Dual Brane Pairs, Chains and the Bekenstein-Hawking Entropy

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Abstract

A proposal towards a microscopic understanding of the Bekenstein-Hawking entropy for D=4 spacetimes with event horizon is made. Since we will not rely on supersymmetry these spacetimes need not be supersymmetric. Euclidean D-branes which wrap the event horizon’s boundary will play an important role. After arguing for a discretization of the Euclidean D-brane worldvolume based on the worldvolume uncertainty relation, we count chainlike excitations on the worldvolume of specific dual Euclidean brane pairs. Without the need for supersymmetry it is shown that one can thus reproduce the D=4 Bekenstein-Hawking entropy and its logarithmic correction.

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1 Introduction

Three decades ago it was proposed by Bekenstein [1] to associate a physical entropy with a black hole. This entropy was argued to be proportional to the area $A_H$ of the black hole’s horizon. Evidence for this proposal came from earlier work of Christodoulou [2] and Hawking [3]. Christodoulou had shown that for physical processes which result in the absorption of a particle by a Kerr black hole its so-called irreducible mass cannot decrease but only increase. The irreducible mass is proportional to $\sqrt{A_H}$. Hawking, then followed with a general proof that $A_H$ cannot decrease in any classical physical process.

For a consistent description of black holes by thermodynamic quantities it is necessary that they emit thermal radiation at a temperature compatible with the laws of thermodynamics. This was indeed found by Hawking [4] and required the inclusion of quantum effects. With the derived value for the Hawking-temperature $T_H$ it was then possible to fix the proportionality constant in the entropy-area relation and to assign to each black hole the Bekenstein-Hawking (BH) entropy

$$S_{BH} = \frac{A_H}{4G_4} \frac{k_B c^3}{\hbar}.$$  (1)

The appearance of Planck’s Constant already points to the fact – as suggested by Statistical Mechanics – that one should better understand and count the microscopic quantum mechanical degrees of freedom leading to the formation of the black hole in order to understand the BH-entropy at a fundamental level.

Later it was shown by Gibbons and Hawking [5] that (1) also applies to cosmological de Sitter event horizons to which one can similarly ascribe a thermal Hawking-temperature. Thus black hole and cosmological event horizons should share a common underlying microscopic property giving rise to the same universal expression for their entropy. It is the aim of this paper to propose a set of microscopic states which can account for the BH-entropy and its logarithmic corrections in a rather universal way. We will lay out here the general framework and reserve the particular application to Schwarzschild black holes and de Sitter cosmologies to future work.

Both string-theory and the quantum geometry program made decisive steps in recent years towards an identification of the microscopic black hole states. While efforts in string-theory, which increased dramatically after the work of [6], more or less focussed on supersymmetric black holes or small deviations thereof, the Quantum Geometry approach led to the derivation of the BH-entropy for the D=4 Schwarzschild case [7] but had to fix
2 Dual Brane Pairs and the BH-Entropy

String-Theory Case: Consider type II string-theory on a D=10 spacetime with Lorentzian signature which factorizes into $\mathcal{M}^{1,3} \times \mathcal{M}^{p-1} \times \mathcal{M}^{7-p} \ (p = 1, \ldots, 5)$ and is described by a metric

$$ds^2 = g_{\mu\nu}^{(1,3)}(x^\rho)dx^\mu dx^\nu + g_{ab}^{(p-1)}(x^c)dx^adx^b + g_{kl}^{(7-p)}(x^m)dx^k dx^l,$$

where

$$\mu, \nu = 0, \ldots, 3; \quad a, b, c = 1, \ldots, p - 1; \quad k, l, m = 1, \ldots, 7 - p.$$

