CP Violation and the CKM Matrix

M. Beneke
Institut für Theoretische Physik E, Sommerfeldstr. 28, RWTH Aachen, D - 52074 Aachen, Germany

This lecture provides a general overview of CP violation, emphasizing CP violation in flavour-violating interactions, such as due to the Kobayashi-Maskawa mechanism.

1. INTRODUCTION

The breaking of CP symmetry ("CP violation"), the composition of parity and charge conjugation, is an interesting phenomenon for several reasons:

a. CP violation together with CPT symmetry implies non-invariance of the microscopic equations of motion under motion-reversal. CP violation rather than C violation implies different physical properties of matter and antimatter. These two facts are of fundamental importance for our understanding of the laws of Nature, and they were perceived as revolutionary, when CP violation was discovered in 1964 [1]. They were also important in the development of the fundamental theory of particles, since the observation of CP violation motivated some early extensions of the Standard Model as it was known at the time (1973), either by extending the Higgs sector [2] or by adding a third generation of quarks and leptons [3]. Nature has opted for the second possibility for certain and the Kobayashi-Maskawa mechanism of CP violation has become part of today’s Standard Model. From today’s perspective motion-reversal non-invariance and the distinction of matter and antimatter, though fundamental, appear no longer surprising and even "natural". What remains surprising, however, is the peculiar way in which CP violation occurs or rather does not occur in the Standard Model and its possible extensions.

b. Assuming that the evolution of the universe began from a matter-antimatter symmetric state, CP violation is necessary [4] to generate the matter-antimatter asymmetric universe that one observes today. At the electroweak phase transition the Standard Model satisfies all the other necessary criteria for baryogenesis (baryon number violation, departure from thermal equilibrium) [5], but CP violation in the Standard Model is too weak to explain the observed baryonto-photon ratio. With the above assumption on the initial condition of the cosmic evolution, our own existence provides evidence for a source of CP violation beyond the Standard Model.

Electroweak baryogenesis has the attractive feature that it couples the required new mechanisms of CP violation to the electroweak scale, therefore making them testable also in particle collider experiments. Nevertheless it now appears more likely that the matter-antimatter asymmetry is not related to the sources of CP violation that one may observe at colliders. Two facts have contributed to this change of perspective: first, the lower limit on the masses of Higgs bosons has been increasing. A heavier Higgs boson implies a weaker (first-order) electroweak phase transition. As a consequence electroweak baryogenesis is already too weak over most of the parameter space of even the minimal supersymmetric extension of the Standard Model. Second, the observation of small neutrino masses through neutrino oscillations is explained most naturally by invoking the seesaw mechanism, which in turn is most naturally realized by postulating massive neutrinos, which are singlets under the Standard Model gauge group. All three necessary

---

conditions for the generation of lepton number are naturally realized in the decay of the massive neutrino(s). Lepton number is then partially converted into baryon number via B+L-violating (but B–L conserving) sphaleron transitions [6]. While the leptogenesis scenario is very appealing, the new sources of CP violation related to the Yukawa couplings of the heavy neutrinos occur at scales of order of the heavy neutrino mass, $M_R \sim (10^{12} - 10^{16})$ GeV (needed to explain the small left-handed neutrino masses), and are not directly testable with collider experiments in the near future. For this reason, CP violation in the context of baryogenesis will not be discussed further in this talk.

c. CP violation in the Standard Model is essentially an electroweak phenomenon originating from the Yukawa couplings of the quarks to the Higgs boson. This implies that probes of CP violation are indirect probes of the electroweak scale or TeV scale, complementary to direct probes such as the observation of Higgs bosons. This is probably the most important reason for the current interest in CP symmetry breaking; in addition to testing the Kobayashi-Maskawa mechanism of CP violation in the Standard Model, experiments directed at CP violation limit the construction of extensions of the Standard Model at the TeV scale. There is an analogy between CP symmetry and electroweak symmetry breaking. Both occur at the electroweak scale and for both the Standard Model provides a simple mechanism. However, neither of the two symmetry breaking mechanisms has been sufficiently tested up to now. Such tests may or may not confirm the Standard Model mechanisms but they may also provide answers to questions that concern the structure of the Standard Model in its entirety, such as the origin of the electroweak scale and the origin of flavour and CP violation.

