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February 2001

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Preprint INLN 2001/08

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ON THE COLLINEAR SINGULARITY PROBLEM OF HOT QCD

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ABSTRACT

The collinear singularity problem of hot QCD is revisited within a perturbative resummation scheme (PR) of the leading thermal fluctuations. On the basis of actual calculations, new aspects are discovered concerning the origin of the singularity plaguing the soft real photon emission rate out of a Quark Gluon plasma at thermal equilibrium, when the latter is calculated by means of the Resummation Program (RP).

PACS: 12.38.Cy, 11.10.Wx

Keywords: Hot QCD , Resummation Program, infrared, mass/collinear singularities,..

1. Introduction

During the past twelve years, a considerable amount of work has been devoted to the study of quantized fields at high temperature and/or chemical potential (see, e.g., ref.[1] for a review of the subject, far from exhaustive though). The inherent non perturbative character of thermal quantum field theories has been recognized [2], and naive perturbation theories accordingly reorganized. This is achieved by means of a *Resummation Program (RP)* [3], which, in the high temperature limit, must be used whenever one is calculating processes involving Green's functions with soft external/internal lines. The soft scale is defined to be on the order of gT where T is the temperature and g some relevant and small enough coupling constant, so as to *decide* of two separate hard (on the order of T) and soft energy scales. The *RP* is given by effective Feynman rules, consisting of effective field propagators and n -points proper vertices, all at a given leading order of approximation which turns out to be g^2T^2 , and is referred to as *HTL* (Hard Thermal Loops). While *HTL* vertices are purely perturbative objects, effective propagators are not, giving rise to pole residues and dispersion laws that cannot be obtained out of pure thermal perturbation theory. In the course of practical calculations, effective propagators are easily handled, relying on analyticity properties and Cauchy's theorem.

Endowed with most beautiful symmetries, the *RP* is an effective theory that has led to a number of satisfying results [1], but has also met two serious obstructions, emanating both from the infrared (*IR*) sector with, first, the infrared (*IR*) problem of *QED* and *QCD* [4], and, second, the soft real photon problem discovered a few years ago [5].

In this article, we take advantage of a so called *Perturbative Resummation* scheme, hereafter denoted *PR* for short, previously introduced in the context of the first obstruction [6], to address the problem of the soft real photon emission rate of thermal *QCD* [5]. This problem is the following. When use is made of the *Resummation Program* to calculate the soft real photon emission rate, out of a Quark-Gluon plasma in thermal equilibrium, the answer comes out affected with a collinear singularity. Several attempts to cure that *IR* disease have been proposed ever since, which, though consistent, encounter further serious difficulties when extended to higher number of loop calculations [7].

Our present analysis is motivated by a recent study of the problem, projected out on a toy model, with the conclusion that things come out very different according to the resummation scheme in use, *RP* or *PR*, [8]. The article is organized as follows. In section 2, for the sake of comparison with *PR* calculations, the basic steps of the *RP* derivation are briefly recalled. The next two sections deal with *PR* calculations: Section 3, at the orders of m^2 and m^4 , and section 4, at the order of m^6 , where m is a thermal mass on the order of the soft scale, gT . In section 5, we show how and why *RP* and *PR* resummation schemes drift apart in their handling of collinear or mass singularities. This section is supported by a short appendix, and our conclusions are presented in section 6.

Throughout the article, we work in the *R/A* real time formalism, which is based on Retarded/Advanced free field functions [9]. Also, we will be using the convention of upper case letters for quadrimomenta and lower case ones for their components, writing, for example, $P = (p_0, \vec{p})$. Our conventions for labelling internal and external momenta can be read off Fig.1, for example.

2. The soft real photon emission rate of hot QCD

We find it convenient to work in the real time formalism with retarded/advanced (R/A) field functions, where a concise and elegant derivation of this famous result can be achieved [10]. The soft photon emission rate is essentially related to the imaginary part of the quantity $\Pi_{RR\mu}^\mu(Q)$, trace of the soft real photon polarization tensor, hereafter written as $\Pi_R(Q)$. At pure one loop order, this imaginary part is zero. However when the photon is soft, this result is incomplete and the *Resummation Program* must be used instead of bare thermal Perturbation Theory. This amounts to keep the one loop diagram of ordinary Perturbation Theory, while replacing bare vertices and propagators by their *HTL*-dressed counterparts. In Feynman gauge, the resulting expression reads (with n_F , the Fermi-Dirac statistical factor, defined without absolute value),

$$\Pi_R(Q) = i \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \text{disc Tr} \left\{ {}^*S_R(P) {}^*\Gamma_\mu(P_R, Q_R, -P'_A) \right. \\ \left. {}^*S_R(P') {}^*\Gamma^\mu(P_R, Q_R, -P'_A) \right\} \quad (2.1)$$

The discontinuity is to be taken in the energy variable p_0 , by forming the difference of R and A -indexed P -dependent quantities. Within standart notations, the fermionic *HTL* self energies, effective propagators and vertices are respectively given by

$$\Sigma_\alpha(P) = m^2 \int \frac{d\hat{K}}{4\pi} \frac{\hat{K}}{\hat{K} \cdot P + i\epsilon_\alpha}, \quad m^2 = C_F \frac{g^2 T^2}{8} \quad (2.2)$$

$${}^*S_\alpha(P) = \frac{i}{\not{P} - \Sigma_\alpha(P)}, \quad \alpha = R, A \quad (2.3)$$

$${}^*\Gamma_\mu(P_\alpha, Q_\beta, P'_\delta) = -ie (\gamma_\mu + \Gamma_\mu^{HTL}(P_\alpha, Q_\beta, P'_\delta)) \quad (2.4)$$

$$\Gamma_\mu^{HTL}(P_\alpha, Q_\beta, P'_\delta) = m^2 \int \frac{d\hat{K}}{4\pi} \frac{\hat{k}_\mu \hat{K}}{(\hat{K} \cdot P + i\epsilon_\alpha)(\hat{K} \cdot P' + i\epsilon_\delta)} \quad (2.5)$$

where $\hat{K}$ is the lightlike four vector $(1, \hat{k})$. In the sequel, it will reveal extremely useful to introduce a "self energy four vector" by writing, instead of standart expression (2.2),

$$\Sigma_\alpha(P) \stackrel{(def)}{=} \not{\Sigma}_\alpha(P) = \gamma \cdot \Sigma_\alpha(P) = \gamma_\mu m^2 \int \frac{d\hat{K}}{4\pi} \frac{\hat{K}^\mu}{\hat{K} \cdot P + i\epsilon_\alpha} \quad (2.6)$$

For the sake of later purpose, it is instructive to recall the *RP* basic steps entering the soft photon emission rate calculation of thermal *QCD*. In the *R/A* formalism being used, this result enjoys a simple and systematic derivation, which we here follow. In view of (2.1) and (2.4), one gets three types of terms: A term with two bare vertices $\Gamma_\mu^{(0)}$, two terms with one bare vertex $\Gamma_\mu^{(0)}$ and the other Γ_μ^{HTL} , and a term with two *HTL* vertices Γ_μ^{HTL} . In *QCD*, the first three terms pose no problem: Terms of second type entail a collinear singularity which, thanks to a $U(1)$ -Ward identity, cancels out with a similar singularity coming from the last

term. A residual collinear singularity remains though, induced by the latter, and we therefore focus on that particular contribution including two vertices Γ_μ^{HTL} . One gets,

$$\Pi_R(Q) = i \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \text{disc} \text{Tr} \left\{ {}^*S_R(P) \Gamma_\mu^{HTL} (P_R, Q_R, -P'_A) \right. \\ \left. {}^*S_R(P') \Gamma^{HTL\mu} (P_R, Q_R, -P'_A) \right\} \quad (2.7)$$

Then substituting the relevant *QCD* expressions, (2.2)-(2.5), one can write, with the convention $\epsilon_R = +\epsilon$,

$$\Pi_R(Q) = -ie^2 m^4 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \\ \text{disc} \frac{\hat{K} \cdot \hat{K}' \text{Tr} \left({}^*S_R(P) \hat{\not{K}} {}^*S_R(P') \hat{\not{K}}' \right)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (2.8)$$

Because of the factor $\hat{K} \cdot \hat{K}'$ appearing in the numerator, there is no double pole but a simple collinear one at $\hat{K} = \hat{Q}$, whose residue just involves the $U(1)$ Ward identity alluded to above, that is,

$$m^2 \int \frac{d\hat{K}'}{4\pi} \frac{[\hat{Q} \cdot \hat{K}'] \hat{\not{K}}'}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} = \frac{1}{q} [\not{V}_R(P) - \not{V}_R(P')] \quad (2.9)$$

and yields for $\Pi_R(Q)$ the expression

$$-i \frac{e^2 m^2}{q} \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \text{disc} \int \frac{d\hat{K}}{4\pi} \frac{1}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \\ \times \text{Tr} \left({}^*S_R(P) \hat{\not{Q}} {}^*S_R(P') [\not{V}_R(P) - \not{V}_R(P')] \right) \quad (2.10)$$

The discontinuity in p_0 can be taken, and an appropriate choice of the integration contour in the p_0 -complex plane allows to write

$$\Pi_R(Q) = -2 \frac{e^2 m^2}{q} \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot Q + i\epsilon} \\ \times \text{Tr} \left({}^*S_A(P) \hat{\not{Q}} {}^*S_R(P') [\not{V}_A(P) - \not{V}_R(P')] \right) \quad (2.11)$$

where a factor of 2 accounts for the two possibilities $\hat{K} = \hat{Q}$ and $\hat{K}' = \hat{Q}$, and where the relation $P' = P + Q$ has been used. The angular integration develops a collinear singularity at $\hat{K} = \hat{Q}$, and is responsible for that singular part of $\Pi_R(Q)$ which can be expressed as

$$-2 \frac{e^2 m^2}{q} \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{Q \cdot \hat{K} + i\epsilon} \right) \int \frac{d^4 P}{(2\pi)^3} \delta(P \cdot \hat{Q}) (1 - 2n_F(p_0)) \\ \times \text{Tr} \left({}^*S_A(P) \hat{\not{Q}} {}^*S_R(P') [\not{V}_A(P) - \not{V}_R(P')] \right) \quad (2.12)$$

