rho = 5$ fm. Both the absolute values of *unphysical* and *physical* amplitudes become almost the same in the figure. The components in the spin pair levels are shown in Figs. 4 and 5. We see a good agreement in the absolute values of *physical* amplitudes (solid and dotted lines), and not in *unphysical* amplitudes (dashed and dot-dashed lines). The numerical analysis supports Eqs. (2.7) and (2.9) works well for the case where the pseudospin symmetry is good and also for the case where the spin symmetry is good. If there were a good degeneracy in pseudospin pair levels labelled with (k, Ω) and $(k', \Omega + 1)$, and also with $(k', \Omega + 1)$ and

$(k', \Omega + 2)$, there would appear triple degeneracy. In the realistic case, it is hard to appear, but Figs. 2 ~ 5 seem to show such candidate.

Now we discuss the relation between the deformed case and the spherical case. At the spherical limit, we have used the polar coordinate (r, θ, φ) instead of the cylindrical coordinate [6–8]. The upper and lower components in the spherical limit are denoted by

$$\begin{aligned} g &= i \frac{G^{\ell j}(r)}{r} \mathcal{Y}_{jm}^{\ell}(\theta, \phi) \\ f &= \frac{F^{\ell j}(r)}{r} (\vec{\sigma} \cdot \hat{r}) \mathcal{Y}_{jm}^{\ell}(\theta, \phi), \end{aligned} \quad (2.11)$$

where $\mathcal{Y}_{jm}^{\ell}(\theta, \phi)$ is a vector spherical harmonics. We rewrite Eq.(2.1) by replacing $k = [\ell, j]$ and then comparing it with Eq. (2.11). Then, we get the relation between $f_{\pm, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta)$ and $g_{\pm, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta)$, and $G^{\ell, j=\ell \pm 1/2}(r)$ and $F^{\ell, j=\ell \pm 1/2}(r)$.

$$\begin{aligned} f_{\pm, \ell, j=\ell+1/2}^{\Omega}(r, \theta) &= \pm \sqrt{2\pi} e^{-i(\Omega \mp 1/2)\varphi} \sqrt{\frac{\ell + 3/2 \mp \Omega}{2\ell + 3}} Y_{\Omega \mp 1/2}^{\ell+1}(\theta, \varphi) \frac{F^{\ell, j=\ell+1/2}(r)}{r} \\ f_{\pm, \ell, j=\ell-1/2}^{\Omega}(r, \theta) &= -\sqrt{2\pi} e^{-i(\Omega \mp 1/2)\varphi} \sqrt{\frac{\ell - 1/2 \pm \Omega}{2\ell - 1}} Y_{\Omega \mp 1/2}^{\ell-1}(\theta, \varphi) \frac{F^{\ell, j=\ell-1/2}(r)}{r} \\ g_{+, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta) &= \pm \sqrt{2\pi} e^{-i(\Omega - 1/2)\varphi} \sqrt{\frac{\ell + 1/2 \pm \Omega}{2\ell + 1}} Y_{\Omega - 1/2}^{\ell}(\theta, \varphi) \frac{G^{\ell, j=\ell \pm 1/2}(r)}{r} \\ g_{-, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta) &= \sqrt{2\pi} e^{-i(\Omega + 1/2)\varphi} \sqrt{\frac{\ell + 1/2 \mp \Omega}{2\ell + 1}} Y_{\Omega + 1/2}^{\ell}(\theta, \varphi) \frac{G^{\ell, j=\ell \pm 1/2}(r)}{r}. \end{aligned} \quad (2.12)$$

Here $f_{\pm, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta)$ and $g_{\pm, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta)$ are functions of (r, θ) and not (ρ, z) , and $Y_{\Omega}^{\ell}(\theta, \varphi)$ is the spherical harmonics. Then, we get the symmetry relation from Eq. (2.12),

$$\begin{aligned} f_{+, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta) &= \mp (-)^{\Omega-1/2} f_{-, \ell, j=\ell \pm 1/2}^{-\Omega}(r, \theta), \\ g_{+, \ell, j=\ell \pm 1/2}^{\Omega}(r, \theta) &= \pm (-)^{\Omega-1/2} g_{-, \ell, j=\ell \pm 1/2}^{-\Omega}(r, \theta). \end{aligned} \quad (2.13)$$

Comparing Eq. (2.5) with Eq. (2.13), $|A_{k, \Omega}|^2$ in Eq. (2.5) equals to $(-)^{\Omega-1/2}$.