d. Leaving aside the matter-antimatter asymmetry in the universe as evidence for CP violation since this depends on a further assumption, CP violation has now been observed in the weak interactions of quarks in three different ways: in the mixing of the neutral kaon flavour eigenstates $\epsilon$ (1964) [1]; in the decay amplitudes of neutral kaons $\epsilon'$ (1999) [7,8]; in the mixing of the neutral $B_d$ meson flavour eigenstates $\sin(2\beta)$, 2001) [9,10]. It will be seen below that these pieces of data together with others not directly related to CP violation suggest that the Kobayashi-Maskawa mechanism of CP violation is most likely the dominant source of CP violation at the electroweak scale. The latest piece of evidence also rules out that CP symmetry is an approximate symmetry. A consequence of this is that generic extensions of the Standard Model at the TeV scale, needed to explain the stability of the electroweak scale, suffer from a CP fine-tuning problem since any such extension implies the existence of many new CP-violating parameters which have no generic reason to be small. Despite the apparent success of the standard theory of CP violation, the problem of CP and flavor violation therefore remains as mysterious as before.

The three quantities listed above establish CP violation unambiguously, but since the observables depend on decays of mesons at low energy, interpreting these quantities in terms of CP-violating fundamental parameters of the theory often involves a very difficult strong interaction problem. Chiral perturbation theory and the heavy quark expansion provide analytic tools to address this problem for kaon and $B$ meson decays, respectively. In addition, lattice QCD can contribute substantially to making the theoretical prediction for many (but not all) quantities relevant to CP violation more precise.

2. CP VIOLATION IN THE STANDARD MODEL

CP violation can occur in the Standard Model in three different ways:

2.1. The $\theta$ term

The strong interactions could be CP-violating [11–13]. The topology of gauge fields implies that the correct vacuum is given by a superposition $|\theta\rangle = \sum_n e^{in\theta} |n\rangle$ of the degenerate vacua $|n\rangle$ in which pure gauge fields have winding number $n$. Correlation functions in the $\theta$-vacuum can be computed by adding to the Lagrangian the term

$$\mathcal{L}_\theta = \theta \cdot \frac{g_s^2}{32\pi^2} G^A_{\mu \nu} \tilde{G}^{A,\mu \nu},$$  (1)
where \( \theta \) now represents a parameter of the theory. Physical observables can depend on \( \theta \) only through the combination \( e^{i\theta} \det M \), where \( M \) is the quark mass matrix. A non-zero value of

\[
\tilde{\theta} = \theta + \arg \det M
\]

(2)

violates CP symmetry. It also implies an electric dipole moment of the neutron of order \( 10^{-16} \tilde{\theta} \text{e cm} \). The non-observation of any such electric dipole moment constrains \( \tilde{\theta} < 10^{-16} \) and causes what is known as the strong CP problem, since the Standard Model provides no mechanism that would require \( \tilde{\theta} \) to vanish naturally. The strong CP problem has become more severe with the observation of large CP violation in \( B \) meson decays since one now knows with more confidence that the quark mass matrix has no reason to be real a priori.

There exist mechanisms that render \( \tilde{\theta} = 0 \) exactly or very small through renormalization effects. None of these mechanisms is convincing enough to provide a default solution to the problem. What makes the strong CP problem so difficult to solve is that one does not have a clue at what energy scale the solution should be sought. Strong CP violation is not discussed further in this talk (see the discussion in [14]).

2.2. The neutrino mass matrix

The Standard Model is an effective theory defined by its gauge symmetries and its particle content. CP violation appears in the lepton sector if neutrinos are massive. The leading operator in the effective Lagrangian is [15]

\[
\frac{f_{ij}}{\Lambda} \cdot [(L^T \epsilon)_i i\sigma^2 H][H^T i\sigma^2 L_j].
\]

(3)

After electroweak symmetry breaking this generates a Majorana neutrino mass matrix with three CP-violating phases. One of these phases could be observed in neutrino oscillations, the other two phases only in observables sensitive to the Majorana nature of neutrinos.

Unless the \( f_{ij} \) are extremely small, the scale \( \Lambda \) must be large to account for small neutrinos masses, which suggests that leptonic CP violation is related to very large scales. For example, the standard see-saw mechanism makes the \( f_{ij} \) dependent on the CP-violating phases in the heavy gauge-singlet neutrino mass matrix. As a consequence one may have interesting model-dependent relations between leptogenesis, CP violation in lepton-flavour violating processes and neutrino physics, but since the observations are all indirect through low-energy experiments, one may at best hope for accumulating enough evidence to make a particular model particularly plausible. Such experiments seem to be possible, but not in the near future and for this reason leptonic CP violation is not discussed further here.