The metric $g_{\mu\nu}$ describes a D=4 spacetime geometry of which we assume that it possesses an event horizon with associated BH-entropy. The 2-surface $H$ will represent the horizon’s boundary (not to be confused with $H^+$, the future event horizon, or $H^-$ the past event horizon; in the case of a D=4 Schwarzschild black hole the boundary $H$ is a 2-sphere $S^2_H$ defined as the intersection of the future event horizon $H^+$ with a partial Cauchy surface ending at spatial infinity $I^0$ in the exterior black hole spacetime). Moreover, to describe a compactification from D=10 down to D=4 we take the two internal $\mathcal{M}^i$ to be compact. A special example with constant internal metric would be a $T^6 = T^{p-1} \times T^{7-p}$ torus-compactification. For these backgrounds the effective D=4 Newton’s Constant is related to the Regge slope $\alpha'$ and the string coupling constant $g_s$ through (we employ conventions as given in [11])

$$G_4 = \frac{G_{10}}{V_{p-1}V_{7-p}} = \frac{(2\pi)^6\alpha'^4 g_s^2}{8V_{p-1}V_{7-p}},$$

where $V_i = vol(\mathcal{M}^i) \equiv \int_{\mathcal{M}^i} d^i x \sqrt{g^{(i)}}$. 

an undetermined multiplicative factor, the Barbero-Immirzi parameter which arises from an ambiguity in the loop quantisation procedure, appropriately. Starting from string-resp. M-theory we want to take here a different route trying to address the problem of understanding the BH-entropy directly. One of the interesting features of our proposal will be that although starting from branes in string-theory we are led to chain-like excitations on the branes’ worldvolume which resemble the polymer-like excitations of Quantum Geometry (see e.g. [8]). This raises at least the hope that there might be some reconciliation between these two major approaches to quantum gravity. We will focus in this work on the case of D=4 spacetimes with event horizons. The generalization to higher dimensions is presented in [9] while further aspects of black holes are addressed in [10].
Imagine now wrapping two orthogonal Euclidean ‘electric-magnetic’ dual branes, $Dp$ and $D(6-p)$, around $H \times M^{p-1}$ and $M^{7-p}$, respectively. So, together the two branes cover the whole internal space plus the area of the exterior D=4 spacetime’s boundary. For such a dual brane pair it follows from the Dirac-quantisation condition \[12\] that the product of their tensions is given by

$$\tau_{Dp} \tau_{D(6-p)} = \frac{1}{(2\pi)^6 \alpha'^4 g_s^2}. \quad (4)$$

Thus we can write

$$\frac{1}{G_4} = 8(\tau_{Dp} V_{p-1}) (\tau_{D(6-p)} V_{7-p}) . \quad (5)$$

Due to the fact that part of the $Dp$ brane wraps the full area of the D=4 spacetime’s boundary we may rewrite the BH-entropy of the D=4 spacetime as

$$S_{BH} = \frac{A_H}{4G_4} = 2S_{Dp} S_{D(6-p)}, \quad (6)$$

where

$$S_{Dp} = \tau_{Dp} \int_{H \times M^{p-1}} d^{p+1} x \sqrt{|\det g|}, \quad S_{D(6-p)} = \tau_{D(6-p)} \int_{M^{7-p}} d^{7-p} x \sqrt{|\det g|}. \quad (7)$$

are the respective Nambu-Goto actions of the dual branes. For our later analysis it turns out to be necessary to get rid of the factor two on the rhs of (6). This can easily be achieved by considering another Euclidean brane pair.

Let us therefore take a second dual Euclidean brane pair $Dq - D(6-q)$, $(q = 1, \ldots, 5)$ and consider a D=10 spacetime background of the form $M^{1,3} \times M^{p-1} \times M^{q-p} \times M^{7-q}$ (without loss of generality we can assume that $p \leq q$) with internal metric

$$g_{\alpha\beta}^{(p-1)} (x^c) dx^\alpha dx^\beta + g_{qr}^{(q-p)} (x^s) dx^q dx^r + g_{uv}^{(7-q)} (x^w) dx^u dx^v. \quad (8)$$