In the spherical limit, it is said that the pseudospin symmetry is hidden in the lower component [3,4,6–8], while the spin symmetry is in the upper component. From Eq. (2.12), we see the following relations,

$$\frac{f_{+, \ell+2, j=\ell+2-1/2}^{\Omega+1}(r, \theta)}{f_{-, \ell, j=\ell+1/2}^{\Omega}(r, \theta)} = \frac{F^{\ell+2, j=\ell+2-1/2}(r)}{F^{\ell, j=\ell+1/2}(r)} \frac{\sqrt{\ell + 5/2 + \Omega}}{\sqrt{\ell + 3/2 + \Omega}}$$

$$\frac{g_{+, \ell, j=\ell+1/2}^{\Omega+1}(r, \theta)}{g_{-, \ell, j=\ell-1/2}^{\Omega}(r, \theta)} = \frac{G^{\ell, j=\ell+1/2}(r)}{G^{\ell, j=\ell-1/2}(r)} \frac{\sqrt{\ell + 3/2 + \Omega}}{\sqrt{\ell + 1/2 + \Omega}}. \quad (2.14)$$

There is no simple relation among the ratio for $g_{+, \ell+2, j=\ell+2-1/2}^{\Omega+1}(r, \theta)$ and $g_{-, \ell, j=\ell+1/2}^{\Omega}(r, \theta)$, and also for $f_{+, \ell, j=\ell+1/2}^{\Omega+1}(r, \theta)$ and $f_{-, \ell, j=\ell-1/2}^{\Omega}(r, \theta)$. It indicates that if we choose polar coordinate at the spherical limit, $f_{\pm, \ell, j}^{\Omega}(r, \theta)$ has pseudospin symmetry while $g_{\pm, \ell, j}^{\Omega}(r, \theta)$ has spin symmetry in itself.

To make our discussion much clearer, we return to the original definition of $g_{\pm, k}^{\Omega}$ and $f_{\pm, k}^{\Omega}$ at the spherical limit without using the representation by r and θ . The relation between the cylindrical coordinate and polar coordinate gives $\partial_r V_1 = \cos\theta \partial_z V_1 + \sin\theta \partial_\rho V_1$, which shows if condition (2.8) is satisfied, automatically $\partial_r V_1 = 0$, i.e. the condition for the pseudospin symmetry [6–8] at the spherical limit. As the potentials have no θ dependence at the spherical limit, $\rho \partial_z V_1 = z \partial_\rho V_1$ and $\rho \partial_z V_2 = z \partial_\rho V_2$, and the quantum number k is replaced by ℓ and j . If $\epsilon_{\ell, j, \Omega} \sim \epsilon_{\ell+2, j+1, \Omega+1}$, we can derive the equation for the linear combination of the *unphysical* amplitudes of the lower components in the pseudospin doublets from Eq. (2.3).

$$\begin{aligned} -V_1^2(f_{+, \ell+2, j+1}^{\Omega+1} \pm f_{-, \ell, j}^{\Omega}) &= \frac{V_1}{V_2} (\partial_\rho^2 + \frac{\partial_\rho}{\rho} - \partial_z^2 \frac{(\Omega + 1/2)^2}{\rho^2}) (f_{+, \ell+2, j+1}^{\Omega+1} \pm f_{-, \ell, j}^{\Omega}) \\ &\quad - \frac{\partial_\rho V_1}{V_2} \left[(\partial_\rho + \frac{z}{\rho} \partial_z) (f_{+, \ell+2, j+1}^{\Omega+1} \pm f_{-, \ell, j}^{\Omega}) - \frac{\Omega + 1/2}{\rho} (f_{+, \ell+2, j+1}^{\Omega+1} \mp f_{-, \ell, j}^{\Omega}) \right. \\ &\quad \left. + (-\partial_z + \frac{z}{\rho} \partial_\rho) (f_{-, \ell+2, j+1}^{\Omega+1} \mp f_{+, \ell, j}^{\Omega}) + \frac{z}{\rho^2} \Omega (f_{-, \ell+2, j+1}^{\Omega+1} \pm f_{+, \ell, j}^{\Omega}) \right. \\ &\quad \left. + \frac{z}{2\rho^2} (3f_{-, \ell+2, j+1}^{\Omega+1} \mp f_{+, \ell, j}^{\Omega}) \right]. \end{aligned} \quad (2.15)$$

