Constraints on Neutrino Mixing Parameters By Observation of Neutrinoless Double Beta Decay

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Abstract

Assuming positive observation of neutrinoless double beta decay together with the CHOOZ reactor bound, we derive constraints imposed on neutrino mixing parameters, the solar mixing angle $\theta_{12}$ and the observable mass parameter $\langle m \rangle_\beta$ in single beta decay experiments. We show that $0.11 \text{ eV} \leq \langle m \rangle_\beta \leq 1.3 \text{ eV}$ at the best fit parameters of the LMA MSW solar neutrino solution by requiring the range of the parameter $\langle m \rangle_\beta$ between $0.11$–$0.56$ eV deduced from recently observed double beta decay events at 95 % CL.

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I. INTRODUCTION

While the good amount of evidences for the neutrino mass and the lepton flavor mixing have been accumulated [1–3], we still lack observational indications of how large is the absolute mass of the neutrinos. To our understanding to date it may show up in only a few places, the single [4] or the double beta decay [5] experiments as well as future cosmological observations [6]. Other potential possibilities for hints of absolute mass of neutrinos include Z-burst interpretation of highest energy cosmic rays [7].

Among these various experimental possibilities the neutrinoless double beta decay experiments seems to have relatively higher sensitivities. The most stringent bound on effective mass parameter $\langle m \rangle_{\beta\beta}$ (see eq. (2) for definition) is now $\langle m \rangle_{\beta\beta} < 0.35$ eV, which comes from Heidelberg-Moscow group [8]. Furthermore, a variety of proposals for future facilities with greater sensitivities are actively discussed. They include GENIUS [9], CUORE [10], MOON [11], XMASS [12], and EXO [13] projects. These high-sensitivity experiments open the enlighting possibility of discovering neutrinoless double beta decay events, not just placing an upper bound on $\langle m \rangle_{\beta\beta}$ by its nonobservation. Therefore, it is of great importance to completely understand what kind of informations can be extracted if such discovery is made.

We discuss in this paper in a generic three flavor mixing framework the constraints on neutrino masses and mixing by positive observation (as well as nonobservation) of neutrinoless double beta decay. The constraints imposed on neutrino mixing parameters by neutrinoless double beta decay have been discussed by many authors. They include the ones in early epoch [14], those in ”modern era” in which real constraints on solar mixing parameters are started to be extracted [15,16], and the ones in ”post-modern era” where the analyses are performed in a comprehensive manner in the framework of generic three flavor neutrino mixing [17].

In a previous paper, we have made a final step in the series of analyses by proposing a way of expressing the constraints solely in terms of observables in single and double beta decay [18]. By using the framework, we discussed the possibility of placing lower bound on $|U_{e3}|^2$ assuming positive observation in direct mass measurement in single beta decay and an upper limit on $\langle m \rangle_{\beta\beta}$ in double beta decay experiments. It is a natural and logical step
for us to examine next the alternative case of positive observation of neutrinoless double beta decay events.

Timely enough, a discovery of the neutrinoless double beta decay has just been announced by Klapdor-Kleingrothaus and collaborators [19]. Since the confidence level of the evidence is about 2 $\sigma$ and 3 $\sigma$ in Bayesian and Particle Data Group methods, respectively, we must wait for confirmation by further data taking, or by other groups to conclude that neutrinos are Majorana particles. Nevertheless, we feel that the peak in the relevant kinematic region in their experiments is too prominent to be simply ignored.

As will become clear as we proceed it is essential to combine the constraint on $|U_{e3}|^2 = s_{13}^2$ imposed by the reactor experiments [20]. One of the key points in our subsequent discussion is that the double beta and the reactor bounds cooperate to produce a stringent constraint on absolute mass scale of neutrinos and the mixing angle $\theta_{12}$ which is responsible for solar neutrino problem.