We wrap the branes of the first pair $Dp$, $D(6-p)$ around $H \times M^{p-1}$ resp. $M^{q-p} \times M^{7-q}$ while the branes of the second pair, $Dq$ and $D(6-q)$ wrap $H \times M^{p-1} \times M^{q-p}$ resp. $M^{7-q}$. By repeating the steps which led to (6) one may now express the D=4 BH-entropy as

$$S_{BH} = S_{Dp} S_{D(6-p)} + S_{Dq} S_{D(6-q)}. \quad (9)$$

Notice further that we are free to exchange any of the appearing D-branes by its anti-D-brane and still arrive at the same expression (9). This option becomes important when one wants to address uncharged non-supersymmetric D=4 spacetimes. We can therefore finally state that for all such dual Euclidean brane configurations the D=4 BH-entropy
can be universally, i.e. irrespective of the particular choice of dual brane pairs, rewritten as

$$ S_{BH} = \sum_{i=1,2} S_{E_i} S_{M_i}, $$

where

$$(E_i, M_i) \in \{(Dp_i, D(6 - p_i)), (\overline{Dp}_i, D(6 - p_i)), (Dp_i, \overline{D}(6 - p_i)), (\overline{Dp}_i, \overline{D}(6 - p_i))\}$$

and $S_{E_i}, S_{M_i}$ are the respective Nambu-Goto actions for these branes. The connection between a specific pair of branes and a specific D=4 spacetime should be established on the basis of the D=10 spacetime which the pair of branes creates as gravitational sources followed by a dimensional reduction to four dimensions.

Let us make two comments. First, until now we have restricted the range of the D-brane dimensions to $p_i = 1, \ldots , 5$. The reason being that a Euclidean D0 brane cannot cover the whole $H$. Therefore, if e.g. $H = S^2$ the metric of the sphere does not factorize into two independent 1-dimensional parts and one therefore cannot write a product of two Nambu-Goto actions in this situation. However, there is no problem with the case where a $D6$ or $\overline{D}6$ wraps $H \times M^5$ and the dual $D0$ or $\overline{D}0$ the remaining internal $M^1 = S^1$. With this subtlety in mind we can extend the range to $p_i = 0, \ldots , 6$. Second, since $\tau_{F1} \tau_{NS5} = \tau_{Dp} \tau_{D(6-p)}$ we could also extend our treatment to incorporate Euclidean ($F1, NS5$) as dual pairs. It is easy to see that wrapping a Euclidean fundamental string $F1$ over $H$ (resp. the internal $M^3$) and the dual Euclidean $NS5$ on the complete internal space $M^6$ (resp. $H \times M^4$) in combination with a second $D1$-$D5$ pair or another ($F1, NS5$) pair leaves (10) intact. Hence we can enlarge the set of dual pairs (11) to include also

$$(F1, NS5).$$

Moreover both $F1$ and $NS5$ can be replaced independently by their NS-NS charge reversed antipartners.

**M-Theory Case:** Since (10) works so universally for all dual pairs of type II string-theory one should expect the formula also to hold true for the unique M-theory dual brane pair, the $(M2, M5)$ pair. This is what we will show now. Let us start again with a single Euclidean $(M2, M5)$ pair where the $M2$ wraps the boundary $H$ associated with the D=4 spacetime’s horizon plus an internal $S^1$ while the Euclidean $M5$ wraps the remaining internal six-space $M_6$. That means we assume the metric of the D=11 spacetime to factorize into the direct product structure $M^{1,3} \times S^1 \times M^6$. 
The tensions of the dual branes satisfy \((l_{Pl})\) is the D=11 Planck-length\)

\[
\tau_{M2}\tau_{M5} = \frac{1}{(2\pi)^7 l_{Pl}^9} \tag{13}
\]

and \(G_4\) can be expressed in terms of the D=11 Newton’s constant as

\[
G_4 = \frac{G_{11}}{L V_6} = \frac{(2\pi)^7 l_{Pl}^9}{8 L V_6} \tag{14}
\]

where \(L = vol(S^1), V_6 = vol(M^6)\). So combining these two equations we obtain

\[
\frac{1}{G_4} = 8(\tau_{M2} L)(\tau_{M5} V_6) \tag{15}
\]