Either of the linear combination of *unphysical* amplitudes, i.e., $f_{+, \ell+2, j+1}^{\Omega+1} \pm f_{-, \ell, j}^{\Omega}$ (– sign in Fig. 2 case) is larger than *physical* amplitudes, i.e. $f_{-, \ell+2, j+1}^{\Omega+1} \pm f_{+, \ell, j}^{\Omega}$ (+ sign in Fig.2 case) for the pseudospin doublets. Moreover, V_2 is large and never goes to zero in its definition. Thus, we can neglect the third and the fourth lines in Eqs. (2.15). This implies that the pseudospin symmetry persists in the lower amplitudes at the spherical limit. Similarly, we rewrite Eq. (2.2) for the linear combination of the *unphysical* amplitudes in the upper components for the pseudospin doublets,

$$-V_1(g_{+, \ell+2, j+1}^{\Omega+1} \pm g_{-, \ell, j}^{\Omega}) = \frac{1}{V_2} (\partial_\rho^2 + \frac{\partial_\rho}{\rho} + \partial_z^2 - \frac{(\Omega + 1/2)^2}{\rho^2}) (g_{+, \ell+2, j+1}^{\Omega+1} \pm g_{-, \ell, j}^{\Omega})$$

$$\begin{aligned}
& -\frac{\partial_\rho V_2}{V_2^2} \left[(\partial_\rho + \frac{z}{\rho} \partial_z)(g_{+, \ell+2, j+1}^{\Omega+1} \pm g_{-, \ell, j}^\Omega) - \frac{\Omega + 1/2}{\rho} (g_{+, \ell+2, j+1}^{\Omega+1} \mp g_{-, \ell, j}^\Omega) \right. \\
& + (-\partial_z + \frac{z}{\rho} \partial_\rho)(g_{-, \ell+2, j+1}^{\Omega+1} \mp g_{+, \ell, j}^\Omega) + \frac{z}{\rho^2} \Omega (g_{-, \ell+2, j+1}^{\Omega+1} \pm g_{+, \ell, j}^\Omega) \\
& \left. + \frac{z}{2\rho^2} (3g_{-, \ell+2, j+1}^{\Omega+1} \mp g_{+, \ell, j}^\Omega) \right] \quad (2.16)
\end{aligned}$$

As V_2 is much larger than V_1 and never becomes zero, we can neglect the terms proportional to $\partial_\rho V_2/V_2^2$. As we see in Fig. 2, the *unphysical* amplitudes in the upper components are small for the pseudospin doublets. They may be small to be negligible order at the spherical limit. Moreover, the relation between the cylindrical coordinate and the polar coordinate gives $\partial_r V_2 = \cos\theta \partial_z V_2 + \sin\theta \partial_\rho V_2$. Subsequently, if condition (2.6) is satisfied, $\partial_r V_2 = 0$, which is not the condition for the pseudospin symmetry as is proved in our paper [7].

Next, we consider of the linear combination of the physical amplitudes for the spin doublets at the spherical limit. If there were a good degeneracy of the spin doublets, i.e., $\epsilon_{\ell, j, \Omega} \sim \epsilon_{\ell, j+1, \Omega+1}$, the linear combination of the *physical* amplitudes in the upper components satisfies the equation

$$\begin{aligned}
-V_1(g_{+, \ell, j+1}^{\Omega+1} \pm g_{-, \ell, j}^\Omega) &= \frac{1}{V_2} (\partial_\rho^2 + \frac{\partial_\rho}{\rho} + \partial_z^2 - \frac{(\Omega + 1/2)^2}{\rho^2}) (g_{+, \ell, j+1}^{\Omega+1} \pm g_{-, \ell, j}^\Omega) \\
& - \frac{\partial_\rho V_2}{V_2^2} \left[(\partial_\rho + \frac{z}{\rho} \partial_z)(g_{+, \ell, j+1}^{\Omega+1} \pm g_{-, \ell, j}^\Omega) - \frac{\Omega + 1/2}{\rho} (g_{+, \ell, j+1}^{\Omega+1} \mp g_{-, \ell, j}^\Omega) \right. \\
& + (-\partial_z + \frac{z}{\rho} \partial_\rho)(g_{-, \ell, j+1}^{\Omega+1} \mp g_{+, \ell, j}^\Omega) + \frac{z}{\rho^2} \Omega (g_{-, \ell, j+1}^{\Omega+1} \pm g_{+, \ell, j}^\Omega) \\
& \left. + \frac{z}{2\rho^2} (3g_{-, \ell, j+1}^{\Omega+1} \mp g_{+, \ell, j}^\Omega) \right] \quad (2.17)
\end{aligned}$$