2.3. The CKM matrix

CP violation can appear in the quark sector of the Standard Model at the level of renormalizable interactions [3]. The quark Yukawa interactions read

\[
L_Y = -y^u_{ij} \bar{Q}'_i H d_j - y^d_{ij} \bar{Q}'_i H^* u_j + \text{h.c.},
\]

(4)

with \( Q' \) the left-handed quark SU(2)-doublets, \( u' \) and \( d' \) the right-handed SU(2) singlets and \( i = 1, 2, 3 \) the generation index. The complex mass matrices that arise after electroweak symmetry breaking are diagonalized by separate unitary transformations \( U_{L,R}^{u,d} \) of the left- and right-handed up- and down-type fields. Only the combination

\[
V_\text{CKM} = U_{L}^{u\dagger} U_{L}^{d} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix},
\]

(5)

referred to as the CKM matrix, is observable, since the charged current interactions now read

\[
-\frac{e}{\sqrt{2} \sin \theta_W} \bar{u}_j \gamma^\mu [V_\text{CKM}]_{ij} d_j W^\mu + \text{h.c.}.
\]

(6)

Flavour and CP violation in the quark sector can occur in the Standard Model only through charged current interactions (assuming \( \tilde{\theta} = 0 \)). With three generations of quarks, the CKM matrix contains one physical CP-violating phase.
Any CP-violating observable in flavour-violating processes must be related to this single phase. The verification or, perhaps rather, falsification of this highly constrained scenario is the primary goal of many current $B$- and $K$-physics experiments. This type of CP violation is therefore discussed in some detail in later sections.

For reasons not understood the CKM matrix has a hierarchical structure as regards transitions between generations. It is therefore often represented in the approximate form [16]

\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]

where $\lambda \approx 0.224$ and $A$, $\rho$, $\eta$ are counted as order unity and corrections are of order $\lambda^4$. It is the great achievement of heavy quark theory of the 1990s to have determined $|V_{ub}|$, i.e. $A$, to the accuracy of a few percent, whereas determining $\rho$ and $\eta$ with this accuracy remains a challenge for this decade. The unitarity of the CKM matrix leads to a number of relations between rows and columns of the matrix. The one which is most useful for $B$-physics is obtained by multiplying the first column by the complex conjugate of the matrix. The one which is most useful for $B$-physics is obtained by multiplying the first column by the complex conjugate of the third:

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.
\]

If $\eta \neq 0$ (which implies CP violation) this relation can be represented as a triangle in the complex plane, called the unitarity triangle. See Figure 1, which also introduces some notation for the angles of the triangle that will be referred to later on.

The hierarchy of the CKM matrix implies that CP violation is a small effect in the Standard Model. More precisely, CP-violating observables are either small numbers, or else they are constructed out of small numbers such as small branching fractions of rare decays. The hierarchy of quark masses and mixing angles represents a puzzle, sometimes called the flavour problem, which will also not be discussed further in this talk. Typical attempts to solve the flavour problem focus on broken generation symmetries.

\[
\begin{array}{c}
\text{(a)} \\
(\rho, \eta)
\end{array}
\]

\[
\begin{array}{c}
\text{(b)} \\
\text{Figure 1. The unitarity triangle.}
\end{array}
\]

\[
\begin{array}{c}
\text{3. CONSTRAINTS ON THE UNITARITY TRIANGLE}
\end{array}
\]

In the following I review the current constraints on $(\bar{\rho}, \bar{\eta})$, the apex of the unitarity triangle, and the prospects for improving these constraints. (The definitions $\bar{\rho}/\rho = \bar{\eta}/\eta = 1 - \lambda^2/2$ render the location of the apex accurate to order $\lambda^5$ [17] and will be used in the following.) It is not the purpose of this talk to go into the details of the theoretical calculations that contribute to these constraints. Recent summaries of the relevant lattice calculations can be found in [18–20].

\[
\begin{array}{c}
\text{3.1. CP-conserving observables}
\end{array}
\]

The lengths of the three sides of the triangle are determined from CP-conserving observables.