Because the Euclidean \(M2\) partly wraps the full boundary area \(A_H = vol(H)\) we can rewrite the D=4 BH-entropy again in terms of the Euclidean Nambu-Goto actions of the respective branes

\[
S_{BH} = \frac{A_H}{4G_4} = 2S_{M2} S_{M5} \tag{16}
\]

In the same manner as for the D-brane case in D=10 we will now add a second dual Euclidean pair \((M2, M5)\) wrapping likewise the complete \(H \times S^1 \times M^6\). This once again allows us to rewrite the D=4 BH-entropy exclusively in terms of the Nambu-Goto actions of the involved M-branes

\[
S_{BH} = \sum_{i=1,2} S_{M2_i} S_{M5_i} \tag{17}
\]

eliminating the prefactor two. Instead of wrapping each \(M2\) around \(H\) we could also consider wrapping one or both of the \(M5\)’s around \(H\) instead. This will lead to internal geometries which factorize like

\[
S^1 \times M^3_{(1)} \times M^3_{(2)} , ~ M^4 \times M^3 \tag{18}
\]

instead of \(S^1 \times M^6\). It is easy to see that following the same reasoning as before we end up again with \((17)\) in these situations. Finally replacing any brane by its anti-brane doesn’t change \((17)\) because the Nambu-Goto actions stay invariant under this replacement. So we can conclude that also in M-theory \((10)\) holds true, this time for

\[
(E_i, M_i) \in \{ (M2, M5), (\overline{M2}, M5), (M2, \overline{M5}), (\overline{M2}, \overline{M5}) \} . \tag{19}
\]
3 Chain-States and Counting of States

So far we have achieved a universal rewriting of the D=4 BH-entropy in terms of dual pair doublets of type II string-theory and M-theory. Let us now see in which way the introduction of the Euclidean pairs can help us to understand the BH-entropy at a microscopic level by counting an appropriate set of states related to these pairs.

To set the stage for a proposal of what the microscopic states in the strongly-coupled regime might be, let us briefly reflect upon the tension of a brane. Usually a brane’s tension $\tau_{Dp}$ (we take a D-brane for definiteness but the following considerations apply as well to the $\overline{Dp}$ plus the Euclidean $F1, NS5, M2, M5$ and their antipartners) is conceived as the brane’s mass per unit $p$-volume. This point of view is natural if the brane’s worldvolume has Lorentzian signature and stresses the split into one time and $p$ space dimensions. However, when dealing with Euclidean branes it is more natural to treat all $p + 1$ space dimensions on an equal footing. To account for this let us write the brane’s tension as a volume $v_{Dp}$ (with corresponding length-scale $l_{Dp}$)

$$\tau_{Dp} \equiv \frac{1}{v_{Dp}} = \frac{1}{l_{Dp}^{p+1}}.$$  \hspace{1cm} (20)

Our basic proposition is that the volume $v_{Dp}$ constitutes a smallest volume unit within the worldvolume of the Euclidean Dp-brane. Indeed, it suffices to assume that it is only the entropy-carrying chains to be introduced shortly which cannot resolve a worldvolume smaller than $v_{Dp}$. This is analogous to the statement that the weakly-coupled fundamental string cannot resolve length-scales shorter than the string-scale $\sqrt{\alpha'}$.