As we commented in Fig. 4, either of the linear combination of *physical* amplitudes, $g_{+, \ell, j+1}^{\Omega+1} \pm g_{-, \ell, j}^\Omega$ (+ sign in Fig. 4) are larger than *unphysical* amplitudes $g_{-, \ell, j+1}^{\Omega+1} \pm g_{+, \ell, j}^\Omega$ for the spin doublets. Thus, we can neglect the terms proportional to $\partial_\rho V_2/V_2^2$ in Eq. (2.17). This indicates that the spin symmetry exists in the upper components at the spherical limit, only when $\epsilon_{\ell, j, \Omega} \sim \epsilon_{\ell, j+1, \Omega+1}$. Similar discussion is carried for the lower components by replacing the index $(\pm, \ell + 2, j + 1)$ by $(\pm, \ell, j + 1)$ in Eq. (2.15). As the *physical* amplitudes in lower components for the spin doublets are small, the spin symmetry is not seen in the lower components at the spherical limit. As for the *physical* amplitudes in the pseudospin

doublets and *unphysical* amplitudes in the spin doublets, there is also no good agreement in the spherical limit, as the centrifugal terms are different as is seen in Eq. (2.10).

In the spherical limit, the pseudospin symmetry is in the small component and the spin symmetry is in the large component. In the deformed case, in contrast to the spherical case, both the pseudospin symmetry and the spin symmetry are found both in the upper (large) and lower (small) components. Both symmetries are closely related, as the condition (2.6) is equivalent to the condition (2.8). This indicates that the deformed field mixes both symmetries just as the deformation mixes ℓ and j . As the pseudospin symmetry is found not only in the small component but also in the large component, the pseudospin symmetry becomes better in the deformed nuclei than in the spherical nuclei.

III. NUMERICAL ANALYSIS

We have performed the numerical analysis for ^{154}Sm with the same code and parameter set as in our last paper [9]. We have rewritten the relativistic Hartre plus BCS approximation code for the deformed nuclei developed by Ring's group [10]. In Fig. 1 we show the relative ratio of the potential derivatives by ρ and z to the potentials $V_V \pm V_S$, which is shown by $V \pm S$ in the figure. As $V_1 = M - \epsilon_{k\Omega} - V_S - V_V$ and $V_2 = M + \epsilon_{k\Omega} + V_S - V_V$, the derivatives of V_1 and V_2 are the same as the derivatives of $V_V \pm V_S$. The figure shows that the region where four kinds of derivatives become zero are nearly the same. For much larger z and ρ , the derivatives of $V_V \pm V_S$ goes to zero, as the potentials are converging to zero which is shown in Fig. 2 and Fig. 3 in Ref. [9]. Thus, the numerical analysis with a realistic parameter supports that the conditions (2.6) and (2.8) are not independent, but satisfied at the same regions.

From Figs. 2 to 5 we show the wave functions for the pseudospin doublets (Figs. 2 and 3), and spin doublets (Figs. 4 and 5). In all four figures, we use dashed lines for the *unphysical* amplitudes corresponding to the level with Ω , and the dot-dashed lines for the level with $\Omega + 1$, respectively. As for the *physical* amplitudes, we use solid lines for the level with Ω

and dotted lines for the level with $\Omega + 1$. Inside the figures, the lines are explained by the asymptotic quantum number $[N, n_z, \ell_z]\Omega$ together with $\pm$ sign in front of $[N, n_z, \ell_z]\Omega$, which corresponds to $\pm \frac{1}{2}$ of s_z . The upper components are shown in the left panel, and the lower components are in the right panel in all figures from Figs. 2 to 5.