The two terms involving one bare vertex γ_μ and a one loop *HTL* correction Γ_μ^{HTL} , entail a similar singularity which, when combined with (2.12), leave uncanceled the $\Pi_R(Q)$ singular contribution

$$-2i \frac{e^2 m^2}{q^2} \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{Q} \cdot \hat{K} + i\epsilon} \right) \int \frac{d^4 P}{(2\pi)^3} \delta(P \cdot \hat{Q}) (1 - 2n_F(p_0)) \times [Tr \left({}^*S_A(P) \hat{Q} \right) - Tr \left({}^*S_R(P') \hat{Q} \right)] \quad (2.13)$$

It is this result which, in the literature is most usually written in the form

$$\frac{C^{st}}{\varepsilon} \int \frac{d^4 P}{(2\pi)^4} \delta(\hat{Q} \cdot P) (1 - 2n_F(p_0)) \sum_{s=\pm 1, V=P, P'} \pi(1 - s \frac{v_0}{v}) \beta_s(V) \quad (2.14)$$

where the overall $1/\varepsilon$ results of a dimensionally regularized evaluation of the factored out angular integration of (2.13), and where $\beta_s(V)$ is related to the effective fermionic propagator usual parametrization [11]

$${}^*S_{R,A}(P) = i \sum_{s=\pm 1} \frac{\hat{P}_s}{D_{R,A}^s(p_0 \pm i\epsilon, \vec{p})} \quad (2.15)$$

with $\hat{P}_s = (1, s\vec{p})$, the label s referring to the two dressed fermion propagating modes. Then one has

$$\frac{1}{D_{R,A}^s(V)} = \alpha_s(V) \mp i\pi \beta_s(V) \quad (2.16)$$

3. *PR* calculations at the orders of m^2 and m^4 .

From now on, we analyze things from the point of view of a *Perturbative Resummation* scheme (*PR*) of the leading thermal fluctuations [6]. For example, with one single insertion of one loop vertex Γ_μ^{HTL} , we are led to consider the hereafter denoted $(N, N'; 1)$ -contributions

$$\Pi_R^{(N, N'; 1)}(Q) \stackrel{(def)}{=} (N, N'; 1) = -ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \frac{Tr \left(S_R^{(N)}(P) \hat{K} S_R^{(N')}(P') \hat{K} \right)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (3.1)$$

where we have defined $S_R^{(N)}(P)$, the so-called "partial effective propagator" corresponding to N insertions of *HTL* self energies along the P -line,

$$S_R^{(N)}(P) = \frac{i \not{P}}{(P^2 + i\epsilon p_0)^{N+1}} (\not{Z}_R(P) \not{P})^N \quad (3.2)$$

Likewise, we have to cope with $(N, N'; 2)$ -contributions, involving two HTL vertices Γ_μ^{HTL} . They read

$$(N, N'; 2) = -ie^2 m^4 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \text{disc} \frac{\hat{K} \cdot \hat{K}' \text{Tr} \left(S_R^{(N)}(P) \hat{K} S_R^{(N')}(P') \hat{K}' \right)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.3)$$

Of course, a full PR answer to the problem requires that contributions (3.1) and (3.3) be summed over N and N' , as well as those of type $(N, N'; 0)$. We will begin with diagrams of order m^2 and m^4 , and make a remark before with go on proceeding with order m^6 diagrams in next section 4.

At the order of m^2 , one has four contributions labelled as $(1, 0; 0)$, depicted in Fig.1, $(0, 1; 0)$ and twice $(0, 0; 1)$. That is, twice $(1, 0; 0)$ by symmetry, and twice $(0, 0; 1)$. Calculations are straightforward and give

$$2 \times (0, 0; 1) = +8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2 + i\epsilon p'_0} \text{disc} \frac{m^2}{P^2 + i\epsilon p_0} \quad (3.4)$$

$$2 \times (0, 1; 0) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2 + i\epsilon p'_0} \text{disc} \frac{2m^2 P \cdot P' - 2P^2 P' \cdot \Sigma_R(P)}{(P^2 + i\epsilon p_0)^2} \quad (3.5)$$

Taking the imaginary parts, $(0, 0; 1)$ is compensated by a similar part in $(1, 0; 0)$, and one is finally left with

$$+8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) (-2i\pi\epsilon(p'_0)\delta(P'^2)) \text{disc} \frac{2 Q \cdot \Sigma_R(P)}{P^2 + i\epsilon p_0} \quad (3.6)$$

Then, integration over the directions $\hat{p}$ can be carried out with the help of $\delta(P'^2)$, so as to yield

$$8e^2 \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2} \right)^1 \int_{-p}^{+p} dp_0 \epsilon(p_0 + q) (1 - 2n_F(p_0)) \text{disc} \frac{1}{P^2 + i\epsilon(p_0)} \left(\hat{q} \cdot \hat{p} + [1 - (\hat{q} \cdot \hat{p}) \frac{p_0}{p}] \frac{1}{2} \ln \left(\frac{p_0 + p}{p_0 - p} \right) \right) \quad (3.7)$$

where we have used

$$Q \cdot \Sigma_R(P) = m^2 \left(\frac{q}{p} (\hat{q} \cdot \hat{p}) + [1 - (\hat{q} \cdot \hat{p}) \frac{p_0}{p}] \frac{q}{2p} \ln \left(\frac{p_0 + p}{p_0 - p} \right) \right) \quad (3.8)$$

and where the angle of unit three-vectors $\hat{q}$ and $\hat{p}$ is determined to be

$$-1 \leq \hat{q} \cdot \hat{p} = \frac{P^2}{2pq} + \frac{p_0}{p} \leq +1 \quad (3.9)$$

As in the moving fermion damping rate problem, it is that very specification of $\hat{q} \cdot \hat{p}$ which, in (3.7), restricts the energy integration range to the interval $-p \leq p_0 \leq +p$. The regular character of (3.7) can be recognized easily: In a neighbourhood of $P^2 = 0$, one has $\hat{q} \cdot \hat{p} = p_0/p$ in view of (3.9), and there is accordingly no singularity attached to the potentially divergent logarithm, since, then, multiplied by a factor of $-P^2/p^2$, and divided by $P^2 + i\epsilon(p_0)$, its subsequent p_0 -integration is singularity free.

At the order of m^4 , eight contributions to $\Pi_R(Q)$ come about, which in our notations can be grouped as follows: $(2, 0; 0)$, $(0, 2; 0)$ which are identical, $(1, 1; 0)$, twice $(1, 0; 1)$, depicted in Fig.2, and twice $(0, 1; 1)$ which are identical, and eventually $(0, 0; 2)$. This case is interesting as it involves the double effective vertex insertion, $(0, 0; 2)$, represented in Fig.2. The full order m^4 is thus given by the sum of the following four contributions, where R -retarded propagator and self energy specifications can be omitted without ambiguity

$$2 \times (2, 0; 0) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{1}{(P^2)^3} (8m^4 P \cdot P' - 4m^2 P^2 P' \cdot \Sigma(P) - 2P^2 (P \cdot P') \Sigma^2(P)) \quad (3.10)$$

$$(1, 1; 0) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{(P'^2)^2} \text{disc} \frac{1}{(P^2)^2} (4m^4 P \cdot P' - 2m^2 P'^2 P \cdot \Sigma(P') - 2m^2 P^2 P' \cdot \Sigma(P) + P^2 P'^2 \Sigma(P) \cdot \Sigma(P')) \quad (3.11)$$

$$4 \times (1, 0; 1) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{1}{(P^2)^2} (-8m^4 + 4P^2 \Sigma^2(P)) \quad (3.12)$$

$$(0, 0; 2) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{1}{P^2} \left(-\Sigma(P) \cdot \Sigma(P') + \frac{1}{2} m^4 (P \cdot P') W(P, P') \right) \quad (3.13)$$

where we have defined W , the function related to the double effective vertex insertion

$$W(P, P') = \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{K} \cdot \hat{K}')^2}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.14)$$

Just as before, some cancellations arise between vertex and self energy graphs [12], so that things can be somehow reduced, the more as, anticipating on what follows, we specialize to imaginary parts. For (3.10), one is left to consider the expression

$$-8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \left(-\frac{4m^2 Q \cdot \Sigma(P)}{(P^2)^2} - \frac{\Sigma^2(P)}{P^2} \right) \quad (3.15)$$

whereas for (3.12), one obtains

$$-8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \left(\frac{-16m^4}{(P^2)^2} + 8 \frac{\Sigma^2(P)}{P^2} \right) \quad (3.16)$$

Eventually the sum of (3.11) and (3.13) gives

$$(3.11) + (3.13) = -8ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{(P'^2)^2} \text{disc} \frac{2m^2}{(P^2)^2} (P'^2 Q \cdot \Sigma(P') - P^2 Q \cdot \Sigma(P)) \\ - 4ie^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{m^4}{P^2} P \cdot P' W(P, P') \quad (3.17)$$

Unlike the previous order m^2 case, mass (or collinear) singularities develop out of the above order m^4 expressions, which we now examine. The contributions of (3.15) and (3.16) to the imaginary part of $\Pi_R(Q)$ are readily seen to be proportional to three basic integrals, the possible mass/collinear singular behaviour of which must be addressed. They are

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \left\{ \frac{1}{(P^2)^2}, \frac{\Sigma^2(P)}{P^2}, \frac{Q \cdot \Sigma(P)}{(P^2)^2} \right\} \quad (3.18)$$

The first one is

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \epsilon(p_0) \delta'(P^2) \quad (3.19)$$

It obviously yields a vanishing contribution, as can be seen relying, for example, on the "mass derivative formula" [13,14]

$$(3.19) = \lim_{\lambda^2=0} \frac{\partial}{\partial \lambda^2} \int \frac{p^2 dp}{(2\pi)^3 2\sqrt{p^2 + \lambda^2}} (1 - 2n_F(p)) \\ \int_{-1}^{+1} dx \left(\delta(x - [1 + \frac{\lambda^2}{2qp}]) + \epsilon(-p + q) \delta(x - [-1 + \frac{\lambda^2}{2qp}]) \right) = 0 \quad (3.20)$$

where, as is customary [1], we assume the final integration, over p , to extend beyond $p = q$, to a typical intermediate scale p^* on the order of $\sqrt{g}T$. The result is otherwise a regular one.