II. CONSTRAINTS FROM NEUTRINOLESS DOUBLE BETA DECAY

Let us start by defining our notations. We use throughout this paper the standard notation of the MNS matrix [21]:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}.$$ (1)

Using the notation, the observable in neutrinoless double beta decay experiments can be expressed as

$$\langle m \rangle_{\beta\beta} = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| = \left| m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta} + m_3 s_{13}^2 e^{i(3\gamma-2\delta)} \right|,$$ (2)

where $m_i$ (i=1, 2, 3) denote neutrino mass eigenvalues, $U_{ei}$ are the elements in the first low of the MNS matrix, and $\beta$ and $\gamma$ are the extra CP-violating phases characteristic to Majorana neutrinos [22], for which we use the convention of Ref. [15].
We define the neutrino mass-squared difference as $\Delta m^2_{ij} \equiv m_j^2 - m_i^2$ in this paper. In the following analysis, we must distinguish the two different neutrino mass patterns, the normal ($\Delta m^2_{23} > 0$) vs. inverted ($\Delta m^2_{23} < 0$) mass hierarchies. We use the convention that $m_3$ is the largest (smallest) mass in the normal (inverted) mass hierarchy so that the angles $\theta_{12}$ and $\theta_{23}$ are always responsible for the solar and the atmospheric neutrino oscillations, respectively. We therefore often use the notations $|\Delta m^2_{23}| \equiv \Delta m^2_{\text{atm}}$ and $\Delta m^2_{12} \equiv \Delta m^2_{\odot}$ to emphasize that they are experimentally measurable quantities. Because of the hierarchy of mass scales, $\Delta m^2_{\odot}/\Delta m^2_{\text{atm}} \ll 1$, $\Delta m^2_{12}$ can be made always positive as far as $\theta_{12}$ is taken in its full range $[0, \pi/2]$ [23].

In order to derive constraint on mixing parameters we need the classification.

**Case I:**
\[
|m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta}| \geq m_3 s_{13}^2
\]  
\(3\)

**Case II:**
\[
|m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta}| \leq m_3 s_{13}^2
\]  
\(4\)

However, examination of the case II reveals that it does not lead to useful bounds. Therefore, we only discuss the case I in the rest of this paper.

**A. Joint constraint by upper bounds on $\langle m \rangle_{\beta\beta}$ and reactor experiments**

Since we try to utilize the experimental upper bound on $\langle m \rangle_{\beta\beta}$, $\langle m \rangle_{\beta\beta} \leq \langle m \rangle_{\beta\beta}^{\text{max}}$, we derive the lower bound on $\langle m \rangle_{\beta\beta}$. It can be obtained in the following way;

\[
\langle m \rangle_{\beta\beta} \geq c_{13}^2 \left| m_1 c_{12}^2 + m_2 s_{12}^2 \right| \cos \beta - i(m_1 c_{12}^2 - m_2 s_{12}^2) \sin \beta \geq m_3 s_{13}^2
\]
\[
= c_{13}^2 \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 \cos 2\beta} - m_3 s_{13}^2.
\]  
\(5\)

Noticing that the right-hand-side (RHS) of (5) has a minimum at $\cos 2\beta = -1$, we obtain the inequality

\[
\langle m \rangle_{\beta\beta} \geq c_{13}^2 \left| m_1 c_{12}^2 - m_2 s_{12}^2 \right| - m_3 s_{13}^2.
\]  
\(6\)

We note that the RHS of (6) is a decreasing function of $s_{13}^2$, and hence takes a minimum value for the maximum value of $s_{13}^2$ which is allowed by the limit placed by the reactor experiments [20]. We denote the maximum value as $s_{13}^2(\text{CH})$ throughout this paper. Numerically,
\[ s_{13}^{2} \simeq 0.03. \] (While the precise value of the CHOOZ constraint actually depends upon the value of \( \Delta m_{\text{atm}}^{2} \) [20], we do not elaborate this point in this paper.) Using the constraint we obtain

\[
\langle m \rangle_{\beta\beta}^{\text{max}} \geq \langle m \rangle_{\beta\beta} \geq |m_1 c_{12}^2 - m_2 s_{12}^2| - \left( m_3 + |m_1 c_{12}^2 - m_2 s_{12}^2| \right) s_{13}^{2} \text{(CH)}. \tag{7}
\]

It can be rewritten as the bound on \( \cos 2\theta_{12} = \cos 2\theta_{\odot} \) as

\[
\frac{m_2 - m_1}{m_2 + m_1} - \frac{\langle m \rangle_{\beta\beta}^{\text{max}} + m_3 s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_2 + m_1)c_{13}^{2}} \leq \cos 2\theta_{12} \leq \frac{m_2 - m_1}{m_2 + m_1} + \frac{\langle m \rangle_{\beta\beta}^{\text{max}} + m_3 s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_2 + m_1)c_{13}^{2}}, \tag{8}
\]

where \( c_{13}^{2} \equiv 1 - s_{13}^{2} \text{(CH)} \).