We adopted the pseudospin doublets with the asymptotic quantum numbers $[400]1/2$ at -9.109 MeV and $[402]3/2$ at -9.289 MeV with the energy splitting of 0.180 MeV. In Fig. 2 we show the behaviour of the upper components for both levels, i.e. $g_{\pm,k}^{\Omega}$ and $g_{\pm,k'}^{\Omega+1}$ in the left panel, and the lower components, i.e. $f_{\pm,k}^{\Omega}$ and $f_{\pm,k'}^{\Omega+1}$ in the right panel as a function of ρ at $z = 3$ fm. We see the values of g_{-k}^{Ω} shown by dashed line and $g_{+k'}^{\Omega+1}$ shown by dot-dashed line in the left panel are almost the same. In the right panel, f_{-k}^{Ω} shown by dashed line is the reversal of $f_{+k'}^{\Omega+1}$ shown by dot-dashed line. They belong to the *unphysical* amplitudes of pseudospin doublets. In contrast to them, the *physical* amplitudes of pseudospin doublets shown by solid lines and dotted lines give quite different behaviour except for larger ρ region, which are more clearly seen in the left panel. The behaviour of g_{+k}^{Ω} (solid line) are quite different from $g_{-k'}^{\Omega+1}$ (dotted line) in the left panel, and f_{+k}^{Ω} (solid line) and $f_{-k'}^{\Omega+1}$ (dotted line) in the left panel are different. It is marked that the longitudinal scale in the left panel is larger than that in the right panel, which makes us call the upper component as the large component and the lower component as the small component. In order to see the behaviour at larger ρ value, we show eight components for pseudospin doublet ($g_{\pm,k}^{\Omega}$ and $g_{\pm,k'}^{\Omega+1}$, $f_{\pm,k}^{\Omega}$ and $f_{\pm,k'}^{\Omega+1}$) at $\rho = 5$ fm as a function of z in Fig. 3. It is seen good agreement in the absolute values of the *unphysical* amplitudes (dashed lines and dot-dashed lines), and also in the *physical* amplitudes (solid and dotted lines) in both panels. This is because the centrifugal term in Eq. (2.9) becomes negligible at larger ρ .

We choose the spin pair levels of $[402]3/2$ at -9.289 MeV and $[402]5/2$ at -10.686 MeV levels. The energy splitting of 1.397 MeV is not small compared with the energy splitting of pseudospin doublet of 0.180 MeV in Fig. 2 and 3, but still small compared with the other spin pair levels shown in Fig. 6. We show the behaviour of the upper components $g_{\pm,k}^{\Omega}$ and

$g_{\pm,k}^{\Omega+1}$ in the left panel, and the lower components $f_{\pm,k}^{\Omega}$ and $f_{\pm,k}^{\Omega+1}$ in the right panel as a function of ρ at $z = 1$ fm in Fig. 4. There is a good agreement in the absolute values of the physical amplitudes of spin doublets, i.e. $g_{-,k}^{\Omega}$ shown by solid lines and $g_{+,k}^{\Omega+1}$ shown by dotted line in the left panel, and $f_{-,k}^{\Omega}$ shown by solid line and $f_{+,k}^{\Omega+1}$ shown by dotted line in the right panel. This figure is comparable to the pseudospin doublet in Fig. 4 in Ref. [9]. In Fig. 5, we show the behaviours of eight amplitudes at $\rho = 5$ fm as a function of z . It is seen there are almost good agreement in the absolute values of *physical* amplitudes (solid and dotted lines) in both panels. As for the *unphysical* amplitudes (dashed lines and dot-dashed lines), the agreement in the absolute values becomes better in the region of $z > 5$ fm. Again it is because of the negligible centrifugal term in Eq. (2.9) at larger ρ . Thus, the numerical analysis supports the conclusion coming from Eqs. (2.7) and (2.9). The absolute values of *unphysical* amplitudes agree for the pseudospin doublets both in upper (large) and lower (small) components. The absolute values of *physical* amplitudes agree for the spin doublets both in upper (large) and lower (small) components.