Now with Σ^2 as given by

$$\Sigma^2(P) = \frac{m^4}{p^2} \left(-1 - \frac{1}{4} \frac{P^2}{p^2} \ln^2 \left(\frac{p_0 + p}{p_0 - p} \right) + \frac{p_0}{p} \ln \left(\frac{p_0 + p}{p_0 - p} \right) \right) \quad (3.21)$$

it is clear that the second integral of (3.18) cannot receive any mass singular contribution from the first and second term in the right hand side of (3.21), and that singular contributions can only be induced by the last term: Its integral is proportional to

$$\frac{1}{2q} \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2} \right)^2 \int_{-p}^{+p} dp_0 (1 - 2n_F(p_0)) \epsilon(p_0 + q) p_0 \\ \times \left(\mathbf{P} \left(\frac{1}{P^2} \right) \mathcal{I}m \ln \left(\frac{p_0 + p}{p_0 - p} \right) - \pi \epsilon(p_0) \delta(P^2) \mathcal{R}e \ln \left(\frac{p_0 + p}{p_0 - p} \right) \right) \quad (3.22)$$

One then has a full compensation of the mass singularities induced by the two terms of (3.22), and this compensation takes place for each sign of p_0 , separately. We introduce the representation used in [6]

$$\epsilon(p_0) \ln\left(\frac{p_0 + p}{p_0 - p}\right) = \lim_{\epsilon=0} \frac{1}{\epsilon} \left(1 - \left(\frac{P^2}{p^2(1 + \sqrt{1 + P^2/p^2})^2} \right)^\epsilon \right) \quad (3.23)$$

Thanks to the $(P^2/p^2)^\epsilon$ factor, this representation is able to provide mass/collinear singularities with the same regularization as a dimensional one would operate, and is endowed with most interesting regularity properties, since, in particular, the limit $\epsilon = 0$, commutes with both the sum over N and integral on p_0 . Considering the case of negative p_0 , for example, one gets, from the term involving the distribution $\delta(p_0 + p)$

$$\begin{aligned} \frac{1}{2q} \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2}\right)^2 \int_{-p}^0 p_0 dp_0 (1 - 2n_F(p_0)) \epsilon(p_0 + q) (-\pi) (-1) \frac{\delta(p_0 + p)}{2p} \frac{1}{\epsilon} \\ = \frac{1}{4q} \frac{\pi}{\epsilon} \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2}\right)^2 (1 - 2n_F(p)) \epsilon(-p + q) \end{aligned} \quad (3.24)$$

The term involving the principal part distribution reads, with x the variable $|P^2|/p^2$,

$$\begin{aligned} \frac{1}{2q} \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2}\right)^2 \int_0^1 \left(-\frac{p^2 dx}{2}\right) \left(\frac{x^\epsilon}{-p^2 x}\right) \\ (1 - 2n_F(-p\sqrt{1-x})) \epsilon(-p\sqrt{1-x} + q) \left[\frac{-\sin(\pi\epsilon)}{\epsilon}\right] \frac{1}{(1 + \sqrt{1-x})^{2\epsilon}} \end{aligned} \quad (3.25)$$

The $x = 0$ mass singular behaviour of (3.25) can be extracted easily to give

$$\frac{-\sin(\pi\epsilon)}{\epsilon} \frac{1}{4q} \frac{2^{-2\epsilon}}{\epsilon} \int \frac{p^2 dp}{(2\pi)^2} \left(\frac{m^2}{p^2}\right)^2 (1 - 2n_F(p)) \epsilon(-p + q) \quad (3.26)$$

It is clear that the limit at $\epsilon = 0$ of (3.26) just compensates the mass singular expression (3.24). As the same generic mechanism applies to the case of positive energies too, one can conclude that the second term in (3.18) yields a regular contribution to the imaginary part of $\Pi_R(Q)$.

The third integral of (3.18) is

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{Q \cdot \Sigma(P)}{(P^2)^2} \quad (3.27)$$

With $Q \cdot \Sigma(P)$ given by (3.8), the potentially dangerous part comes from the integral

$$\begin{aligned} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \\ \times \left(-\frac{q}{2p}\right) \text{disc} \frac{1}{(P^2)^2} \left[1 - (\hat{q} \cdot \hat{p}) \frac{p_0}{p}\right] \ln\left(\frac{p_0 + p}{p_0 - p}\right) \end{aligned} \quad (3.28)$$

Now, as observed at the level of (3.8), in a neighbourhood of $P^2 = 0$, the cosine $\hat{q} \cdot \hat{p}$ behaves as p_0/p because of the $\delta(P'^2)$ constraint, and accordingly, the whole integrand of (3.28) as

$$(1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \left(-\frac{q}{2p}\right) \frac{-1}{p^2} \text{disc} \frac{1}{P^2} \ln\left(\frac{p_0 + p}{p_0 - p}\right) \quad (3.29)$$

which expression, we have just proven to yield a regular behaviour. The imaginary parts of both (3.15) and (3.16) are therefore devoid of any collinear intricacies.

The singularity structure of the imaginary part of (3.17) remains to be analyzed. The first term of (3.17) is left the same in the exchange of Q and $-Q$, whereas the imaginary part of $\Pi_R(Q)$ is an odd function of q_0 at all orders of bare thermal perturbation theory [14]. This term has therefore imaginary part zero, and we now come to the last term of (3.17) whose imaginary part is proportional to the integral

$$\int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{Q \cdot P}{P^2} W(P, P') \quad (3.30)$$

where use has been made of the relation $P \cdot P' \delta(P'^2) = -Q \cdot P \delta(P'^2)$. One has also, due to the same $\delta(P'^2)$ constraint

$$\text{disc} \frac{Q \cdot P}{P^2} W(P, P') \stackrel{\delta(P'^2)}{=} -\frac{1}{2} \text{disc} W(P, P') - 2i\pi \epsilon(p_0) \delta(P^2) Q \cdot P \text{Re} W(P, P') \quad (3.31)$$

with the discontinuity of W given by

$$\text{disc} W(P, P') = -4i\pi \epsilon(p_0) \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{K} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.32)$$

This equation is non trivial and requires some explanation, because it cannot be obtained out of previous defining expression (3.14). As should be clear in effect, the discontinuity of (3.14) would not produce the factor $\epsilon(p_0)$ appearing in the right hand side of (3.32). Indeed, while we have

$$\text{disc} (3.14) = \text{disc} \left(\text{Tr} \left(\Gamma_\mu^{HTL}(P_R, Q_R, -P'_A) \Gamma^\mu{}^{HTL}(P_R, Q_R, -P'_A) \right) \right) \quad (3.33)$$

the expression (3.32) is actually relevant of a different sequence of the two *discontinuity* and *HTL-restriction* operations since, denoting by $\Gamma_\mu^{(1)}$ the full one loop vertex correction, and not just its *HTL* approximated form (2.5), we have

$$(3.32) = \left(\text{disc} \text{Tr} \left(\Gamma_\mu^{(1)}(P_R, Q_R, -P'_A) \Gamma^{\mu(1)}(P_R, Q_R, -P'_A) \right) \right)^{HTL} \quad (3.34)$$

The two sequences differ the above sign of p_0 distribution, and for reasons to become clear shortly, it is this latter sequence that we retain as the W -function discontinuity proper definition.

At $\hat{K} = \hat{Q}$, (3.32) develops a collinear singularity, and we have, the suffix S denoting that singular part

$$\text{disc } W(P, P')|_S = -4i\pi\epsilon(p_0) \delta(\hat{Q} \cdot P) \int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.35)$$

The second term in the right hand side of (3.31) yields

$$\int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) (-2i\pi) \epsilon(p_0) \delta(P^2) Q \cdot P \text{Re} W(P, P') \quad (3.36)$$

Recalling (3.14), the integrand involves the expression

$$q \text{Re} \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \frac{[\hat{Q} \cdot P] (\hat{K} \cdot \hat{K}')^2}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.37)$$

together with constraints which satisfy

$$2q \delta(P'^2) \delta(P^2) = \delta(P'^2) \delta(\hat{Q} \cdot P) \quad (3.38)$$

It results that (3.37) is zero, unless we are in either of the two collinear configurations $\hat{K} = \hat{Q}$ and $\hat{K}' = \hat{Q}$ with residue

$$2\epsilon(p'_0) \delta(P'^2) \epsilon(p_0) \delta(P^2) q \text{Re} \int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \quad (3.39)$$

where an overall factor of 2 accounts for the two collinear possibilities just mentioned. Eventually, omitting for short a common integration factor of

$$\int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \epsilon(p'_0)$$

we can see out of (3.31), (3.35) and (3.39), that we have, on the one hand, a contribution of

$$\delta(P'^2) \frac{1}{2} [-4i\pi\epsilon(p_0)] \delta(\hat{Q} \cdot P) \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \right) \quad (3.40)$$

and in the other hand, a contribution of

$$\delta(P'^2) [+4i\pi\epsilon(p_0)] \delta(P^2) q \text{Re} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \right) \quad (3.41)$$

Because of the common factor of $\delta(P'^2)$, the last factorized $\hat{K}$ -integral of (3.40) and (3.41) develops a collinear singularity at $\hat{K} = \hat{P}'$ (where $\hat{P}'$ is the light-like vector $(\epsilon(p'_0), \hat{p}')$), and both expressions are therefore singular. However, relation (3.38) between constraints, just

provides us with that very factor of $1/2$ which makes the two singular contributions compensate each other exactly.

Had we relied on disc (3.14) instead of (3.32), we missed that $\epsilon(p_0)$ factor which makes it possible for the two collinear singularities (3.40) and (3.41) to cancel out for positive as well as negative p_0 energies. Here terminates the proof that order m^4 contributions to the imaginary part of $\Pi_R(Q)$ are collinear singularity free, as are order m^2 ones.