*Semileptonic decays.* $|V_{cb}| = 0.041 \pm 0.002$ sets the scale of the sides of the triangle and is determined from exclusive [21] and inclusive semileptonic $B$ decays [22, 23]. Both methods rely on the heavy quark expansion. The current error on $|V_{cb}|$ is not a limiting factor in the determination of $(\bar{\rho}, \bar{\eta})$, but it may become important for rare kaon decays, which depend on $A$ to a high power. The inclusive method has probably reached its intrinsic limits [24]. Further improvement then depends on how well the $B \rightarrow D^{(*)}$ form factors can be computed with lattice QCD.

The determination of $|V_{ub}|$ uses semileptonic $b \rightarrow u$ decays and gives

\[
\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{1 - \lambda^2/2}{\lambda} \frac{|V_{ub}|}{|V_{cb}|}.
\]

However, $|V_{ub}/V_{cb}| \approx 0.085$ is currently known only within an error of about $\pm 20\%$. $|V_{ub}|$ can also be determined from inclusive or exclusive de-
cays. The inclusive treatment would parallel that of $|V_{ub}|$ if not the background from $b \to c$ transitions had to be suppressed. Distributions in various kinematic variables (lepton energy, hadronic invariant mass) have been considered at the price of a more complicated and uncertain theory. A cut on the leptonic and hadronic invariant mass avoids the kinematic region, where the heavy quark expansion (in local operators) is invalid, but the scale of the expansion is now around 2 GeV rather than $m_b$ [25–27]. While the ultimate accuracy of this method is not known, it should be possible to halve the error on $|V_{ub}|$. The exclusive determination of $|V_{ub}|$ must rely on lattice QCD for the $B \to M$ form factor. $|V_{ub}|$ is extracted by comparing the lepton invariant mass spectrum at large $q^2$ with the form factor in this region, where it is computed most reliably at present. The exclusive method is not yet competitive, but it will certainly play an important role in an accurate determination of $|V_{ub}|$ in the future.

The $|V_{ub}|$-constraint on $(\bar{\rho}, \bar{\eta})$ is particularly important, since it is the only constraint that is based on a tree decay and therefore arguably insensitive to non-Standard Model interactions. It constrains the parameters of the CKM matrix even in the presence of new physics and helps to define the Standard Model reference point. The error on $|V_{ub}|$ is also a major component of the error in the indirect determination of $\sin 2\beta$ from the global fit to $(\bar{\rho}, \bar{\eta})$ discussed below. Its reduction would therefore sharpen the consistency test with the direct measurement of $\sin 2\beta$.

$B B$ mixing. In the Standard Model the $B B$ mass difference is dominated by the top quark box diagram, proportional to $|V_{tq}V_{tb}|^2$ ($q = d, s$). This determines the length of the remaining side of unitarity triangle. The large mass of the top quark implies that the mass difference can be calculated up to the matrix element of the local operator $(\bar{q}b)_{V-A}(\bar{q}b)_{V-A}$, conventionally parameterized by $f_{B_s} B_{B_s}$. For $\Delta M_{B_s}$ one obtains

$$\sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = (0.83 \pm 0.03) \times \frac{f_{B_s} B_{B_s}^{1/2}}{230 \text{MeV}} \quad (10)$$

The use of this result is limited by an error of about $\pm 15\%$ on the quantity $f_{B_s} B_{B_s}^{1/2}$. Only lattice QCD can possibly improve upon this error.

Since $V_{ts}$ is already determined by the unitarity of the CKM matrix, $\Delta M_{B_s}$ alone does not constrain the unitarity triangle further. However, the ratio $(\Delta M_{B_d}/\Delta M_{B_s})^{1/2}$ also determines $(1 - \bar{\rho})^2 + \bar{\eta}^2$ and involves only the ratio $\xi = f_{B_s} B_{B_s}^{1/2}/f_{B_d} B_{B_d}^{1/2}$, which is believed to be known from lattice QCD with an error ($\pm 6\%$) smaller than the error on each of the hadronic parameters individually. The current lower limit on $\Delta M_{B_s}$ then provides an important upper limit on the length of the relevant side of the unitarity triangle, which turns into the most important restriction on the upper limit for the angle $\gamma$ in the combined $(\bar{\rho}, \bar{\eta})$ fit.