As is seen from Figs. 2 and 3, the *unphysical* amplitudes in upper components of the pseudospin doublets are smaller than the *physical* amplitudes, and the *unphysical* amplitudes in lower components of the pseudospin doublet are larger than the *physical* amplitudes. In contrast to the pseudospin doublets, as is seen in Figs. 4 and 5, the *physical* amplitudes in upper components of the spin doublets are larger than the *unphysical* amplitudes, and the *physical* amplitudes in lower components of the spin doublets are smaller than the *unphysical* amplitudes. The pseudospin doublets and spin doublets, i.e. $[400]1/2$, $[402]3/2$ and $[402]5/2$ levels are a good candidate for the triple degeneracy. Although 1.397 MeV and 0.180 MeV are not negligible order, there is no such good candidate with a good triple degeneracy in all the other levels shown in Figs. 6 and 7.

If we adopt the Nilsson Hamiltonian as a model at the non-relativistic limit, we can relate $\epsilon_{k,\Omega}$ with the classical energy given by Eq. (5-10) in Bohr-Mottelson's text book [11]. In Fig. 6 we show the value of $\Delta\epsilon$ as a function of ϵ . Their definitions in Fig. 6 are followings;

$$\Delta\epsilon = \frac{(\epsilon_{Nn_z\ell_z,\Omega} - \epsilon_{Nn_z\ell_z+2,\Omega+1})}{\tilde{\ell}_z}$$

$$\epsilon = \frac{1}{2\tilde{\ell}_z}(\epsilon_{Nn_z\ell_z,\Omega}(\tilde{\ell}_z + 1) + \epsilon_{Nn_z\ell_z+2,\Omega+1}(\tilde{\ell}_z - 1)). \quad (3.1)$$

By using the same notation as in Eq. (5-10) in Ref. [11], the asymptotic value at the non-relativistic limit of $\Delta\epsilon \rightarrow \hbar\bar{\omega}\tilde{\ell}_z(v_{\ell s} - 4v_{\ell\ell})$. Here $v_{\ell s}$ is the strength of the spin-orbit interaction, and $v_{\ell\ell}$ is the strength of orbit-orbit interaction at the non-relativistic limit. In the non-relativistic treatment, the pseudospin symmetry appears when $v_{\ell s} \sim 4v_{\ell\ell}$. The asymptotic value of ϵ is independent of $v_{\ell s}$ at the classical limit. The numerals inside the figure are $(\Omega^{\text{parity}}, n_z)$, which denote the pair levels with $[N, n_z, \ell_z]\Omega$ and $[N, n_z, \ell_z + 2](\Omega + 1)$ are used for $\Delta\epsilon$ and ϵ in (3.1). The doublets in $N = 4$ are shown by plus signs, and the pseudospin doublets for $[400]1/2$ and $[402]3/2$ are in the bottom of the chain $N = 4$ with suffix $(1/2^+, 0)$. The doublets shown by square with $(7/2^-, 0)$ are for the levels with $[503]7/2$ and $[505]9/2$. The figure shows $\Delta\epsilon$ for this doublet is smallest, but $\Delta\epsilon$ is defined as the real energy difference divided by $\tilde{\ell}_z$ as is seen Eq. (3.1). The real energy difference between $[503]7/2$ and $[505]9/2$ is 0.279 MeV, which is much larger than the energy difference for $[400]1/2$ and $[402]3/2$ of 0.180 MeV. The doublets $[400]1/2$ and $[402]3/2$ have the smallest energy splitting in this nucleus.

Similarly, we show the energy difference $\Delta\epsilon$ versus ϵ for the spin doublets in Fig. 7. In this case, the definition of $\Delta\epsilon$ and ϵ are followings;

$$\Delta\epsilon = \frac{1}{\ell_z}(\epsilon_{Nn_z\ell_z,\Omega} - \epsilon_{Nn_z\ell_z,\Omega+1}),$$

$$\epsilon = \frac{1}{2}(\epsilon_{Nn_z\ell_z,\Omega} + \epsilon_{Nn_z\ell_z,\Omega+1}) \quad (3.2)$$

The asymptotic value of $\Delta\epsilon$ becomes $\hbar\bar{\omega}v_{\ell s}$ at the non-relativistic limit, and the asymptotic value of ϵ is independent from $v_{\ell s}$. Inside the figure, the numerals are also $(\Omega^{\text{parity}}, n_z)$, which denote the pair levels with $[N, n_z, \ell_z]\Omega$ and $[N, n_z, \ell_z](\Omega + 1)$ for $\Delta\epsilon$ and ϵ in (3.2). The spin pair levels of $[402]3/2$ and $[402]5/2$ are in the bottom of the $N = 4$ chain denoted by plus sign with the suffix of $(3/2^+, 0)$. In the figure, the smaller $\Delta\epsilon$ than the doublets $(3/2^+, 0)$ is seen for the doublets in the $N = 5$ chain denoted by squares with the sign of $(1/2^-, 0)$ and

