Abstract

We study large flavor violating signals at future $\sqrt{s} = 5$ TeV linear colliders. Decays at future $\sqrt{s} = 5$ TeV linear colliders.
There are stringent constraints on lepton flavor violation (LFV) in the charged lepton sector, the strongest coming from the decay branching ratio of $\mu^- \rightarrow e^-\gamma$, $BR(\mu^- \rightarrow e^-\gamma) < 1.2 \times 10^{-11}$ [1]. Others are $BR(\mu^- \rightarrow e^-e^+e^-) < 10^{-12}$, $BR(\tau^- \rightarrow e^-\gamma) < 2.7 \times 10^{-6}$, $BR(\tau^- \rightarrow \mu^-\gamma) < 1.1 \times 10^{-6}$, see [2].

On the other hand, recent experiments [3] indicate that at least $\nu_\mu$ and $\nu_\tau$ have an almost maximal mixing angle, $\sin^2 2\theta_{\text{atm}} > 0.88$. The latest results of SNO [4] suggest that also the $\nu_e - \nu_\mu$ sector contains a large mixing, whereas the third mixing angle has to be small [5]. The Standard Model can account for the lepton flavor conservation in the charged lepton sector, but has to be extended to account for neutrino masses and mixings, e.g. by the see-saw mechanism and by introducing heavy right-handed Majorana neutrinos [6].

In general, a gauge and supersymmetric invariant theory does neither conserve total lepton number $L = L_e + L_\mu + L_\tau$ nor individual lepton number $L_e$, $L_\mu$ or $L_\tau$. One usually invokes R-parity symmetry, which forces total lepton number conservation but still allows the violation of individual lepton number, e.g. due to loop effects in $\mu^- \rightarrow e^-\gamma$ [7]. The Minimal Supersymmetric Standard Model (MSSM) with R-parity conservation embedded in a GUT theory induces LFV [8, 9, 10] at the weak scale. This is a consequence of having leptons and quarks in the same GUT multiplet and of the quark flavor mixing due to the CKM matrix. A general analysis of flavor changing neutral (FCNC) effects in K- and B-meson as well as in lepton physics was recently performed in [11].

Moreover, in the MSSM a large $\nu_\mu - \nu_\tau$ mixing can lead to a large $\tilde{\nu}_\mu - \tilde{\nu}_\tau$ mixing via renormalisation group equations [12]. This leads to clear LFV signals in slepton and sneutrino production and in the decays of neutralinos and charginos into sleptons and sneutrinos at the LHC [13] and in $e^+e^-$ and $\mu^+\mu^-$ collisions [14, 15]. Signatures due to $\tilde{e}_R - \tilde{\mu}_R$ mixing were discussed in [16]. In all these studies, it has been assumed that only one lepton flavor violating term dominates.

In this letter, we study the consequences of LFV in the sfermion sector at future $e^+e^-$ colliders, respecting present bounds on rare lepton decays. Assuming the most general mass matrices for sleptons and sneutrinos, we demonstrate that large signals are expected.

The most general charged slepton mass matrix including left-right mixing as well as flavor mixing is given by:

$$
M_i^2 = \begin{pmatrix}
M_{LL}^2 & M_{LR}^2 \\
M_{LR}^2 & M_{RR}^2
\end{pmatrix},
$$

(1)
where the entries are $3 \times 3$ matrices. They are given by

\[
M_{LL,ij}^2 = M_{L,ij}^2 + \frac{v_d^2 Y_{F_1}^E Y_{F_2}^E}{2} \left( \frac{g^2 - g_\nu^2}{v_d^2 - v_u^2} \right),
\]

\[
M_{LR,ij}^2 = \frac{v_d A_{ji}^2 - \mu v_u Y_{H_2}^E}{\sqrt{2}},
\]

\[
M_{RR,ij}^2 = M_{E,ij}^2 + \frac{v_d^2 Y_{F_1}^E Y_{F_2}^E}{2} \left( \frac{g^2 - g_\nu^2}{v_d^2 - v_u^2} \right),
\]

(2)

The indices $i, j, k = 1, 2, 3$ characterize the flavors $\epsilon, \mu, \tau$. $M_{LL}^2$ and $M_{RR}^2$ are the soft SUSY breaking mass matrices for left and right sleptons, respectively. $A_{ij}$ are the trilinear soft SUSY breaking couplings of the sleptons and Higgs boson. The physical mass eigenstates states $\tilde{t}_n$ are given by $\tilde{t}_n = R_{nm} \tilde{t}_m$ with $R_{nm} = (\tilde{\epsilon}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{\epsilon}_R, \tilde{\mu}_R, \tilde{\tau}_R)$. Similarly, one finds for the neutrinos

\[
M_{E,ij}^2 = M_{L,ij}^2 + \frac{g^2 + g_{\nu}^2}{8} \left( \frac{v_d^2 - v_u^2}{v_d^2 - v_u^2} \right),
\]