3.2. CP violation in kaon decays

CP violation in mixing (indirect, 1964). Due to CP violation in $K \bar{K}$ mixing, the neutral kaon mass eigenstates are superpositions of CP-even and CP-odd components. The long-lived kaon state, $K_L \approx K_2 + \epsilon K_1$, is predominantly CP-odd, but decays into two pions through its small CP-even component $K_1$. The decay $K_L \to \pi\pi$ constituted the first observation of CP violation ever [1]. The quantity $|\epsilon| = 2.27 \cdot 10^{-3}$ (equal to $|\epsilon|$ in the standard phase convention to very good accuracy) has now been measured in many different ways.

$K \bar{K}$ mixing is dominated by top and charm box diagrams. The long-distance contributions are encoded in the matrix element of a four-fermion operator similar to $B \bar{B}$ mixing, conventionally parameterized by $B_K$ and computed in lattice QCD. There is currently an uncertainty of at least $\pm 15\%$ on this parameter. $|\epsilon|$ determines $\tilde{\eta}(1.3 \pm 0.05 - \bar{\rho})$. This is the fourth and last constraint that enters the “standard” unitarity triangle fit.

CP violation in decay (direct, 1999). CP-violating effects can also be seen in the interference of two decay amplitudes with different CKM phases. The double ratio

$$\frac{\Gamma(K_L \to \pi^0\pi^0)\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^0\pi^0)\Gamma(K_L \to \pi^+\pi^-)} \approx 1 - 6\text{Re}\left(\frac{\epsilon'}{\epsilon}\right), \quad (11)$$

where $\epsilon'$ is the upper CKM unitarity triangle constraint $\Delta M_{B_d}$.
if different from unity, implies such an effect, since both ratios would equal \( |\epsilon| \), if CP violation occurred only in mixing. The existence of this effect has been conclusively demonstrated by two experiments in 1999, following the first hints of a non-vanishing \( \epsilon'/\epsilon \) in 1992 [28]. The new results of 2001 have further clarified the situation, which is now summarized by [29,30]

\[
\frac{\epsilon'}{\epsilon} = \begin{cases} 
(15.3 \pm 2.6) \times 10^{-4} & \text{NA48 (97-99)} \\
(20.7 \pm 2.8) \times 10^{-4} & \text{KTeV (96/97)}.
\end{cases}
\]

The theory of \( \epsilon'/\epsilon \) is more complicated than that of any other quantity discussed so far. The short-distance contributions are many-fold, but have been worked out to next-to-leading order [31,32]. The following, approximate representation of the result,

\[
\frac{\epsilon'}{\epsilon} = 16 \times 10^{-4} \left[ \frac{\text{Im} V_{us}^* V_{ud}}{1.2 \times 10^{-4}} \right] \left( \frac{110 \text{MeV}}{m_s(2 \text{GeV})} \right)^2
\]

\[
\times \left\{ B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 \left( \frac{\bar{m}_K}{165 \text{GeV}} \right)^{2.5} B_8^{(3/2)} \right\},
\]

illustrates the difficulty that arises from a cancellation between strong and electroweak penguin contributions and the need to know the hadronic matrix elements \( B_i \propto \langle \pi \pi | O_i | K \rangle \), which involve a two-pion final state, accurately. Before 1999 it was commonly, though not universally, assumed that \( B_{6,8} \approx 1 \) near their vacuum saturation value, and with the isospin breaking factor \( \Omega_{IB} \approx 0.25 \), this gives only about \( 6 \cdot 10^{-4} \). The experimental result has triggered a large theoretical activity directed towards understanding better the hadronic matrix elements. Different approaches continue to disagree by large factors, but it appears now certain that serious matrix element calculations must in one way or another account for final state interactions of the two pions. Chiral perturbation theory combined with a large-\( N_c \) matching of the non-leptonic operators can probably go furthest towards this goal with analytic methods. The calculation reported in [33] finds \( B_6^{(1/2)} \) enhanced by a factor 1.55 through rescattering and this, together with a reevaluation of isospin-breaking, may account for the experimental result within theoretical uncertainties.

It has also been demonstrated that, in principle, lattice QCD can settle the matrix element issue definitively, since \( K \to \pi \pi \) matrix elements computed in a lattice of finite volume, can be matched to continuum, infinite-volume matrix elements including all information on rescattering [34,35]. Due to the potential cancellations the matrix elements are needed with high precision. They are also needed soon, since further CP-violating observables that will be measured in the future will diminish the importance of \( \epsilon'/\epsilon \).