$(3/2^-, 1)$. The doublets $(1/2^-, 0)$ correspond to the pair levels $[501]1/2$ and $[501]3/2$, and $(3/2^-, 1)$ to $[512]3/2$ and $[512]5/2$. They seem to have a good spin symmetry degeneracy in the wave functions, as energy splitting of $(1/2^-, 0)$ is 0.407 MeV and $(3/2^-, 1)$ is 1.034 MeV less than 1.397 MeV for $(3/2^+, 0)$ in $N = 4$ chain. However, their pseudospin partner levels have worse energy degeneracy. For example, the pseudospin doublets of $[501]3/2$ and $[503]5/2$, which are pseudospin partner of $(1/2^-, 0)$ in Fig. 7, has 0.413 MeV energy splitting. The pseudospin doublets of $[510]1/2$ and $[512]3/2$, which are pseudospin partner of $(3/2^-, 1)$ in Fig. 7 has 0.252 MeV energy splitting. Both values are much larger than 0.180 MeV for $[400]1/2$ and $[402]3/2$. Thus, the candidate for the triple degeneracy in this nucleus is only $[400]1/2$, $[402]3/2$ and $[402]5/2$. The spin symmetry is closely related with the pseudospin symmetry. It is because both the pseudospin symmetry and the spin symmetry are caused as a result of the common spin-orbit interaction.

Now we compare the results in Fig. 6 and 7 with those in the spherical nuclei. In the spherical nuclei, as is seen in Fig. 1 in our old paper [6], $\Delta\epsilon$ is decreasing for the pseudospin splitting, while $\Delta\epsilon$ is increasing for the spin-orbit splitting according with increasing ϵ . For the pseudospin doublets in Fig. 6, we see $\Delta\epsilon$ is decreasing according with increasing ϵ in the fixed N chain. For example, $\Delta\epsilon$ in $N = 4$ chain decreases with increasing ϵ starting from $(1/2^+, 2)$ to $(1/2^+, 0)$. Moreover, if the doublets with the same Ω and its highest n_z chain are connected over different N , their $\Delta\epsilon$ decrease with increasing ϵ . For example, $(1/2^-, 1)$ in $N = 3$, $(1/2^+, 2)$ in $N = 4$, $(1/2^-, 3)$ in $N = 5$ and $(1/2^+, 4)$ in $N = 6$, they are decreasing with increasing energy. Similarly, $(3/2^-, 0)$ in $N = 3$, $(3/2^+, 1)$ in $N = 4$, and $(3/2^-, 2)$ in $N = 5$, and $(3/2^+, 3)$ in $N = 6$ are decreasing. This trend agrees with the spherical case. For the spin doublets $\Delta\epsilon$ in the fixed N chain decreases with increasing ϵ . However, if we connect the doublets with the same Ω and its highest n_z over different N , their $\Delta\epsilon$ increases with increasing ϵ . For example, $(1/2^-, 2)$ in $N = 3$, $(1/2^+, 3)$ in $N = 4$ and $(1/2^-, 4)$ in $N = 5$, their $\Delta\epsilon$ are increasing. Similarly, $(3/2^-, 1)$ in $N = 3$, $(3/2^+, 2)$ in $N = 4$, and $(3/2^-, 3)$ in $N = 5$, and $(3/2^+, 4)$ in $N = 6$ are increasing. Their energy dependence is smaller than the pseudospin doublets. This trend is also found in the spherical case [6].

IV. CONCLUSION

We discussed the extreme case where exact pseudospin symmetry and spin symmetry are realized. In that case, the absolute values of *unphysical* amplitudes (with *unnatural* ℓ_z in the centrifugal term compared with that estimated from the asymptotic quantum number j_z and s_z) agree each other both in upper (large) and lower (small) components for the pseudospin doublets. On the other hand, for the spin doublets the absolute values of *physical* amplitudes (with *natural* ℓ_z in the centrifugal term compared with that estimated from the asymptotic quantum number j_z and s_z) agree each other both in upper (large) and lower (small) components. If there were the extreme case, the triple degeneracy appears among the levels with (k, Ω) , $(k', \Omega + 1)$ and $(k', \Omega + 2)$. In the realistic case, we found this situation is almost satisfied for the pseudospin doublets of $[400]1/2$ and $[402]3/2$, and for the spin doublets of $[402]3/2$ and $[402]5/2$ in ^{154}Sm .