Let us stress that this conclusion is interesting and suggestive enough for the following reason. We have proven that, for example, $(1, 0; 1)$ is singularity free. Now, at $N = 1$, $N' = 0$ and one *HTL* vertex insertion, (3.1) gives

$$(1, 0; 1) = -ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \text{disc} \frac{\text{Tr} \left(S_R^{(1)}(P) \hat{K} S_R^{(0)}(P') \hat{K} \right)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (3.42)$$

If, as usual, the discontinuity is taken by forming the difference of *R* and *A*-indexed *P*-quantities, an integration contour in the p_0 -complex plane may be chosen so as to write

$$(1, 0; 1) = -ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) (-2i\pi) \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \text{Tr} \left(S_A^{(1)}(P) \hat{K} S_R^{(0)}(P') \hat{K} \right) \quad (3.43)$$

Again, because of the constraint $\delta(\hat{K} \cdot P)$, the angular integration is singular at $\hat{K} = \hat{Q}$, and is responsible for a singular expression which reads

$$(1, 0; 1)|_s = -e^2 m^2 \left[\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right] \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \delta(\hat{Q} \cdot P) \text{Tr} \left(S_A^{(1)}(P) \hat{Q} S_R^{(0)}(P') \hat{Q} \right) \quad (3.44)$$

while we have just proven that $(1, 0; 1)$ is singularity free! Of course, (3.44) is only a formal, deceptive result because the trace it involves is itself trivially zero. Indeed we will have to wait until the next m^6 - order, before we can exhibit a sounder example concerning the origin of the difference separating *RP* and *PR* resummation schemes. In order to conclude this section, though, we can observe that passing from (3.42) to (3.44), the special values $N = 1$ and $N' = 0$ played no role at all. The same reasoning can therefore be applied to any value of N and N' , and the result summed upon from $N, N' = 0$ to infinity,

$$\Pi_R(Q)|_s = -e^2 m^2 \left[\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right] \int \frac{d^3 p}{(2\pi)^3} \sum_{N, N'} \int dp_0 (1 - 2n_F(p_0)) \delta(\hat{Q} \cdot P) \text{Tr} \left(S_A^{(N)}(P) \hat{Q} S_R^{(N')}(P') \hat{Q} \right) \quad (3.45)$$

so as to yield

$$\Pi_R(Q)|_s = -e^2 m^2 \left[\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right] \int \frac{d^3 p}{(2\pi)^3} \int dp_0 (1 - 2n_F(p_0)) \delta(\hat{Q} \cdot P) \text{Tr} \left({}^*S_A(P) \hat{Q} {}^*S_R(P') \hat{Q} \right) \quad (3.46)$$

where we have ignored the intricacies related to the non commutativity of the sum (N, N') and integral (p_0) operations [6], as they do not affect the point we are interested in. Comparing with Eq.(66) of [9], or Eq.(16) of [5], one readily recognizes the singular result obtained by using the RP , for diagrams involving two effective propagators and one HTL vertex.

4. PR calculations at the order of m^6 .

Results obtained so far are instructive, but do not involve the interplay of double HTL vertex with HTL self energy insertions. However, this interplay is crucial in view of the irreducible collinear singularity (2.13) which, along the RP sequence, is precisely related to the double HTL -vertex insertion. Inspection shows that along a PR sequence, the double HTL vertex topologies are also, potentially, the singular most ones, and may retain peculiarities that are important to analyze from the point of view of collinear singularities. At the order of m^6 , things begin with the terms $(1, 0; 2)$ of Fig.3, and $(0, 1; 2)$, and one finds

$$\begin{aligned} (1, 0; 2) + (0, 1; 2) = & +8ie^2 m^4 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \\ & \text{disc} \frac{1}{(P^2)^2} \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \frac{\hat{K} \cdot \hat{K}'}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \\ & \left(4m^2 \hat{K}' \cdot P' \hat{K} \cdot P - 2m^2 P \cdot P' \hat{K} \cdot \hat{K}' - 2P^2 \hat{K}' \cdot P' \hat{K} \cdot \Sigma(P) + P^2 \hat{K} \cdot \hat{K}' P' \cdot \Sigma(P) \right) \end{aligned} \quad (4.1)$$

Second and fourth terms can be written in term of the function W introduced in (3.14),

$$(-2m^2(P \cdot P') + P^2(P' \cdot \Sigma(P))) W(P, P') \quad (4.2)$$

The contribution of (4.1) to the imaginary part of $\Pi_R(Q)$, involves the distribution $\delta(P'^2)$, and one has accordingly

$$(4.2) \stackrel{\delta(P'^2)}{=} (Q \cdot \Sigma(P)) P^2 W(P, P') \quad (4.3)$$

One is therefore led to consider the quantity

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{Q \cdot \Sigma(P)}{P^2} W(P, P') \quad (4.4)$$

The scalar product $Q \cdot \Sigma(P)$ is given in (3.8) and is composed of two terms, the second of them being the simpler to deal with. For this term we have in effect to consider the expression

$$-(\frac{m^2 q}{2p}) \text{disc} \left\{ \frac{1}{P^2} [1 - (\hat{q} \cdot \hat{p}) \frac{p_0}{p}] \ln(\frac{p_0 + p}{p_0 - p}) W(P, P') \right\} \quad (4.5)$$

An important observation is in order. Expression (4.5) involves the quantity $\text{disc } W$ given in (3.32), which we have seen to exhibit a collinear singularity, generated in a neighbourhood of $\hat{K} = \hat{Q}$. At first glance it appears that this singularity would plague the non zero P^2 contributions of the functions which, inside the discontinuity prescription of (4.5), multiplies the term $W(P, P')$. But it is not so. As displayed by (3.35), the singular behaviour of (3.32) is proportional to a factor of $\delta(\hat{Q} \cdot P)$, and because of the overall $\delta(P'^2)$ constraint, we realize that mass singular behaviours are indeed relevant of the light cone region, $P^2 \simeq 0$. Now, it is straightforward to derive the following equivalence of light cone behaviours

$$-m^2 [1 - (\hat{q} \cdot \hat{p}) \frac{p_0}{p}] \frac{q}{2p} \ln(\frac{p_0 + p}{p_0 - p}) \simeq -\frac{qm^2}{p^3} (Q \cdot P) \ln(\frac{p_0 + p}{p_0 - p}) \simeq -\frac{qm^2}{p^3} (Q \cdot P) \frac{1}{\varepsilon} \quad (4.6)$$

where we have used the determination of $\hat{q} \cdot \hat{p}$ provided by the $\delta(P'^2)$ constraint, equation (3.9), as well as the regularization offered by (3.23) for the light cone divergent logarithm. For this very part of (3.8), we get eventually

$$\text{disc} \frac{Q \cdot \Sigma(P)}{P^2} W(P, P') \longrightarrow -\frac{qm^2}{p^3} \frac{1}{\varepsilon} \text{disc} \frac{Q \cdot P}{P^2} W(P, P') \quad (4.7)$$

which expression has been proven to display a full compensation of mass singularities, through previous equations (3.30)-(3.41). Note that this compensation arises, here, for order ε^{-2} mass singularities, that are the higher order mass singularities that can be expected from order m^6 contributions.

With the first term of (3.8), proportional to $(\hat{q} \cdot \hat{p})$, a new situation sets in, since, plugged into (4.4), it leads to the expression

$$(\frac{m^2}{p^2}) \text{disc} \frac{-Q \cdot P + qp_0}{P^2} W(P, P') \quad (4.8)$$

In (4.8), the part $Q \cdot P$ is, again, the same as dealt with from (3.30) to (3.41), but the second is new, and involves the integral

$$q \int \frac{d^3 p}{(2\pi)^3} \frac{m^2}{p^2} \int_{-p}^{+p} \frac{p_0 dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{W(P, P')}{P^2} \quad (4.9)$$

The discontinuity of (4.9) reads

$$\begin{aligned} -4i\pi\epsilon(p_0) \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{K} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \left\{ \frac{\mathbf{P}}{P^2} \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \right. \\ \left. + \delta(P^2) \int \frac{d\hat{K}}{4\pi} \frac{1}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \right\} \quad (4.10) \end{aligned}$$

Again, the first term appearing in the quantity written between brackets, develops a collinear singularity at $\hat{K} = \hat{Q}$, which can be regularized using (3.23), to yield

$$-4i\pi\epsilon(p_0) \left(\int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P)(\hat{K}' \cdot P' + i\epsilon)} \right) \frac{\mathbf{P}}{P^2} \delta(\hat{Q} \cdot P) \frac{1}{2q} \frac{1}{\epsilon} \quad (4.11)$$

For the second piece of (4.10), P and P' are on the light cone because of the product of constraints $\delta(P'^2)$ and $\delta(P^2)$. One therefore gets collinear singularities at $\hat{K} = \hat{P}$ and $\hat{K} = \hat{P}'$ whose sum of residues reads

$$\begin{aligned} -2i\pi\epsilon(p_0)\delta(P^2) \left\{ \frac{1}{2\epsilon} \frac{1}{p'} \frac{1}{\hat{P}' \cdot P + i\epsilon} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{P}' \cdot \hat{K}')^2}{(\hat{K}' \cdot P)(\hat{K}' \cdot P' + i\epsilon)} \right. \\ \left. + \frac{1}{2\epsilon} \frac{1}{p} \frac{1}{\hat{P} \cdot P' + i\epsilon} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{P} \cdot \hat{K}')^2}{(\hat{K}' \cdot P)(\hat{K}' \cdot P' + i\epsilon)} \right\} \quad (4.12) \end{aligned}$$

where, again, collinear singularities have been regularized using (3.23). Having $\delta(P'^2)\delta(P^2) = \delta(P'^2)\delta(2Q \cdot P')$, one has also $\hat{P}' = \hat{Q}$, and likewise for $\hat{P}$, so that (4.12) can be written as

$$-4i\pi\epsilon(p_0)\delta(P^2) \frac{1}{2\epsilon} \frac{1}{Q \cdot P + i\epsilon} \left(\int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P + i\epsilon)(\hat{K}' \cdot P' + i\epsilon)} \right) \quad (4.13)$$

Summing up (4.11) and (4.13), one gets

$$\frac{-4i\pi\epsilon(p_0)}{2\epsilon} \left(\int \frac{d\hat{K}'}{4\pi} \frac{(\hat{Q} \cdot \hat{K}')^2}{(\hat{K}' \cdot P)(\hat{K}' \cdot P' + i\epsilon)} \right) \left(\frac{\mathbf{P}}{P^2} \frac{\delta(\hat{Q} \cdot P)}{q} + \frac{\delta(P^2)}{Q \cdot P + i\epsilon} \right) \quad (4.14)$$

and thanks to the overall multiplicative $\delta(P'^2)$ constraint, we have eventually

$$\frac{\mathbf{P}}{P^2} \frac{\delta(\hat{Q} \cdot P)}{q} + \frac{\delta(P^2)}{Q \cdot P + i\epsilon} = \frac{\mathbf{P}}{P^2} \delta(-\frac{P^2}{2}) + \frac{\delta(P^2)}{-P^2/2 + i\epsilon} = 0 \quad (4.15)$$

which completes the proof that (4.2), leads to singularity free expressions. Note that in order to obtain a perfect cancellation of singularities it has been crucial to rely again on the definition (3.32) for the discontinuity of the function $W(P, P')$. Like in the previous order m^4 situation, dealing with *disc*(3.14) had led to incomplete cancellations of mass singularities.

