Genuine Three-Body Bose-Einstein Correlations and Percolation of Strings

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Abstract

Recent data on the genuine three-body Bose-Einstein correlations are consistent with fully coherent pion production in S-Pb collisions but with uncoherent pion production in central Pb-Pb collisions. These results, unexpected from conventional approaches, are naturally explained by the percolation of colour strings produced in the collisions and subsequent incoherent fragmentation of the formed clusters.

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The strength of the genuine three-particle Bose-Einstein correlations can be measured by the weight factor $\omega$ introduced in [1]:

$$\omega = \frac{C_3(q) - 1 - C_2(q_{12}) - 1 - C_2(q_{23}) - 1 - C_2(q_{31}) - 1}{2\sqrt{C_2(q_{12}) - 1\{C_2(q_{23}) - 1\}{C_2(q_{31}) - 1}}}$$

(1)

Here $C_2$ and $C_3$ are the two- and three-body correlation functions [3] and

$$q_{ij} = q_i - q_j, \quad q^2 = q_{12}^2 + q_{23}^2 + q_{31}^2$$

(2)

The weight-factor $\omega$ has been experimentally studied in $e^+e^-$ collisions by L3 collaboration, with the result consistent with $\omega = 1$ [2]. It has also been studied in heavy-ion collisions. NA44 collaboration [3,4] has obtained $\omega = 0.20 \pm 0.02 \pm 0.19$ for SPb collisions, i.e. no genuine three-body correlations have been found. This is interpreted as evidence of coherence. On the other hand, the same experiment with the same methodology has found $\omega = 0.85 \pm 0.02 \pm 0.21$ for central Pb-Pb collisions. This value is compatible with $\omega = 0.606 \pm 0.005 \pm 0.178$ earlier reported by WA98 collaboration [5].

It has been recently suggested [6] that an explanation of these data seems to be in line with the behaviour of the chaoticity parameter $\lambda$ which measures the strength of the two-body Bose-Einstein correlations

$$\lambda = C_2(q = 0) - 1$$

(3)

Experimental data in heavy-ion collisions show that for moderate atomic numbers of colliding nuclei $\lambda$ decreases with atomic number, as expected from the corresponding increase of the number of independent incoherent sources [7,8]. Indeed going from O-C to O-Cu, O-Ag and O-Au, $\lambda$ falls from 0.79 to 0.32 [9]. However for heavier nuclei $\lambda$ no longer decreases and eventually starts to increase. For S-Pb and Pb-Pb collisions NA44 obtains $\lambda = 0.56$ and 0.59 respectively [10]. Similar values have been found at RHIC for Au-Au collisions at $\sqrt{s} = 130$ GeV [11].

This behaviour can be understood assuming that in a collision colour strings are formed stretched between the projectile and target, which then break due to formation of quark-antiquark pairs. Each colour string is assumed to have a finite transverse dimension of area
$S_1 = \pi r_0^2$ \((r_0 \simeq 0.2 \text{ fm})\). As the energy and/or atomic number of the projectile and target increase the number and density of strings grows, so that they start to overlap, forming clusters, which act as new effective sources of particle production. Both the strings and their clusters can be assumed to be totally chaotic sources with $\lambda = 1$ [12]. Assuming that for particles coming from different strings there are no Bose-Einstein correlations [13] one then obtains [14]

$$\lambda = \frac{n_S}{n_T}$$

where $n_S$ and $n_T$ are the average numbers of particle pairs produced in a given rapidity and transverse momentum range from the same cluster and from all the clusters respectively.

It is clear from (4) that $\lambda$ decreases with the number of incoherent sources (clusters). For very large energies and/or atomic numbers the clusterization process will diminish the number of independent sources (asymptotically to unity). As a consequence the chaoticity parameter $\lambda$ will grow.

This approach may be realized in the framework of different scenarios depending on the assumed form of the interaction between strings at close distances [15,16]. If the area of a cluster is formed by the geometrical sum of overlapping strings then a phase transition is observed when the string density $\eta$ reaches a critical value $\eta_c$ [17]. This percolation phase transition corresponds to the appearance of at least one cluster which spans the whole interaction transverse area. The value of $\eta_c$ lies in the interval 1.17-1.5 depending on the form of the profile functions of the colliding nuclei. Staying within this percolation scenario, the dynamics of the string interaction still admits different possibilities. The observed behaviour of $\lambda$ favours considering each cluster as a single string with a higher colour given by the vectorial sum of colours of the overlapping strings times a factor which takes into account the degree of overlapping. As a result, the number of particles $\mu_n$ produced by a cluster of area $S_n$ formed by $n$ strings is given by

$$\mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1$$

(5)
where $\mu_1$ is the number of particles produced by a simple string. In the case of total overlapping $S_n = S_1$ and $\mu_n = \sqrt{n}\mu_1$. In the opposite case when strings just touch each other $S_n = nS_1$ and one gets $\mu_n = n\mu_1$ as expected.