We discussed the relation between the deformed case and the spherical case. In the spherical case, if we transform the cylindrical coordinate to the polar coordinate, the lower (small) component has the pseudospin symmetry, while the upper (large) component has spin symmetry in itself. In contrast to the spherical case where the pseudospin symmetry is in the lower (small) component and the spin symmetry is in the upper (large) component, the pseudospin and spin symmetries are found both in the upper and lower components in the deformed case. Just as the deformation mixes ℓ and j , the deformation mixes the pseudospin and the spin symmetries. As the pseudospin symmetry is found both in the upper and lower components, it may be the main reason why the pseudospin approximation becomes better in the deformed case than in the spherical case.

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FIGURES

FIG. 1. The ratio of the potential derivatives to the potentials as a function of ρ at $z = 3$ fm (left panel), and as a function of z at $\rho = 3$ fm (right panel). V corresponds to V_V and S to V_S in the text.

FIG. 2. The upper components (left panel) and lower components (right panel) for pseudospin doublet of $[400]1/2$ and $[402]3/2$ as a function of ρ at $z = 3$ fm. The dashed and the dot-dashed lines are for the *unphysical* amplitudes, and the solid and the dotted lines are for the *physical* amplitudes. The dashed and solid lines are for $[400]1/2$ and the dot-dashed and the dotted lines are for $[402]3/2$. The $\pm$ sign in front of $[N, n_z, \ell_z]\Omega$ corresponds to $\pm \frac{1}{2}$ of s_z .

FIG. 3. The upper components (left panel) and lower components (right panel) for pseudospin doublet of $[400]1/2$ and $[402]3/2$ as a function of z at $\rho = 5$ fm. The lines are the same as defined in Fig. 2.

FIG. 4. The upper components (left panel) and lower components (right panel) for spin doublet of $[402]3/2$ and $[402]5/2$ as a function of ρ at $z = 1$ fm. The lines for the *unphysical* and *physical* amplitudes are the same as defined in Fig. 2. The dashed and the solid lines are for $[402]3/2$ and the dot-dashed and the dotted lines are for $[402]5/2$.

FIG. 5. The upper components (left panel) and lower components (right panel) for spin doublet of $[402]3/2$ and $[402]5/2$ as a function of z at $\rho = 5$ fm. The lines are the same as defined in Fig. 4.

FIG. 6. The energy difference $\Delta\varepsilon$ versus average energy ε for the pseudospin pair levels. The longitudinal axis is for $\Delta\varepsilon = (\epsilon_{\Omega, \tilde{\ell}_z-1} - \epsilon_{\Omega+1, \tilde{\ell}_z+1})/\tilde{\ell}_z$, and the horizontal axis is for $\varepsilon = \frac{1}{2}(\epsilon_{\Omega, \tilde{\ell}_z-1}(\tilde{\ell}_z + 1) + \epsilon_{\Omega+1, \tilde{\ell}_z+1}(\tilde{\ell}_z - 1))/\tilde{\ell}_z$ with $\tilde{\ell}_z = \ell_z + 1$. The numerals inside parentheses is $(\Omega^{\text{parity}}, n_z)$. The doublets belonging to $N = 3$ is shown by diamonds, those to $N = 4$ by pluses, those to $N = 5$ by squares and those to $N = 6$ by crosses.

FIG. 7. The energy difference $\Delta\varepsilon$ versus average energy ε for the spin pair levels. The longitudinal axis is for $\Delta\varepsilon = (\epsilon_{\Omega,\ell_z} - \epsilon_{\Omega+1,\ell_z})/\ell_z$. The horizontal axis is for $\varepsilon = \frac{1}{2}(\epsilon_{\Omega,\ell_z} + \epsilon_{\Omega+1,\ell_z})/2$. The numerals inside parentheses is $(\Omega^{\text{parity}}, n_z)$. The symbols are the same as defined in Fig. 6.

fig 2.

fig3

fig 6.

fig 7.