The third term in the last line of (4.1) must be analyzed now. It is

$$\begin{aligned} -16ie^2 m^4 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \\ \text{disc} \frac{1}{P^2} \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \frac{(\hat{K} \cdot \hat{K}') (\hat{K} \cdot \Sigma(P))}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)} \quad (4.16) \end{aligned}$$

Integration on $\hat{K}'$ can be carried out, and the imaginary part of (4.16) is proportional to the integral

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{1}{P^2} \int \frac{d\hat{K}}{4\pi} \frac{(\hat{K} \cdot \Sigma(P))^2}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (4.17)$$

A first contribution is given by the expression

$$\int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \frac{\mathbf{P}}{P^2} \text{disc} \int \frac{d\hat{K}}{4\pi} \frac{(\hat{K} \cdot \Sigma(P))^2}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (4.18)$$

the discontinuity of which reading

$$-2i\pi\epsilon(p_0) \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{(\hat{K} \cdot P' + i\epsilon)} (\hat{K} \cdot \Sigma_A(P))^2 \quad (4.19)$$

where we have taken the $\epsilon(p_0)$ of (3.32) into account, and appended the relevant *advanced* suffix A to the self energy. Since the four-momentum P' is on the light cone, the angular integration will develop a collinear singularity in a neighbourhood of $\hat{K} = \hat{P}'$, with residue

$$\delta(P'^2) \frac{\mathbf{P}}{P^2} \delta(P^2) (-2i\pi\epsilon(p_0)) \frac{1}{\epsilon} \frac{(P' \cdot \Sigma_A(P))^2}{p'^2} \quad (4.20)$$

The contribution of (4.17), completing (4.18) is

$$\delta(P'^2) \delta(P^2) (-2i\pi\epsilon(p_0)) \mathcal{R}e \int \frac{d\hat{K}}{4\pi} \frac{(\hat{K} \cdot \Sigma(P))^2}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (4.21)$$

Again, P' is on the light cone and the angular integration develops a collinear singularity at $\hat{K} = \hat{P}'$, with residue

$$\delta(P'^2) \delta(P^2) (-2i\pi\epsilon(p_0)) \frac{1}{2\epsilon} \frac{1}{p'} \mathcal{R}e \frac{(\hat{P}' \cdot \Sigma_R(P))^2}{\hat{P}' \cdot P + i\epsilon} \quad (4.22)$$

where, because of (4.20) where the advanced prescription comes about, we have recalled the retarded character of the involved self energy function; (4.22) can be cast into the form

$$\delta(P'^2) \frac{\delta(P^2)}{-P^2 + i\epsilon} (-2i\pi\epsilon(p_0)) \frac{1}{\epsilon} \frac{(P' \cdot \Sigma_R(P))^2}{p'^2} \quad (4.23)$$

Eventually, gathering pieces, we see that the net expression (4.17) contributes the imaginary part of $\Pi_R(Q)$ an amount,

$$\begin{aligned} & -16ie^2 m^4 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \delta(P'^2) (-2i\pi\epsilon(p'_0)) \\ & (-2i\pi\epsilon(p_0)) \frac{\mathbf{P}}{P^2} \delta(P^2) \frac{1}{\epsilon} \frac{1}{p'^2} \mathcal{R}e[(P' \cdot \Sigma_A(P))^2 - (P' \cdot \Sigma_R(P))^2] \end{aligned} \quad (4.24)$$

Now, the last term of (4.24) has real part zero as can be seen writing, for example,

$$\begin{aligned} (P' \cdot \Sigma_A(P))^2 - (P' \cdot \Sigma_R(P))^2 &= Q \cdot (\Sigma_A(P) - \Sigma_R(P)) (2m^2 + Q \cdot (\Sigma_A(P) + \Sigma_R(P))) \\ &= -2i \mathcal{I}m Q \cdot \Sigma_R(P) (2m^2 + 2\mathcal{R}e Q \cdot \Sigma_R(P)) \end{aligned} \quad (4.25)$$

This achieves the proof of a cancellation of collinear or mass singularities that otherwise, are on the order of ε^{-2} since, expanding the expression $(P' \cdot \Sigma_R(P))^2$ in (4.23) for example, one gets,

$$\delta(P'^2) \frac{\delta(P^2)}{-P^2 + i\varepsilon} \frac{1}{\varepsilon} \frac{1}{p'^2} \frac{m^4 q}{p} \left(-\frac{P^2}{p^2}\right) \ln\left(\frac{p_0 + p}{p_0 - p}\right) \longrightarrow \delta(P'^2) \delta(P^2) \frac{m^4 q}{p} \frac{1}{\varepsilon} \frac{1}{p'^2} \frac{1}{\varepsilon} \frac{1}{p^2} \quad (4.26)$$

where (3.8) and (3.23) have been used.

Other mass singularities show up, attached to the "non residual" parts of both (4.19) and (4.21). In the former case, that "non residual" part reads

$$(-2i\pi\epsilon(p'_0))\delta(P'^2)(-2i\pi\epsilon(p_0)) \frac{\mathbf{P}}{P^2} \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{(\hat{K} \cdot P' + i\varepsilon)} [(\hat{K} \cdot \Sigma_A(P))^2 - (\hat{P}' \cdot \Sigma_A(P))^2] \quad (4.27)$$

By construction, this expression is no longer singular at $\hat{K} = \hat{P}'$, but instead in a neighbourhood of $P^2 = 0$, where $\hat{K} = \hat{P}$ is hereafter selected by the angular integration to give (positive p_0 is chosen for the sake of illustration)

$$(-2i\pi)^2 \delta(P'^2) \frac{\mathbf{P}}{P^2} \frac{1}{p} \frac{1}{p'} \frac{(\hat{P} - \hat{P}') \cdot \Sigma_A(P)}{\hat{P} \cdot \hat{P}' + i\varepsilon} (\hat{P} + \hat{P}') \cdot \Sigma_A(P) \quad (4.28)$$

and this reduces to

$$(-2i\pi)^2 \delta(P'^2) \frac{\mathbf{P}}{P^2} \frac{1}{p} \frac{1}{p'} \left(-\frac{m^2}{p} \left[\frac{p_0}{2p} \ln\left(\frac{p_0 + p}{p_0 - p}\right) - 1 \right] \right) \frac{2m^2}{p} \quad (4.29)$$

Likewise, the "non residual" part of (4.21) reads

$$(-2i\pi)^2 \delta(P'^2) \delta(P^2) \mathcal{R}e \int \frac{d\hat{K}}{4\pi} \frac{(\hat{K} \cdot \Sigma(P))^2 - (\hat{P} \cdot \Sigma(P))^2}{(\hat{K} \cdot P + i\varepsilon)(\hat{K} \cdot P' + i\varepsilon)} \quad (4.30)$$

Angular integration develops a collinear behaviour by $\hat{P}'$, with residue

$$2 \times (-2i\pi)^2 \delta(P'^2) \delta(P^2) \frac{1}{2p'\varepsilon} \frac{1}{p} \left(-\frac{m^2}{p} \left[\frac{p_0}{2p} \ln\left(\frac{p_0 + p}{p_0 - p}\right) - 1 \right] \right) \frac{2m^2}{p} \quad (4.31)$$

where an overall factor of 2 accounts for the two symmetric possibilities $\hat{K} = \hat{P}, \hat{P}'$, withdrawing either $(\hat{P} \cdot \Sigma(P))^2$ or $(\hat{P}' \cdot \Sigma(P))^2$ from the term $(\hat{K} \cdot \Sigma(P))^2$. The subsidiary integration over p_0 or more conveniently P^2 , [6], provides (4.29) with an extra factor of $1/\varepsilon$, because of the *Principal Part* distribution of $1/P^2$, and the P^2 -integrated sum of (4.29) and (4.31) comes out

devoid of any order ε^{-2} and ε^{-1} contributions, the former attached to the logarithmic functions written in terms of (3.23). Finally, one can conclude that no mass singular behaviour is generated by the third term in the last line of (4.1).