(3)

with the physical mass eigenstates $\tilde{\nu}_i = R_{ij} \tilde{\nu}_j$ and $\tilde{\nu}_j = (\tilde{\nu}_\epsilon, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$. The relevant interactions for this study are given by:

\[
\mathcal{L} = \bar{\chi}_i (d_{ikm}^L P_L + c_{ikm}^R P_R) \chi^0_k \tilde{t}_m
\]

\[
+ \bar{\chi}_i (d_{ijr}^L P_L + d_{ijr}^R P_R) \chi_r \tilde{\nu}_r
\]

\[
+ \bar{\tilde{\nu}_i} (c_{ikm}^L P_L + c_{ikm}^R P_R) \chi^+_i \tilde{t}_m.
\]

(4)

The specific form of the couplings $d_{ikm}^L, c_{ikm}^L, d_{ikm}^R, c_{ikm}^R, d_{ijr}^L, d_{ijr}^R$ and $c_{ikm}^R$ will be given elsewhere [17]. The first two terms in Eq. (6) give rise to the signals whereas the last one will give rise to the SUSY background.

As mentioned above, most studies so far consider the case where only one of the flavor mixing entries in the slepton (Eq. (1)) and sneutrino mass (Eq. (5)) matrices is non-zero, as for instance, $M_{LL,23}^2 \neq 0$. It is the purpose of this study to allow for all possible flavor violating entries in Eqs. (1) and (5) which are compatible with the present bounds on lepton number violating processes, such as $\mu^{-} \to e^{-} \gamma$, $e^{-} e^{+} e^{-}$, $\tau^{-} \to e^{-} \gamma$, $\tau^{-} \to \mu^{-} \gamma$ and $Z \to e\mu\tau\nu$. For definiteness, we have taken the first of the mSUGRA points of Snowmass’ 01 [18] as reference point which is characterized by $M_{1/2} = 250$ GeV, $M_0 = 100$ GeV, $A_0' = -100$ GeV, $\tan \beta = 10$ and sign($\mu$) = +. Note that $A_0'$ has to be multiplied by the Yukawa couplings
TABLE I: SUSY parameters at the scale $Q = \sqrt{M_{Q3} M_{U3}}$ for $M_{1/2} = 250$ GeV, $M_0 = 100$ GeV, $A'_0 = -100$ GeV, $\tan \beta = 10$ and $\text{sign}(\mu) = +$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>107.9</td>
</tr>
<tr>
<td>$M_2$</td>
<td>208.4</td>
</tr>
<tr>
<td>$M_3$</td>
<td>611.6</td>
</tr>
</tbody>
</table>

$M_{E3} = 138.7 \quad ML = 202.3 \quad A_\tau/Y_\tau = -257.3$

$M_{E3} = 136.3 \quad ML = 201.5 \quad A_\tau/Y_\tau = -257.3$

$M_{D1} = 536 \quad ML = 540 \quad MQ_1 = 562$

$M_{D2} = 534 \quad ML = 427 \quad Q_3 = 509$

$A_h = -863 \quad A_t = -503$

TABLE II: SUSY spectrum for $M_{1/2} = 250$ GeV, $M_0 = 100$ GeV, $A_0 = -100$ GeV, $\tan \beta = 10$ and $\text{sign}(\mu) = +$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\tilde{\chi}_1^0}$</td>
<td>111</td>
</tr>
<tr>
<td>$m_{\tilde{t}_1^0}$</td>
<td>395</td>
</tr>
<tr>
<td>$m_{\tilde{A}_1^0}$</td>
<td>395</td>
</tr>
<tr>
<td>$m_{H^+}$</td>
<td>403</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>618</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_1^+}$</td>
<td>193.6</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_2^0}$</td>
<td>376.2</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_2^0}$</td>
<td>103.1</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_2^0}$</td>
<td>194.6</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_2^0}$</td>
<td>355.1</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_2^0}$</td>
<td>376.0</td>
</tr>
<tr>
<td>$m_{\tilde{\tau}_R}$</td>
<td>146.9</td>
</tr>
<tr>
<td>$m_{\tilde{\tau}_L}$</td>
<td>214.7</td>
</tr>
<tr>
<td>$m_{\tilde{\tau}_R}$</td>
<td>138.6</td>
</tr>
<tr>
<td>$m_{\tilde{\tau}_L}$</td>
<td>217.7</td>
</tr>
<tr>
<td>$m_{\tilde{e}_R}$</td>
<td>199.4</td>
</tr>
<tr>
<td>$m_{\tilde{e}_R}$</td>
<td>198.5</td>
</tr>
</tbody>
</table>