Evidence for such a smallest volume unit on a brane’s worldvolume comes from the ‘worldvolume uncertainty relation’ for D-branes [13] as we will now explain (cf. also the discussion in [15]). It is well-known that a D-brane in the presence of a background magnetic flux along its worldvolume acquires a non-commutative geometry [14]. The non-trivial commutator

$$[X^i, X^j] = 2\pi i \alpha' F^{ij}$$  \hspace{1cm} (21)

(with $F = B - dA$ the difference of the NS-NS 2-form potential and the $U(1)$ gauge field strength on the brane) of the D-brane’s longitudinal coordinates $X^i$ already suggests a worldvolume uncertainty relation. Furthermore, it was shown in [13] that a non-trivial expectation value for $F$ can also arise from integrating out quantum fluctuations around a classical background, even when a background flux $F$ is absent. The proper framework
to describe this result is string field theory, where one describes the D-brane through a normalized wave-function $\Psi(X)$ with $X$ the brane’s target space coordinates. The quadratic deviation is then given by the functional integral

$$(\Delta X^i)^2 = \int [DX] \Psi(X) \Psi^*(X) (X^i - \bar{X}^i)^2 \Psi(X)$$

with average value

$$\bar{X}^i = \int [DX] \Psi(X) \Psi^*(X) X^i \Psi(X) .$$

Moreover, one notices that $F$ as a background field is independent of $\Psi$. From the commutator $[X^i, X^j] = 2\pi i \alpha' F^{ij}$ one therefore obtains via the standard quantum mechanical procedure the relation

$$\Delta X^i \Delta X^j \geq 2 \pi \alpha' |F^{ij}| .$$

Let us now come to the uncertainty in $X^i$ which is defined through

$$\delta X^i = \langle (\Delta X^i)^2 \rangle^{1/2} ,$$

where the expectation value is determined via the string field path integral

$$\langle (\Delta X^i)^2 \rangle = \frac{1}{Z} \int [DB] e^{-S} (\Delta X^i)^2 .$$

Here $B$ comprises all component fields contained in $\Psi$ which includes the metric and $B$. From the Cauchy-Schwarz inequality one obtains $\langle \delta X^i \delta X^j \rangle \geq |\langle \Delta X^i \Delta X^j \rangle|^2$. Therefore the product of the uncertainties of two different brane coordinates becomes lower-bounded by the expectation value for $|F^{ij}|$

$$\delta X^i \delta X^j \geq 2 \pi \alpha' \langle |F^{ij}| \rangle .$$

In [13] the expectation value $\langle |F^{ij}| \rangle$ has been calculated for the $D1$-brane case by using for the action $S$ in eq. (26) the supergravity action which is a valid approximation at small string coupling. It was found that even in the absence of a background $F$ field, quantum fluctuations lead to a non-trivial expectation value and thus an uncertainty relation among the $D1$-brane worldvolume coordinates. In this case with no explicit background flux the expectation value becomes a simple expression in terms of the string coupling constant $g_s$ and $\alpha'$. The result for other $Dp$-branes plus the M-theory $M2$ and $M5$-branes was then inferred by string-duality arguments. When one expresses all these worldvolume uncertainty relations for various branes in terms of their tensions, as was done in [13], [16], it leads to the following result for any brane with $p + 1$ dimensional worldvolume

$$\delta X^0 \ldots \delta X^p \gtrsim \frac{1}{\tau} ,$$

7
valid for all $Dp$-branes and $M2$, $M5$ with $\tau$ the respective brane’s tension. One therefore sees that the smallest volume allowed by this brane worldvolume uncertainty principle is indeed given by the inverse of the brane’s tension which provides nice evidence for our assumption.

Equipped with this notion of a smallest world volume unit which we will term a cell henceforth, we are naturally led to think of the Euclidean $Dp$-brane as being composed out of $N_{Dp}$ cells, where $N_{Dp}$ is measured by the brane’s Nambu-Goto action

$$N_{Dp} = \tau_{Dp} \int d^{p+1}x \sqrt{\det g} .$$

More precisely, since the Nambu-Goto action is ordinarily assumed to take smooth continuous values, one should regard the Nambu-Goto action as an approximation to a more fundamental microscopic integer-valued function. This latter function would give the number of cells contained in the brane’s worldvolume but will be well approximated by the smooth Nambu-Goto action when the number of cells becomes large and the discrete cell structure becomes quasi-continuous. This large-cell limit is the case we are interested in here.