Of its own, the first term appearing in the last line of (4.1), does not pertain to the double *HTL* vertex topologies. Indeed, in agreement with pretty general mechanisms [12], it is withdrawn by a similar contribution coming from order m^6 pure self energy diagrams (2, 1; 0) plus (1, 2; 0). Now, since we are restricting our order m^6 analysis to the most interesting case of double *HTL*-vertex topologies, this term must be considered either. It is

$$+32ie^2m^6 \int \frac{d^4P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{1}{(P^2)^2} \int \frac{d\hat{K}}{4\pi} \int \frac{d\hat{K}'}{4\pi} \frac{\hat{K} \cdot \hat{K}'}{(\hat{K} \cdot P' + i\epsilon)(\hat{K}' \cdot P + i\epsilon)} \\ = +32ie^2m^2 \int \frac{d^4P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2} \text{disc} \frac{1}{(P^2)^2} \Sigma_R(P) \cdot \Sigma_R(P') \quad (4.32)$$

and its contribution to the imaginary part of $\Pi_R(Q)$ is given by the integral

$$32e^2m^2 \int \frac{d^3p}{(2\pi)^3} \int dp_0 (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) (\Sigma_R(P'))_\mu \text{disc} \frac{1}{(P^2)^2} \Sigma_R^\mu(P) \quad (4.33)$$

that is,

$$\frac{16e^2m^2}{q} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p} \int_{-p}^{+p} dp_0 (1 - 2n_F(p_0)) \epsilon(p'_0) (\Sigma_R(P'))_\mu \text{disc} \frac{1}{(P^2)^2} \Sigma_R^\mu(P) \quad (4.34)$$

with $\Sigma_R^\mu(P)$, the self energy four-vector

$$\Sigma_R^\mu(P) = m^2 \left(\frac{1}{2p} \ln\left(\frac{p_0 + p}{p_0 - p}\right), \frac{\hat{p}}{p} \left[\frac{p_0}{2p} \ln\left(\frac{p_0 + p}{p_0 - p}\right) - 1 \right] \right) \quad (4.35)$$

the retarded (or advanced) character of which being consistently encoded in the logarithmic determinations. In the integrand, the scalar product of self energy functions involves an angle constrained by the relations

$$\hat{q} \cdot \hat{p} = \frac{P^2}{2pq} + \frac{p_0}{p}, \quad \hat{p} \cdot \hat{p}' = \frac{1}{|p_0 + q|} \left(\frac{P^2}{2p} + \frac{qp_0}{p} + p \right) \quad (4.36)$$

Now, the discontinuity in (4.34) entails a contribution attached to the integrand

$$\epsilon(p'_0) (\Sigma_R(P'))_\mu \frac{\mathbf{P}}{(P^2)^2} \left(\text{disc} \Sigma_R^\mu(P) = -\frac{2i\pi\epsilon(p_0)\Theta(-P^2)m^2}{2p} \left(1, \hat{p}\left(\frac{p_0}{p}\right) \right)^\mu \right) \quad (4.37)$$

whose scalar product with $\Sigma_R(P')$ reads

$$\Sigma_R(P') \cdot \left(1, \frac{p_0}{p} \hat{p} \right) = \frac{m^2}{p'} \frac{p_0}{p} (\hat{p} \cdot \hat{p}') + \frac{m^2}{2p'} \left(1 - (\hat{p} \cdot \hat{p}') \frac{p_0}{p} \frac{p'_0}{p'} \right) \mathcal{R}e \ln\left(\frac{p'_0 + p}{p'_0 - p}\right) \quad (4.38)$$

With $\hat{p} \cdot \hat{p}'$ as given in (4.36), it is easy to realize that the scalar product (4.38) fails to be proportional to P^2 , and that (4.37) will accordingly display mass singularities of the *Lebesgue non integrable* type: Dimensional regularization, as well as representation (3.23), are unable to take care of them. Likewise, one may verify that the counterpart of (4.37), proportional to a distribution of $\delta'(P^2)$, is responsible for terms that are non differentiable at $P^2 = 0$. These singularities are specific to the thermal context, where they show up in $N \geq 3$ loop calculations. They were first discovered in a hot scalar field model [15], but can be predicted on the basis of purely axiomatic arguments [16], and have recently been the subject of a most rigorous analysis in the moving fermion damping rate problem [6].

We follow the route proposed in [6], and provide the internal P -line with a mass μ , which, divided by p , defines the auxiliary dimensionless IR regulator $\lambda = \mu/p$, in the presence of which one is entitled to take the mass shell limit [16]. We specialize (4.37) to the case of positive energies, p_0 , for the sake of as simple an illustration as possible, and get the integral

$$-\frac{8ie^2m^4}{q} \frac{\sin(\pi\varepsilon)}{\varepsilon} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p^5} \mathbf{P} \int_0^1 dx \frac{x^\varepsilon}{(x-\lambda)^2} F_+(\varepsilon, x) \quad (4.39)$$

where we have used the representation (3.23), the integration variable $x = |P^2|/p^2$, and defined $F_+(\varepsilon, x)$, the function

$$F_+(\varepsilon, x) = \frac{(1 - 2n_F(p\sqrt{1-x})) [\Sigma_R^+(P') \cdot (1, \hat{p}\sqrt{1-x})]}{\sqrt{1-x} (1 + \sqrt{1-x})^{2\varepsilon}} \quad (4.40)$$

where the symbol " + " is to remind the case of positive energies. Now, we claim that with x in the interval $]0, 1[$, the function F_+ admits a Taylor series expansion

$$F_+(\varepsilon, x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} F_+^{(n)}(\varepsilon, 0) \quad (4.41)$$

The above statement is clearly true for the the squared root, the statistical factor, and the scalar product (4.38), whereas it is proved in [6], appendix C, that it holds true also, for the last factor entering the definition of $F_+(\varepsilon, x)$, the function $(1 + \sqrt{1-x})^{-2\varepsilon}$. The same steps as taken in the detailed proof of [6], can be taken here also to assert that both *Lebesgue non integrable* and *Lebesgue integrable* mass singularities do cancel out, so as to leave a perfectly regular quantity. We shall not develop the full demonstration here again, because it is a bit lengthy and just a particular case of the more general proof given in [6]. We may therefore content ourselves of verifying the cancellation of the *Lebesgue non integrable* mass singularities, as they are definitely the new objects arising in our present calculation.

Out of (4.39), one gets

$$-\frac{8ie^2m^4}{q} \frac{\sin(\pi\varepsilon)}{\varepsilon} \sum_{n=0}^{\infty} \frac{F_+^{(n)}(\varepsilon, 0)}{n!} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p^5} \mathbf{P} \int_0^1 dx \frac{x^{\varepsilon+n}}{(x-\lambda)^2} \quad (4.41)$$

where we have taken advantage of the commutativity (proved in [6], appendix C) of the sum over n and integral on x . It is clear that the only occurrence of a *Lebesgue non integrable* mass

singularity is at $n = 0$, with the result (Eq.(7.9) of [6])

$$+ \frac{8ie^2 m^4}{q} \frac{\sin(\pi\epsilon)}{\epsilon} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p^5} \frac{F_+^{(0)}(\epsilon, 0)}{0!} \times \left(\mathbf{P} \int_0^1 dx \frac{x^{\epsilon+0}}{(x-\lambda)^2} = \cos(\pi\epsilon) \frac{B(1+\epsilon, 1-\epsilon)}{\lambda^{1-\epsilon}} - \frac{1}{1-\epsilon} + \mathcal{O}(\lambda) \right) \quad (4.42)$$

where $B(x, y)$ is the Euler beta-function [18]. Since we have

$$\frac{F_+^{(0)}(\epsilon, 0)}{0!} = \frac{m^2}{2^{2\epsilon}(p+q)} (1 - 2n_F(p)) \quad (4.43)$$

the above result just exhibits a previously *Lebesgue non integrable* mass singularity, which is now taken care of by the auxiliary *IR* regulator λ , is on the order of $\lambda^{-1+\epsilon}$, and reads eventually

$$+ \frac{8ie^2}{q} \frac{\sin(\pi\epsilon)}{\epsilon} \frac{\cos(\pi\epsilon)}{2^{2\epsilon}} \frac{B(1+\epsilon, 1-\epsilon)}{\lambda^{1-\epsilon}} \int \frac{p^2 dp}{(2\pi)^2} (1 - 2n_F(p)) \left(\frac{m^2}{p^2} \right)^3 \frac{p}{p+q} \quad (4.44)$$

The term completing, for (4.34), the contribution of (4.37) is, within the same variables and for positive energies

$$\frac{8i\pi e^2 m^4}{q\epsilon} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p^5} \int_0^1 dx \delta(x-\lambda) \frac{d}{dx} \frac{(1 - 2n_F(p\sqrt{1-x}))}{\sqrt{1-x}} \times \left(1 - \frac{x^\epsilon e^{i\pi\epsilon}}{(1 + \sqrt{1-x})^{2\epsilon}} \right) [\Sigma_R^+(P') \cdot ((1, \hat{p}\sqrt{1-x}) + (0, -\hat{p}))] \quad (4.45)$$

It contains four terms, out of them we recognize a piece

$$- \frac{8i\pi e^2 m^4}{q\epsilon} \cos(\pi\epsilon) \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p^5} \int_0^1 dx \delta(x-\lambda) \frac{d}{dx} x^\epsilon F_+(\epsilon, x) \quad (4.46)$$

Then, in the λ -vanishing limit, only the term $n = 0$, in the Taylor series expansion of $F_+(\epsilon, x)$, will contribute to give (Eq.(7.4) of [6])

$$- \frac{8i\pi e^2}{q\epsilon} \frac{\cos(\pi\epsilon)}{2^{2\epsilon}} \frac{\sin(\pi\epsilon)}{\pi} \frac{B(1+\epsilon, 1-\epsilon)}{\lambda^{1-\epsilon}} \int \frac{p^2 dp}{(2\pi)^2} (1 - 2n_F(p)) \left(\frac{m^2}{p^2} \right)^3 \frac{p}{p+q} \quad (4.47)$$

As advertised, both *Lebesgue non integrable* mass singularities can be seen to disappear in the sum of (4.44) and (4.47). The other three pieces of (4.45) are of the *integrable* type, on the order of ϵ^{-1} at most, and cancel out, in particular with a contribution of (4.41) obtained at $n = 1$. Of course, the same results are obtained also in the case of negative energies.

Actually, taking advantage of the present results, as well as of results rigorously established in [6], it seems possible to foresee that for all N and N' , any of the diagrams of types $(N, N'; 0)$, $(N, N'; 1)$ and $(N, N'; 2)$ contribute mass singularity free quantities to the soft photon emission rate. A general proof of that statement could be worth presenting elsewhere [19].

5. Illustration of the RP versus PR difference

Here, as in [8], we conclude that : *The calculational steps that are followed within the RP evaluation of $\Pi_R(Q)$, have led to single out and isolate a collinear singularity in the HTL vertex, from the ones also induced by partial effective propagators. Overall detailed balance compensations of mass singularities (that are very likely to hold at any number N, N' of self energies and HTL vertex insertions) are accordingly violated, and an uncanceled collinear singularity factors out.. as displayed by (3.44) and (3.46).*

Indeed, while (3.44) was recognized a deceptive example, at present order m^6 , it becomes possible to exhibit an example which allows to understand how RP and PR resummation schemes drift apart in the domain of mass singularities.