**B. Joint constraint by lower bounds on \( \langle m \rangle_{\beta\beta} \) and reactor experiments**

A positive observation of neutrinoless double beta decay will lead to the experimental lower bound on \( \langle m \rangle_{\beta\beta} \), \( \langle m \rangle_{\beta\beta} \geq \langle m \rangle_{\beta\beta}^{\text{min}} \), which we use to place new bound on neutrino mixing parameters. Toward the goal we note, similarly as (5), that

\[
\langle m \rangle_{\beta\beta} \leq c_{13}^{2} \sqrt{m_1^{2}c_{12}^{4} + m_2^{2}s_{12}^{4} + 2m_1 m_2 c_{12}^{2}s_{12}^{2} \cos \beta + m_3^{2}}, \tag{9}
\]

whose the RHS is maximized by taking \( \cos 2\beta = +1 \) and \( s_{13}^{2} = s_{13}^{2} \text{(CH)} \) in the last term and \( c_{13}^{2} = 1 \) in front of the square root. One can then derive a similar inequality as (8);

\[
\langle m \rangle_{\beta\beta}^{\text{min}} \leq \langle m \rangle_{\beta\beta} \leq \left( m_1 c_{12}^{2} + m_2 s_{12}^{2} \right) + m_3 s_{13}^{2} \text{(CH)}. \tag{10}
\]

By rewriting (10) we obtain the other upper bound on \( \cos 2\theta_{12} \);

\[
\cos 2\theta_{12} \leq \frac{m_2 + m_1}{m_2 - m_1} - \frac{\langle m \rangle_{\beta\beta}^{\text{min}} - m_3 s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_2 - m_1)}. \tag{11}
\]

To summarize, we have derived in this section the two kinds of upper bound on \( \cos 2\theta_{12} \) (lower bound for \( \cos 2\theta_{12} < 0 \)) by using the assumed experimental constraint \( \langle m \rangle_{\beta\beta}^{\text{min}} \leq \langle m \rangle_{\beta\beta} \leq \langle m \rangle_{\beta\beta}^{\text{max}} \) imposed by neutrinoless double beta decay experiments.

**III. CONSTRAINTS EXPRESSED BY EXPERIMENTAL OBSERVABLES**

We rewrite the bounds on solar mixing angle in terms of measurable quantities. Toward the goal we note that three neutrino masses \( m_i \) (i=1,2,3) can be expressed by the two
\[ \Delta m^2 \] and a remaining over-all scale \( m_H \). We assign \( m_H \) to the mass of the highest-mass state, \( m_3 \) in the normal mass hierarchy (\( \Delta m^2_{23} > 0 \)), and \( m_2 \) in the inverted mass hierarchy (\( \Delta m^2_{23} < 0 \)), respectively.

We have argued in our previous paper [18] that in a reasonable approximation one can regard \( m_H \) as the observable \( \langle m \rangle_\beta \) in direct mass measurements in single beta decay experiments.* We have noticed that the identification is exact in two extreme cases of degenerate and hierarchical mass spectra. Then, the three mass eigenvalues of neutrinos can be represented solely by observables; \( \Delta m^2_{\text{atm}}, \Delta m^2_{\odot}, \) and \( \langle m \rangle_\beta \) in a good approximation.

In each neutrino mass pattern, we have the expressions of three mass eigenvalues:

Normal mass hierarchy (\( \Delta m^2_{23} > 0 \));

\[
m_1 = \sqrt{m_H^2 - \Delta m^2_{\odot} - \Delta m^2_{\text{atm}}}, \quad m_2 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}}}, \quad m_3 = m_H \simeq \langle m \rangle_\beta. \quad (13)
\]

Inverted mass hierarchy (\( \Delta m^2_{23} < 0 \));

\[
m_1 = \sqrt{m_H^2 - \Delta m^2_{\odot}}, \quad m_2 = m_H \simeq \langle m \rangle_\beta, \quad m_3 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}}}. \quad (14)
\]

It is instructive to work out the form of constraint in the degenerate mass approximation, \( m_i \simeq m^2 > \Delta m^2_{\text{atm}}, \Delta m^2_{\odot} \). It is easy to show that in the degenerate mass limit the bound (8) becomes

\[
| \cos 2\theta_{12} | \leq \sec^2 \theta_{13}(\text{CH}) \left[ \frac{\langle m \rangle_{\beta\beta}^\text{max}}{\langle m \rangle_\beta} + s_{13}^2(\text{CH}) \right].
\]

(15)

On the other hand, the bound (9) gives the inequality \( \langle m \rangle_\beta \geq \langle m \rangle_{\beta\beta}^\text{min} \) in the degenerate mass limit. (To show this one may go back to (9), rather than using (11).