So each brane pair $(Dp,D(6-p))$ having mutually orthogonal branes possesses $N_{Dp} \times N_{D(6-p)}$ cells. Thus altogether the doublet of dual Euclidean pairs used for the reformulation of the BH-entropy exhibits a total of

$$N = \sum_{i=1,2} N_{Ei} N_{Mi}$$

cells on its combined worldvolume (counting cells on $(E_1, M_1)$ and $(E_2, M_2)$ separately). Therefore, by virtue of (29) and its generalization to $F1, NS5, M2, M5$ plus antipartners we see that (10) becomes

$$S_{BH} = \sum_{i=1,2} N_{Ei} N_{Mi} \equiv N .$$

Let us now conceive on the lattice of the combined ‘$(E_1, M_1) + (E_2, M_2)$’ brane worldvolume an $(N-1)$-chain, i.e. a chain composed out of $N-1$ successive links where we allow all links to start and end on any occurring cell (see fig.1). In particular a link might start and end on the same cell thus creating a loop. Altogether the number of such chains is $N^N$.

Our prime motivation to consider long (having about the same number of links as there are cells in the lattice) chains comes from the following heuristic reasoning. The
‘worldvolume uncertainty principle’ for branes led to a smallest resolvable worldvolume \( v_{Dp} = 1/\tau_{Dp} \) given by the inverse of the brane’s tension. Hence it implies a minimal resolvable length-scale \( l_{Dp} = v_{Dp}^{1/(p+1)} \). Now, such a minimal length-scale leads via the ordinary Heisenberg uncertainty principle of quantum mechanics, involving space coordinates and their conjugate momenta, to a non-vanishing momentum \( \Delta P \simeq 1/l_{Dp} \). For any relativistic object, for which we can equate energy with momentum, we are therefore led to a corresponding energy of magnitude

\[
\Delta E \simeq \Delta P \simeq \frac{1}{l_{Dp}} = (\tau_{Dp})^{1/(p+1)} = \frac{1}{\sqrt{\alpha'(2\pi)^p} (g_s(2\pi)^p)^{1/(p+1)}}. \tag{32}
\]

If we associate with \( \Delta E \) a temperature, we see that in the strong-coupling regime, where \( g_s \simeq 1 \), this temperature is of the size of the Hagedorn-temperature. Close to this temperature we know from experience with weakly coupled string-theory that it is entropically favourable to allocate the energy of the system to just one single long string instead of distributing the energy more democratically in smaller portions to lots of small strings (for a review see \cite{17}; the relation between entropy and the length of a string is discussed in \cite{18}). This motivates us to consider as candidates for microscopic excitations chains which are long\(^2\).

\(^2\)While long chains are important for gravitational aspects, short chains composed of just two links showed up in standard model like constructions \cite{19} based on warped backgrounds \cite{20}.

The chain-counting until now assumed that all cells were distinguishable. This however is likely to be changed in a quantum treatment of the problem. Here the cells would have to be regarded as ‘partons’, i.e. indistinguishable bosonic degrees of freedom. The cure

\[ \text{Figure 1: Constructive view of the } (N-1)\text{-chain where we arrange all cells of the lattice in a column and use } N \text{ copies of them. We allow each link to connect any cell of a column with any cell of the succeeding column. Horizontal links correspond to loops.} \]
for this is well-known from statistical mechanics. We have to divide the classical number of states through the quantum mechanical Gibbs-correction factor $N!$ to account for the indistinguishability of the bosonic cells. Thus, with this quantum-mechanical correction we obtain the number of chain-states

$$\Omega(N) = \frac{N^N}{N!}.$$  \hfill (33)

Let us now determine from $\Omega(N)$ the entropy $S_c$ of the chain-states in the thermodynamic large $N$ limit. Using Stirling’s approximation, $\ln(N!) = N \ln N - N + \mathcal{O}(\ln N)$, we obtain

$$S_c = \ln \Omega(N) = N + \mathcal{O}(\ln N).$$  \hfill (34)

By virtue of (31) $N$ is however nothing else but the BH-entropy such that finally we get

$$S_c = S_{BH} + \mathcal{O}(\ln S_{BH}).$$  \hfill (35)

Thus the entropy of the proposed chain-states coincides exactly, up to a logarithmic correction, with the semiclassical D=4 BH-entropy. Notice that in this approach there is no need to fix the proportionality constant as is the case in many other approaches which derive the BH-entropy.