Let us write down the contribution to the imaginary part of $\Pi_R(Q)$, of the term $(1, 1; 1)$, whose imaginary part is singularity free, as shown in the appendix. We have

$$(1, 1; 1) = +e^2 m^2 \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \frac{1}{(P'^2 + i\epsilon p'_0)^2} \frac{1}{(P^2 - i\epsilon p_0)^2} \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \text{Tr} \left(\not{P} \not{\Sigma}_A(P) \not{P}' \not{\hat{K}} \not{P}' \not{\Sigma}_R(P') \not{P}' \not{\hat{K}} \right) \quad (5.1)$$

As usual, we observe that a collinear singularity develops out of the angular integration at $\hat{K} = \hat{Q}$ and yields the singular contribution

$$(1, 1; 1)|_s = e^2 m^2 \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right) \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \delta(\hat{Q} \cdot P) \frac{1}{(P'^2 + i\epsilon(p'_0))^2} \frac{1}{(P^2 - i\epsilon(p_0))^2} \text{Tr} \left(\not{P} \not{\Sigma}_A(P) \not{P}' \not{\hat{Q}} \not{P}' \not{\Sigma}_R(P') \not{P}' \not{\hat{Q}} \right) \quad (5.2)$$

Now, among the four terms coming from the evaluation of the trace, it is elementary to verify that, due to the constraint $\delta(\hat{Q} \cdot P)$, the only surviving term is

$$+8 P^2 P'^2 \hat{Q} \cdot \Sigma_A(P) \hat{Q} \cdot \Sigma_R(P') \quad (5.3)$$

all of the other three terms being proportional to the scalar product $(\hat{Q} \cdot P)$. We thus get

$$(1, 1; 1)|_s = 8e^2 m^2 \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right) \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \delta(\hat{Q} \cdot P) \times \left(\frac{\hat{Q} \cdot \Sigma_R(P')}{P'^2 + i\epsilon p'_0} \frac{\hat{Q} \cdot \Sigma_A(P)}{P^2 - i\epsilon p_0} = \frac{\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P)}{(P^2 + i\epsilon)^2} \right) \quad (5.4)$$

where in the last line, we have used the relation $P'^2 = P^2$, which, again, is due to the constraint $\delta(\hat{Q} \cdot P)$. Performing the angular integration on $\hat{p}$, one gets

$$8e^2 m^2 \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \right) \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p} \int_{-p}^{+p} dp_0 (1 - 2n_F(p_0)) \frac{\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P)}{(P^2 + i\epsilon)^2} \quad (5.5)$$

whose imaginary part reads

$$8ie^2 m^2 \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} \int \frac{p^2 dp}{(2\pi)^2} \frac{1}{p} \int_{-p}^{+p} dp_0 (1 - 2n_F(p_0)) \right. \\ \left. \left(\mathbf{P} \frac{Im \left(\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \right)}{(P^2)^2} - \pi \delta'(P^2) Re \left(\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \right) \right) \right) \quad (5.6)$$

At this stage, it is possible to rely on the relations

$$\hat{Q} \cdot \Sigma_A(P) = \frac{m^2}{p} \left(p_0 - \frac{P^2}{2p^2} \left[\ln \left| \frac{p_0 + p}{p_0 - p} \right| + i\pi \Theta(-P^2) \right] \right) \\ \hat{Q} \cdot \Sigma_R(P') = \frac{m^2}{p'} \left(p'_0 - \frac{P^2}{2p'^2} \left[\ln \left| \frac{p'_0 + p'}{p'_0 - p'} \right| - i\pi \Theta(-P^2) \right] \right) \quad (5.7)$$

in order to verify once more the robustness of *integrable* and *non integrable* mass singularity cancellations, in terms of $\mathbf{P}/(P^2)^n$ versus $\delta^{(n-1)}(P^2)$ detailed balance contributions. Calculations, the same as developed throughout the present article, are tedious but straightforward, displaying no novel features or peculiarities. The matter however, is that the detailed balance leaves us with an order $\mathcal{O}(\epsilon^0)$, regular result, which gets subsequently multiplied by the collinear singularity factor (using (3.23))

$$\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot Q + i\epsilon} = \frac{1}{2q\epsilon} \quad (5.8)$$

That is, the imaginary part of $(1, 1; 1)$ turns out to be plagued with a collinear singularity, in contradiction with our claim that it is singularity free.

This contradiction is valuable as it illustrates the difference we are looking at. We get back to the *PR* handling of $(1, 1; 1)$, reproducing here for the sake of close comparison, that very part of equation (A.3), to wit,

$$+8ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{P'^2 + i\epsilon p'_0} \\ \text{disc} \frac{1}{P^2 + i\epsilon p_0} \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P') \hat{K} \cdot \Sigma(P)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (5.9)$$

out of which (5.4) can be readily obtained, if one proceeds along the *RP* calculational steps. In (5.9), the discontinuity is taken the usual way, so as to give

$$+8e^2 m^2 \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \\ \frac{1}{P'^2 + i\epsilon p'_0} \frac{1}{P^2 - i\epsilon p_0} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \hat{K} \cdot \Sigma_R(P') \hat{K} \cdot \Sigma_A(P) \quad (5.10)$$

Now, in a neighbourhood of $\hat{Q}$, where the angular integration develops a collinear singularity, the whole integrand behaves like

$$(1 - 2n_F(p_0(P^2))) \frac{1}{P'^2 + i\epsilon p'_0} \frac{1}{P^2 - i\epsilon p_0} \frac{\delta(\hat{Q} \cdot P)}{\hat{Q} \cdot P + i\epsilon} \hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \quad (5.11)$$

and the point is that partial effective propagators mass singularities, developing by the light cone region at $P^2 = 0$, cannot be disentangled from the collinear singularity coming from the *HTL* vertex at $\hat{K} = \hat{Q}$. They come about mixed up, as can be seen by re-expressing (5.11) as

$$(1 - 2n_F(p_0(P^2))) \frac{1}{P'^2 + i\epsilon p'_0} \frac{1}{(P'^2 - 2q \hat{Q} \cdot P) - i\epsilon p_0} \frac{2q \delta(\hat{Q} \cdot P)}{P'^2 - P^2 + i\epsilon} \hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \quad (5.12)$$

which, by the light cone region, behaves like

$$(1 - 2n_F(\epsilon(p_0)p)) \frac{1}{(P'^2)^2 + i\epsilon p P'^2} \frac{2q \delta(\hat{Q} \cdot P)}{P'^2 + i\epsilon} \hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \quad (5.13)$$

that is, like

$$(1 - 2n_F(\epsilon(p_0)p)) \left(\frac{2q \delta(\hat{Q} \cdot P)}{(P'^2 + i\epsilon)^3} = \frac{2q \delta(\hat{Q} \cdot P)}{(P^2 + i\epsilon)^3} \right) \hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P) \quad (5.14)$$

and for the imaginary part, like

$$2iq \delta(\hat{Q} \cdot P) (1 - 2n_F(\epsilon(p_0)p)) \left\{ \frac{\mathbf{P}}{(P^2)^3} \text{Im} [\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P)] - \pi \frac{(-1)^2}{2!} \delta^{(2)}(P^2) \text{Re} [\hat{Q} \cdot \Sigma_R(P') \hat{Q} \cdot \Sigma_A(P)] \right\} \quad (5.15)$$

In agreement with the singularity free character of $(1, 1; 1)$, we know that, when integrated on P^2 (or p_0), the structure (5.15) is generic in granting both *integrable* and *non integrable* mass singularities, with robust cancellation patterns. Here is illustrated the content of the statement we wrote in italics in the beginning of this section, which statement had been foreseen on a scalar model simpler basis [8].

It should appear clear now, that, if *a fortiori*, one deals with full, $*S_\alpha(P)$, instead of partial effective propagators, the above subtlety is necessarily overlooked because, in the present *Linear Response Theory* context, full effective propagators have only timelike poles, outside the light cone region : The possibility that effective propagators and vertices combine their singular residues into singularity free expressions, is excluded by construction. Note that the *RP* versus *PR* difference we have just choosen to illustrate in the case of the one *HTL* vertex diagram $(1, 1; 1)$, can be extended (as quickly outlined in passing from (4.5) to (4.6)), to a diagram like $(1, 0; 2)$ which involves two *HTL* vertex insertions. In order to avoid overcharging the present article, this extra technical development, which brings no new insight, will be omitted here.

6. Conclusion

Given the matter, we could not avoid dealing with a certain amount of purely technical calculations, and so it is useful to begin with summarizing the results that have finally been obtained. The collinear singularity problem of hot QCD has been analyzed within a Perturbative Resummation (PR) scheme of the leading thermal fluctuations, that had been previously introduced and explored in the case of the moving fermion damping rate problem [6].

Calculations are thus purely perturbative, though rapidly growing in number and complexity. In this article we have carefully examined the full m^2 and m^4 order contributions to the soft real photon emission rate from a quark-gluon plasma in thermal equilibrium, and found that they come out mass (or collinear) singularity free.

At order m^4 , we have seen that the derivation of this result is already a non trivial one. In particular, it requires a proper identification of the discontinuity of a function specific to the double HTL vertex insertion. If the discontinuity in the energy is calculated right from the product of HTL vertex standard expression, then, mass singularity cancellations fail to be complete at negative energies. Inspection shows, though, that this subtlety cannot be invoked so as to cure the collinear singularity disease which plagues the emission rate Resummation Program (RP) calculation.

Then we found worthwhile analyzing the first diagrams exhibiting an interplay of HTL self energy and double vertex insertions, since the hot QCD collinear singularity problem is precisely due to these very topologies. Though a bit involved technically, we have been able to prove that order m^6 potentially singular most topologies are collinear singularity free either, and only display the novel feature of *Lebesgue non integrable* mass singularities, that are common to most $n \geq 3$ loop calculations: First put forth in a hot scalar field three loop study [15], this new type of IR singularities can be predicted on the basis of general axiomatic developments [16], and have been the matter of a recent and most rigorous analysis, in the moving fermion damping rate context [6]. In this order m^6 case also, the proper discontinuity definition alluded to above, has been crucial for granting mass singularity cancellations with completeness. Indeed, considering the long noticed, robust and generic character of singularity compensations, in terms of $\mathbf{P}/(P^2)^n$ versus $\delta^{(n-1)}(P^2)$ detailed balance contributions, we think that this conclusion is very likely to hold true at any higher order m^n , $n > 6$. A proof of this statement could be given elsewhere [19].