$M_{\tilde{d}_R} = 560 \quad M_{\tilde{d}_L} = 585 \quad M_{\tilde{u}_R} = 561 \quad M_{\tilde{u}_L} = 579$

$M_{\tilde{b}_1} = 530 \quad M_{\tilde{b}_2} = 559 \quad M_{\tilde{t}_1} = 407 \quad M_{\tilde{t}_2} = 600$

To get the $A$ parameter as given in Eq. (3). The corresponding parameters at the scale $Q = \sqrt{M_{Q3} M_{U3}}$ are given in Table I and the physical masses (computed at one-loop) in Table II. We keep all parameters fixed except for the slepton parameters $M_{\tilde{L}}$, $M_{\tilde{R}}$ and $A_t$ where all entries are varied in the whole range compatible with the experimental constraints.

We find values for $|M_{\tilde{R}_{i,j}}|$ up to $8 \cdot 10^3$ GeV$^2$, $|M_{\tilde{L}_{i,j}}|$ up to $6 \cdot 10^3$ GeV$^2$ and $|A_{i,j} v_d|$ up to 650 GeV$^2$ compatible with the constraints. In most cases, one of the mass squared parameters is at least one order of magnitude larger than all the others. However, there is a sizable part in parameters where at least two of the off-diagonal parameters have the same order of magnitude as shown in Fig. 1.

In what follows, we concentrate on possible LFV signals at a 500 GeV $e^+e^-$ collider: $e_{\mu} \tilde{\nu}_{\tau}$, $e_{\tau} \tilde{\nu}_{\mu}$, $\mu_{\tau} \tilde{\nu}_{\tau}$, as well as the possibility of two additional jets. We consider the following SUSY
FIG. 1: Ranges for parameters inducing lepton number violation.

processes: $e^+ e^- \rightarrow \tilde{l}_i^- \tilde{l}_j^+, \tilde{\nu}_i \tilde{\nu}_j, \chi^0_m, \tilde{\chi}_n^+ \tilde{\chi}_n^- \rightarrow \chi^0_m$, as well as stop and Higgs production. We take into account all possible SUSY and Higgs cascade decays. We have taken into account ISR- and SUSY-QCD corrections for the production cross sections.

The main sources for the LFV signal stem from production of sleptons, sneutrinos and their decays, for example:

$$e^+ e^- \rightarrow \tilde{l}_i^- \tilde{l}_j^+ \rightarrow \tilde{l}_m^- \tilde{l}_m^+ \tilde{\chi}_1^0.$$  

(7)

We have also included the oscillation between flavors, being important in the case that $\Delta m^2 < m \Gamma$ [14, 19].

For the background we take into account all possible SUSY cascade decays faking the signal and the Standard Model background from $W$-boson pair production, $t$-quark pair production and $\tau$-lepton pair production. The SM background has been calculated with the program Whizard [20]. A SUSY background reaction is, for example, the chain $\tilde{\chi}_r^0 \rightarrow \tilde{l}_j^- \nu_i \tilde{\chi}_l^+ \rightarrow \tilde{l}_m^- \nu_i \nu_j \chi_m^0$. We have generated 8000 points consistent with the experimental, varying the parameters randomly on a logarithmic scale: $10^{-8} \leq |A_{ij}| \leq 50$ GeV, $10^{-8} \leq M_{ij}^2 \leq 10^4$ GeV$^2$. About 1200 of these have at least one signal larger than 0.1 fb. In
TABLE III: Maximal cross section in fb for various signals at a 500 GeV $e^+e^-$ collider. The $b$-jets have been excluded.