To calculate $\omega$ we have to know

$$\lambda_3 = C_3(q = 0) - 1. \quad (6)$$

Under the above mentioned assumptions, similarly to (3), we have

$$\lambda_3 = 5\frac{n'_S}{n'_T} \quad (7)$$

where now $n'_S$ and $n'_T$ are the average numbers of particle triplets produced in a given rapidity and transverse momentum range from the same cluster and from all the clusters respectively.

A completely chaotic cluster of $n$ strings will produce $(1/2)\mu^2_n$ pairs of particles and $(1/6)\mu^3_n$ triplets of particles with $\mu_n$ given by (5). Summing over all formed clusters $i = 1, 2, ... M$ one obtains $n_S$ and $n'_S$. The total number of particles produced from all clusters is obviously

$$\mu = \sum_{i=1}^{M} \mu_n. \quad (8)$$

The total numbers of pairs and triplets are $(1/2)\mu^2$ and $(1/6)\mu^3$ respectively. Thus we find

$$\lambda = \frac{<\sum_{i=1}^{M} n_i S_n/S_1>}{<\left(\sum_{i=1}^{M} \sqrt{n_i S_n/S_1}\right)^2>}, \quad \lambda_3 = 5\frac{<\sum_{i=1}^{M} (n_i S_n/S_1)^{3/2}>}{<\left(\sum_{i=1}^{M} \sqrt{n_i S_n/S_1}\right)^3>} \quad (9)$$

To calculate (9) a Monte-Carlo simulation was performed. We generated $N$ discs of radius $r_0$ inside a circle of radius $R$ corresponding to the interaction area. For sufficiently large number of discs (strings) and large $R$ both $\lambda$ and $\lambda_3$ result depending only on the string density $\eta$ determined by

$$\eta = \frac{N r_0^2}{R^2} \quad (10)$$

Identifying all the formed clusters and determining their areas we obtained $\lambda$ and $\omega$ as functions of $\eta$. Our results are presented in Figs. 1 and 2. The two experimental points
of Fig. 2 correspond to central S-Pb and Pb-Pb collisions at SPS energies. The number of strings for these collisions is obtained from a Monte-Carlo code [19] based on the Quark-Gluon string model with \( r_0 \simeq 0.2 \text{ fm} \) and \( R \) corresponding to the required centrality.

As to the chaoticity parameter \( \lambda \), the obtained behaviour for it is similar to our previous calculations in [18]. Note that there a more elaborate approach was chosen in which we took into account energy-momentum conservation. The energy-momentum of each string was determined from that of the partons at its ends, which in its turn was given by the corresponding structure functions. Energy-momentum conservation limits the number of formed strings with energy sufficient to produce particles. As a result, it shifts the minimum of the curve of Fig. 1 to the right and makes its rise somewhat slowlier.

The dependence of \( \omega \) on \( \eta \) is found to be stronger than for \( \lambda \), which was to be expected due to stronger dependence on the number of sources. This explains why the values of \( \omega \) are measured to be so different for S-Pb and Pb-Pb collisions at the same energy, and this is the reason of the good agreement obtained.

We expect that inclusion of energy-momentum conservation will only slightly modify the shape of the dependence of \( \omega \) on \( \eta \), as is the case of \( \lambda \). Experimental information on \( \omega \) at lower values of \( \eta \) (e.g. from light nuclei or peripheral heavy ion collisions) would be most welcome to verify the predicted change of sign of \( \omega \).

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FIG. 1. The chaoticity parameter $\lambda$ as a function of $\eta$ from Eq. (9). The experimental points are for semi-central S-Pb collisions [3] (filled triangle), 18% central Pb-Pb collisions [4] (nonfilled box) and 10% central Pb-Pb collisions [20] (filled box) at SPS.

FIG. 2. The weight factor $\omega$ as a function of $\eta$ from Eqs. (1) an (9). The experimental points are for semi-central S-Pb collisions [3] (filled triangle) and 9% central Pb-Pb collisions [4] (nonfilled circle) at SPS.