*While we used the linear formula derived by Farzan, Peres and Smirnov [24]

\[
\langle m \rangle_\beta = \frac{\sum_{j=1}^{n} m_j |U_{ej}|^2}{\sum_{j=1}^{n} |U_{ej}|^2}
\]

with \( n \) being the dimension of the subspace of (approximately) degenerate mass neutrinos, this point remains valid even if we use an alternative quadratic expression [25].
IV. ANALYSIS OF THE DOUBLE BETA-REACTOR JOINT CONSTRAINTS

We analyze in this section the joint constraints derived in the foregoing sections and try to extract the implications. Let us start by examining the case of recent discovery announced in [19] which gives rise to $0.11 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.56 \text{ eV}$ at 95 % CL. In Fig. 1 we present on $\langle m \rangle_{\beta} - \cos 2\theta_{12}$ plane the constraint (8) by the thick solid lines and (11) by the thick dashed line, respectively. The region surrounded by these lines are allowed. The slope of $\langle m \rangle_{\beta}$-dependence of (11) is so large that the dashed line looks like a vertical line, which implies the inequality $\langle m \rangle_{\beta} \geq \langle m \rangle_{\beta\beta}^{\text{min}}$. We have derived it earlier in the degenerate mass limit, but it is generically true if $\Delta m_{23}^2$ is smaller than other relevant mass squared scales. Only the case of normal mass hierarchy ($\Delta m_{23}^2 > 0$) is shown in Fig. 1; the case of inverted hierarchy ($\Delta m_{23}^2 < 0$) gives an almost identical result except for a slight shift of the dashed line toward smaller $\langle m \rangle_{\beta}$ by $\sim s_{13}^2(CH) \simeq 3 \%$.

Superimposed in Fig. 1 is the 95 % CL allowed regions of $\cos 2\theta_{12}$ for the large mixing angle (LMA) MSW solution (indicated by the shaded region between thin solid lines) and the low (LOW) MSW solution (indicated by the shaded region between thin dashed lines) of the solar neutrino problem [26]. There are several up to date global analyses of the solar neutrino data with similar results of allowed region of mixing parameters [27]. Therefore, we just quote the result obtained by Krastev and Smirnov in the last reference in [27].

Figure 1 indicates that for a given value of $\cos 2\theta_{12}$ the single beta decay observable $\langle m \rangle_{\beta}$ has to fall into a region bounded by $\langle m \rangle_{\beta}^{\text{min}} \simeq \langle m \rangle_{\beta\beta}^{\text{min}}$, and $\langle m \rangle_{\beta}^{\text{max}}$ which is dictated by (8). Thus, we have a clear prediction for direct mass measurement using single beta decay with observation of double beta decay events. For example, the observable $\langle m \rangle_{\beta}$ must fall into the region $0.11 \text{ eV} \leq \langle m \rangle_{\beta} \leq 1.3 \text{ eV}$ at the best fit parameters of the LMA MSW solution. (The best fit value is $\tan^2 \theta_{12} = 0.35$, or $\cos 2\theta_{12} = 0.48$ in the last reference in [27].) Within the allowed region the cancellation between three mass eigenstates can take place for appropriate values of Majorana phases that allow (typically) a factor of 2-3 larger values of $\langle m \rangle_{\beta}$ compared with the measured value of $\langle m \rangle_{\beta\beta}$. At around maximal mixing ($\cos 2\theta_{12} \simeq 0$), which is allowed by 95 % CL in the LOW solution, the cancellation is so efficient that much larger values of $\langle m \rangle_{\beta}$ is allowed. Therefore, there are still ample room for hot dark matter mass neutrinos both in the LMA and the LOW solutions.
It should be emphasized that a finite value of $\langle m \rangle_{\beta\beta}$ does imply a lower bound on $\langle m \rangle_{\beta}$, as indicated in Fig. 1; a vanishingly small $\langle m \rangle_{\beta}$ cannot be consistent with finite $\langle m \rangle_{\beta\beta}$ in double beta decay experiments. The sensitivity of the proposed KATRIN experiment is expected to extend to $\langle m \rangle_{\beta} \leq 0.3$ eV [28]. On the other hand, the present 68 % CL limit quoted in [19] is $0.28 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.49 \text{ eV}$. Therefore, if the limit is further tightened by additional data taking in the future, both experiments can become inconsistent, giving another opportunity of cross checking.