Strikingly enough, we found that the same order m^4 , m^6 quantities that have been proven to be singularity free within the framework of a PR calculation, are found to display the same famous collinear singularity as found in the RP case, if standart RP 's calculational steps are taken!

This fact is interesting. From a purely logical point of view, it indicates that, when adopted within a PR scheme, the RP calculational steps entail a so called *surreptitious element*, which we have been able, in section 5, to identify without ambiguity. Reciprocally, this identification makes it possible to get a clear understanding of how and why RP and PR schemes drift apart in their handling of mass (or collinear) singularities. Though relying on a somewhat different mechanism, we note that a very close conclusion could be reached in a simple scalar field model either [8].

In our opinion, this enforces and refines, the proposition we made then, concerning the mechanism responsible for the RP collinear singularity's generation, and by the same token,

elucidates the questionable nature [20] of that very singularity. Things can be put as follows :

In the course of the soft photon emission rate calculation, the *RP* trades *PR* partial effective propagators, $S_\alpha^{(N)}(P)$, for fully resummed ones, $^*S_\alpha(P) = \sum_N S_\alpha^{(N)}(P)$, and then, takes the discontinuity in the energy, of functions that are related to one or two *HTL* vertex insertions. Now, the matter is that the latter exhibit a collinear singularity by the light cone, whereas the former does not. This divergence, purely perturbative and exclusively related to the light cone singular behaviour of *HTL* vertices, remains accordingly isolated in an *RP* scheme, with no other singular counterpart to cancel against.

This gets particularly enlightened by comparing with what happens in a *PR* handling of the problem, since the *HTL* vertex light cone singularity arises in a phase space domain where inspection shows that it cannot be disentangled from partial effective propagators own mass singularities. The whole “boils down” to a structure in terms of $\mathbf{P}/(P^2)^n$ versus $\delta^{(n-1)}(P^2)$ detailed balance contributions, which guarantees both *integrable* and *non integrable* mass singularities with robust cancellation patterns [6], as checked once more throughout this article. Of course, in an emission rate *RP* calculation, this point is necessarily overlooked since, contrarily to partial effective propagators, $S_\alpha^{(N)}(P)$, full effective propagators, $^*S_\alpha(P)$, have no poles in the light cone region.

By the way, the same mechanism elucidates also the reason why the *RP* collinear singularity is a typical high temperature effect: Calculating the same diagrams at zero temperature, one deals with dressed fermionic propagators which, for massless fields, only differ bare propagators, a multiplicative renormalization constant and display, otherwise, the same pole structure by the light cone. Then, contrarily to the thermal *RP* situation, vertex potential mass singularities (depending in that case, on the gauge choice) can mix up with dressed fermionic propagators own mass singularities, into patterns which grant their overall compensations.

Out of [8] and the present analysis, it is tempting to conclude, in agreement with a former interpretation [21], that the collinear singularity plaguing the soft photon emission rate *RP* calculation, is an *artefact* of the *RP* resummation scheme itself. In full rigour though, such a conclusion could be premature: In effect, performing the same rate *PR* calculation, one needs be sure that any of the $(m^2)^n$ contribution is mass singularity free, and that their sum is free of any further pathology either. Work in this sense is in progress [19].

After all, whenever Resummation is required by the context, a guiding principle could very well be that it be conceived and taken of finite, well defined elements, and, in particular, of mass singularity free terms. In this respect, it is instructive to come back to the original article where the *RP* was mostly founded, and to realize that the authors were conscious of difficulties that could be inherited from the fact that the *RP* did not necessarily comply with this requirement [22].

The full *PR* result for the soft photon emission rate will be worth investigating elsewhere [19], though possibly not an easy task! As R.D. Pisarski put it, some years ago, “It’s really quite surprising, that it’s so difficult to compute damping rates in hot gauge theories !” [23].

Appendix

In order to avoid overwhelming the main text of the present article with even more technical pieces, we give here the proof that the imaginary part of $(1, 1; 1)$ is singularity free. One has

$$(1, 1; 1) = -ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \int \frac{d\hat{K}}{4\pi} \text{disc} \frac{\text{Tr} \left(S_R^{(1)}(P) \hat{K} S_R^{(1)}(P') \hat{K} \right)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (A.1)$$

That is

$$+8ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{(P'^2)^2} \text{disc} \frac{1}{(P^2)^2} \int \frac{d\hat{K}}{4\pi} \frac{1}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \\ \left\{ 4m^4 (\hat{K} \cdot P)(\hat{K} \cdot P') - 2m^2 P'^2 (\hat{K} \cdot P) \hat{K} \cdot \Sigma(P') - (P \leftrightarrow P') + P^2 P'^2 \hat{K} \cdot \Sigma(P') \hat{K} \cdot \Sigma(P) \right\} \quad (A.2)$$

or,

$$+8ie^2 m^2 \int \frac{d^4 P}{(2\pi)^4} (1 - 2n_F(p_0)) \frac{1}{(P'^2)^2} \text{disc} \frac{1}{(P^2)^2} \\ \left\{ 4m^4 - 2m^2 (P^2 \Sigma^2(P) + (P \rightarrow P')) + P^2 P'^2 \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P') \hat{K} \cdot \Sigma(P)}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \right\} \quad (A.3)$$

Relying on "mass derivative formulae" [13,14], such as used in section 3, we have checked that first and second terms do not contribute any singular quantity to the imaginary part of $\Pi_R(Q)$. The same holds for the third term, symmetric of the second in the exchange $P \leftrightarrow P'$. Moreover, in agreement with general mechanisms alluded to above [12], it is straightforward to check that second and third terms do not really enter the order m^6 imaginary part of $\Pi_R(Q)$: First doubled by similar contributions coming from the two one *HTL* vertex diagrams $(2, 0; 1)$ and the two diagrams $(0, 2; 1)$, their sum is eventually cancelled out by the sum of pure *HTL* self energy diagrams $(3, 0; 0)$, $(0, 3; 0)$, $(1, 2; 0)$ and $(2, 1; 0)$. On the contrary, the fourth term belongs to $(1, 1; 1)$ in proper, and its imaginary part reads

$$16e^2 m^2 \int \frac{d^4 P}{(2\pi)^3} (1 - 2n_F(p_0)) \epsilon(p'_0) \delta(P'^2) \text{disc} \frac{1}{P^2} \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P) \hat{K} \cdot \Sigma(P')}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (A.4)$$

with

$$\text{disc} \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P) \hat{K} \cdot \Sigma(P')}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} = \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P')}{\hat{K} \cdot P' + i\epsilon} \left(\frac{\hat{K} \cdot \Sigma_R(P)}{\hat{K} \cdot P + i\epsilon} - \frac{\hat{K} \cdot \Sigma_A(P)}{\hat{K} \cdot P - i\epsilon} \right) \quad (A.5)$$

One is led to compare the two following expressions,

$$-2i\pi\epsilon(p_0) \delta(P'^2) \frac{\mathbf{P}}{P^2} \int \frac{d\hat{K}}{4\pi} \frac{\delta(\hat{K} \cdot P)}{\hat{K} \cdot P' + i\epsilon} \hat{K} \cdot \Sigma_A(P) \hat{K} \cdot \Sigma(P') \quad (A.6)$$

on the one hand, and on the other hand

$$-2i\pi\epsilon(p_0) \delta(P'^2)\delta(P^2) \int \frac{d\hat{K}}{4\pi} \frac{\hat{K} \cdot \Sigma(P) \hat{K} \cdot \Sigma(P')}{(\hat{K} \cdot P + i\epsilon)(\hat{K} \cdot P' + i\epsilon)} \quad (A.7)$$

A collinear singularity arises at $\hat{K} = \hat{P}'$, with, for (A.6), the residue

$$-2i\pi\epsilon(p_0) \delta(P'^2) \frac{\mathbf{P}}{P^2} \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \right) \delta(Q \cdot P) \frac{m^2}{p'} (m^2 + Q \cdot \Sigma_A(P)) \quad (A.8)$$

and for (A.7), the residue

$$-2i\pi\epsilon(p_0) \delta(P'^2)\delta(P^2) \left(\int \frac{d\hat{K}}{4\pi} \frac{1}{\hat{K} \cdot P' + i\epsilon} \right) \frac{m^2}{Q \cdot P + i\epsilon} \left(\frac{m^2 + Q \cdot \Sigma_R(P)}{p'} \right) \quad (A.9)$$

The sum of the above two singular residues comes out proportional to the quantity

$$\delta(P^2 + 2Q \cdot P) \left(\frac{\mathbf{P}}{P^2} \delta(Q \cdot P) + \delta(P^2) \frac{\mathbf{P}}{Q \cdot P} \right) = 0 \quad (A.10)$$

as long as real parts of both $\Sigma_R(P)$ and $\Sigma_A(P)$ are involved in (A.8) and (A.9). The latter, (A.9), also contributes the imaginary part of $\Pi_R(Q)$, an amount involving, as part of the integrand, the product

$$\delta(P'^2) \text{Im} (Q \cdot \Sigma_R(P)) \delta(Q \cdot P) = 0 \quad (A.11)$$

in view of (3.8) and (3.9), and the cancellation of singular residues is complete. "Non residual" parts also enter the game, that is parts which do not display collinear behaviours at $\hat{K} = \hat{P}'$, and are defined as in (4.27) and (4.30). It is easy to check that the same treatment as given there, from (4.27) to (4.31), allows to conclude that there is no singularity attached to these "non residual" pieces either, and that the fourth term of (A.2) is eventually singularity free.

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Figure caption

Fig.1: A graph of order m^2 , denoted by $(1,0;0)$, with one insertion of *HTL* self energy along the *P*-line.

Fig.2: Some graphs of order m^4 : With two insertions of *HTL* vertex correction, $(0,0;2)$, with one insertion of *HTL* vertex correction, $(1,0;1)$.

Fig.3: Graph of order m^6 , with one self energy and two vertex *HTL* insertions.

(1,0;0)

Fig.1

(0,0;2)

(1,0;1)

Fig.2

$(1,0;2)$

Fig.3