<table>
<thead>
<tr>
<th>signal</th>
<th>$(0,0)$</th>
<th>(-0.8,-0.6)</th>
<th>(-0.8,0.6)</th>
<th>(0.8,-0.6)</th>
<th>(0.8,0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^\pm \mu^\mp H_T$</td>
<td>149</td>
<td>67</td>
<td>208</td>
<td>231</td>
<td>71</td>
</tr>
<tr>
<td>$e^\pm \tau^\mp H_T$</td>
<td>178</td>
<td>93</td>
<td>220</td>
<td>248</td>
<td>72</td>
</tr>
<tr>
<td>$\mu^\pm \tau^\mp H_T$</td>
<td>61</td>
<td>56</td>
<td>127</td>
<td>115</td>
<td>42</td>
</tr>
</tbody>
</table>

Table III we present the maximal cross section for various signals with a cross section larger than $10^{-2}$ fb. The cross section for $e^\pm H_T$ can go up to 250 fb leading to about $10^5$ events with a luminosity of 500 fb$^{-1}$. In the case of two leptons with different flavors and 2 jets, we have put a veto on $b$-jets because of the large background stemming from $t$-quark production. One observes that the cross section for $\mu^\pm \tau^\mp H_T$ is somewhat smaller than the cross section for $e^\pm \mu^\mp H_T$ and $e^\pm \tau^\mp H_T$. The reason for this is that $\tilde{e}\tilde{e}$ production is larger than $\tilde{\mu}\tilde{\mu}$ ($\tilde{\tau}\tilde{\tau}$) due to the additional t-channel contribution.

In Fig. 2a we show the cross section in fb of $e^+e^- \rightarrow e^\pm \tau^\mp H_T$ as a function of $BR(\tau^- \rightarrow e^-\gamma)$ and in Fig. 2b the ratio signal over square root of the background ($S/\sqrt{B}$) as a function $BR(\tau^- \rightarrow e^-\gamma)$ assuming an integrated luminosity of 100 fb$^{-1}$. Although no cuts have been applied, there is in most cases a spectacular signal. The cases where the ratio $S/\sqrt{B}$ is of order 1 or smaller should clearly improve, once appropriate cuts are applied. For example, a cut on the angular distribution of the final state leptons will strongly reduce the $WW$ background. Further cuts as applied in the study of slepton production [21] will enhance the ratio $S/\sqrt{B}$.

There is an accumulation of points in Fig. 2 along a band. These points are characterized by large $\tilde{e}_R\tau_R$ mixing which is less constraint by $\tau^- \rightarrow e^-\gamma$ than the corresponding left-left or left-right mixing.

In Fig. 3 we study the dependence of the signal on the collider energy for different beam polarizations. The various kinks are due to the onset of the different production cross
FIG. 2: (a) Cross section in fb for the signal $e^\pm\tau^\mp$ + missing transverse momentum and (b) the ratio signal over square root of background as a function of $\text{BR}(\tau \rightarrow e\gamma)$ for $\sqrt{s} = 500$ GeV, $P_{\tau^-} = 0$ and $P_{\tau^+} = 0$. In the latter case we have assumed an integrated luminosity of 100 fb$^{-1}$.

sections. One observes a strong dependence on the beam polarization. Beam polarization is not only useful for a possible reduction of the background, but might also serve as a possible tool to disentangle different contributions to the signal. We have chosen a point giving rise to a LFV signal in several channels. The largest flavor violation entries are in this case $M_{E,13}^2 = 3440$ GeV$^2$, $M_{L,12}^2 = -2.4$ GeV$^2$, $v_1 A_{31} = 1.8$ GeV$^2$. In this particular example, the branching ratios for the rare lepton decays are within the reach of the next generation of experiments, e.g. $\text{BR}(\tau^- \rightarrow e^-\gamma) = 2.6 \cdot 10^{-7}$.

For other points of the parameter space there will be only one or two channels with large LFV signals. However, the point chosen demonstrates the general behavior of strong beam polarization dependence of the various signals and is therefore quite representative. In the case of additional jets in the final state, the cross section is lower as can be seen
FIG. 3: The cross sections in fb for the signals \( \sigma(e^\pm \mu^\mp H_f) \) \( \text{(a)} \), \( \sigma(e^\pm \tau^\mp H_f) \) \( \text{(b)} \), and \( \sigma(\mu^\pm \tau^\mp H_f) \) \( \text{(c)} \) as a function of \( \sqrt{s} \) for different beam polarizations. The different lines correspond to \( (P_-, P_+) = (0, 0) \) (full), \( (P_-, P_+) = (-0.8, -0.6) \) (dashed), \( (P_-, P_+) = (-0.8, 0.6) \) (dashed dotted), \( (P_-, P_+) = (0.8, -0.6) \) (long dashed) and \( (P_-, P_+) = (0.8, 0.6) \) (dotted).

From Table III. At large values of \( \sqrt{s} \) there are of course more open channels due to the production of squarks. The corresponding LFV signals also show a pronounced dependence on beam polarization \([17] \).

In conclusion, we have shown that the most general flavor violating structure of the slepton and sneutrino mass matrix may lead to large lepton flavor violating signals at a future \( e^+ e^- \) collider – despite the strong constraints on rare lepton decays.

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