In Fig. 2 and Fig. 3, we present the similar allowed regions for hypothetical discovery of neutrinoless double beta decay events which would produce the experimental bounds (H1); $0.1 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.3 \text{ eV}$ (Fig. 2), and (H2); $0.01 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.03 \text{ eV}$ (Fig. 3).

It is to examine how the constraint changes in some other situation of discovery with different mass parameter ranges. We note that even the region of sensitivity in (H2) will be explored by several experiments [9,11–13].

The most stringent bound to date on $\langle m \rangle_{\beta}$ is from the Mainz collaboration [29], $\langle m \rangle_{\beta} \leq 2.2 \text{ eV}$ (95 % CL). (A similar bound $\langle m \rangle_{\beta} \leq 2.5 \text{ eV}$ (95 % CL) is derived by the Troitsk group [30].) As we can see in Fig. 2 that the double beta bound with the (H1) parameter becomes stronger than the Mainz bound for the LMA MSW solution but not for the LOW MSW solution in their 95 % CL regions.

For the (H2) case, the bounds for the normal and the inverted mass hierarchies start to split as shown in Fig. 3. In the case of inverted mass hierarchy the lower bound on $\langle m \rangle_{\beta}$ is replaced by the trivial bound $\langle m \rangle_{\beta} \geq \sqrt{\Delta m^2_{\text{atm}}}$ which is more restrictive. The latter is indicated by the dash-dotted line in Fig. 3b. It is also evident that the constraint from double beta decay is so stringent that the limit on $\langle m \rangle_{\beta}$ is tightened to be $\langle m \rangle_{\beta} \lesssim 0.2 \text{ eV}$ for the LMA MSW solution.

Finally, some remarks are in order:

(1) If the LMA MSW solution is the case and if the KamLAND experiment [31] that just started data taking can measure $\cos 2\theta_{12}$ within 10 % level accuracy, the upper limit of $\langle m \rangle_{\beta}$ can be accurately determined with $\sim 20$ % accuracy.

(2) In this paper we have derived constraints imposed on neutrino mixing parameters by
observation of neutrinoless double beta decay events and the CHOOZ reactor bound on $|U_{e3}|^2$ in the generic three flavor mixing framework of neutrinos. Suppose that neutrinoless double beta decay events are confirmed to exist and the single beta decay experiments detect neutrino mass outside the region of the bound derived in this paper. What does it mean? It means either that double beta decay would be mediated by some mechanisms different from the usual one with Majorana neutrinos, or the three flavor mixing framework used in this paper is too tight.

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FIG. 1. Presented are the constraints imposed on neutrino mixing parameters $\theta_{12}$ and the observable mass parameter $\langle m \rangle_\beta$ in single beta decay experiments by recent observation of neutrinoless double beta decay events with mass parameter $0.11 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.56 \text{ eV}$. The thick solid and the thick dashed lines are from the bounds (8) and (11), respectively; the allowed region is inside these three lines. The mixing parameters are fixed as $\Delta m^2_{\text{atm}} = 3 \times 10^{-3} \text{ eV}^2$ and $\Delta m^2_{12} = 4.8 \times 10^{-5} \text{ eV}^2$. Also shown as shaded region are the allowed regions of $\cos 2\theta_{12}$ at 95 % CL for the LMA (the region between thin solid lines) and LOW (the region between thin dashed lines) MSW solutions.
FIG. 2. The same as in Fig. 1 but with assumed observed mass parameter $\langle m \rangle_{\beta\beta}$ in the range $0.1 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.3 \text{ eV}$.
FIG. 3. The same as in Fig. 2 but with assumed observed mass parameter $\langle m \rangle_{\beta\beta}$ in the range $0.01 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.03 \text{ eV}$. Fig. 3a and 3b for the normal and the inverted mass hierarchies, respectively. The dash-dotted line in Fig. 3b denotes the trivial bound $\langle m \rangle_{\beta} \geq \sqrt{\Delta m_{atm}^2}$. 