## 4 Corrections to the BH Area Law

Recently also corrections to the BH area-law became available. Corrections for D=4 black holes have been determined in supersymmetric cases from string-theory (see e.g. [21]) while results in non-supersymmetric cases came from the Quantum Geometry approach [22] or the Conformal Field Theory (CFT) approach of Carlip [23]. The general result is a logarithmic correction, $-k \ln S_{BH}$ with a positive constant $k > 0$. The appearance of a negative correction can be attributed to the Holographic Principle [24] as emphasized in [25]. For example in the CFT approach by determining corrections to the Cardy formula [26] one arrives at an entropy

$$S_{CFT} = S_{BH} - \frac{3}{2} \ln S_{BH} + \ln c + \text{const}$$  \hfill (36)

for a class of D=4 black holes [23]. Now $c$ the central charge is given by

$$c = \frac{3A_H \gamma}{2\pi G_4 \kappa},$$  \hfill (37)
where $\kappa$ is the black hole’s surface gravity and $\gamma$ an undetermined periodicity parameter. If one could assume that $\gamma$ could be chosen such that $c$ is a constant, independent of $A_H$, then $k = 3/2$. However, it has been demonstrated in [27] that $\gamma$ should equal $2\pi T_H$ with $T_H$ the Hawking-temperature. Using $T_H = \frac{\kappa}{2\pi}$ one then arrives at the value $k = 1/2$.

Before addressing corrections to the BH-entropy let us emphasize that we are working throughout this paper with a microcanonical ensemble in equilibrium. That means we are considering the chains as quantum microstates which share the same fixed energy. It can be easily seen e.g. for the Schwarzschild black hole that its energy/mass depends like $M_{BH} \propto \sqrt{N}$ on $N$ (see [28]). Hence fixing the energy amounts to fixing $N$. The microscropic entropy coming from chain states $S_c$ is then simply defined as the logarithm of the number of chains with same fixed energy $E$ resp. same $N$. This number is given by $\Omega(N)$. The corrections which we will study will arise from using more accurate approximations to $\Omega(N)$ in the large $N \gg 1$ regime.

We had found the expression (33) for the number of states and derived from it the chain’s entropy at leading order in $N$ by using Stirling’s approximation for $N!$. This gave agreement with the BH-entropy at this leading order. Obviously a more accurate evaluation of the chain’s entropy and hence corrections to the BH-entropy will arise from taking a more accurate approximation for $N!$ using the Stirling series to higher orders. This will give a more accurate evaluation of $\Omega(N)$. For instance if we take [29]

$$N! = \sqrt{2\pi N}N^Ne^{-N} \left(1 + \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right)\right)$$

we obtain for the microcanonical chain-entropy

$$S_c = \ln \Omega(N) = N - \frac{1}{2} \ln N - \ln \sqrt{2\pi} - \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right).$$

By means of the identification (31) this leads to the corrected chain-entropy formula

$$S_c = S_{BH} - \frac{1}{2} \ln S_{BH} - \ln \sqrt{2\pi} - \frac{1}{12S_{BH}} + \mathcal{O}\left(\frac{1}{S_{BH}^2}\right).$$

The chain-entropy gives therefore not only the expected leading logarithmic correction term but also agrees quantitatively with $k = 1/2$. Moreover, the first three correction terms are negative in accord with the aforementioned restriction from the Holographic Principle.
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