**Rare kaon decays (future).** There exist several proposals to measure the very rare decays \( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K_L \to \pi^0 \nu \bar{\nu} \) with expected branching fractions of about \( 7 \times 10^{-11} \) and \( 3 \times 10^{-11} \), respectively. The first of these decays is CP-conserving and constrains \( \epsilon'/\epsilon \) to lie on a certain ellipse in the \((\rho, \eta')\)-plane. The second decay is CP-violating and determines \( \eta' \). The branching fractions are predicted theoretically with high precision, so that these two kaon modes alone can in principle fix the shape of the unitarity triangle, or uncover inconsistencies with other constraints [36]. Two \( K^{+} \to \pi^{+} \nu \bar{\nu} \) events have in fact been observed [37] resulting in a branching fraction somewhat larger than expected but consistent with expectations within the experimental error.

### 3.3. Summing up

The four quantities \( |V_{ub}/V_{cb}|, \Delta M_{B_d,s}, \) and \( |\epsilon| \) are usually combined into a global fit of \((\rho, \eta')\). Different groups use different statistical methods, but since the dominant errors of all input quantities are theoretical, no sophisticated procedure can conceal the fact that there is a difficulty in quantifying such errors objectively. Currently the various procedures appear to give similar results when the same inputs are used. Figure 2 shows the result of one such global fit [38].

The four quantities are in remarkable agreement. This results in an indirect determination of the angles of the unitarity triangle, in particular

\[
\sin(2\beta) = 0.68 \pm 0.21, \quad (14)
\]

\[
\gamma = (58 \pm 24)^\circ. \quad (15)
\]

As discussed above the precision of the indirect
3.4. \( \sin(2\beta) \)

In 2001 CP violation has been observed also in \( B \) meson decays, more precisely in the interference of mixing and decay. Assume that both, \( B^0 \) and \( \bar{B}^0 \), can decay into a CP eigenstate \( f \), call the amplitude of the former decay \( A \), the latter \( \bar{A} \) and define \( \lambda = e^{-2i\beta} \bar{A}/A \) with \( 2\beta \) the phase of the \( B\bar{B} \) mixing amplitude (standard phase convention). A \( B \) meson identified as \( B^0 \) at time \( t = 0 \) can decay into \( f \) at a later time \( t \) either directly or indirectly through its \( \bar{B}^0 \) component acquired by mixing. If there is CP violation, the amplitude for the CP conjugate process will be different, resulting in a time-dependent asymmetry

\[
A_{\text{CP}}(t) = \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)}
\]  

\[
= \frac{2\text{Im}\lambda}{1 + |\lambda|^2} \sin(\Delta M_B t) - \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M_B t).
\]

In the special case that \( A \) is dominated by a single weak phase, \( A = |A|e^{i\delta_W} \) (so that \( |\lambda| = 1 \)), the asymmetry is proportional to \( \pm \sin 2(\beta + \delta_W) \), the sign depending on the CP eigenvalue of \( f \).

The final state \( J/\psi K_S \) (and related ones) satisfies this special condition to the accuracy of a percent. Furthermore \( \delta_W \approx 0 \) for \( b \to c\bar{c}s \). Hence the mixing-induced CP asymmetry in \( B \to J/\psi K \) decay determines the \( B\bar{B} \) mixing phase (relative to \( b \to c\bar{c}s \)), or \( \sin(2\beta) \) in the Standard Model, with little theoretical uncertainty \([39,40]\). It determines the \( B\bar{B} \) mixing phase also beyond the Standard Model, since it is unlikely that the CKM-favoured \( b \to c\bar{c}s \) transition acquires a large CP-violating phase from new flavour-changing interactions.

The asymmetry is now precisely measured by the two \( B \) factories. The central values reported by both experiments have been increasing over the past year as the statistics of the experiments improved and now reads \([9,10]\)

\[
\sin(2\beta) = \begin{cases} 
0.59 \pm 0.15 & \text{BaBar} \\
0.99 \pm 0.15 & \text{Belle}, \end{cases}
\]

yielding the world average \( \sin(2\beta) = 0.79 \pm 0.10 \). The fact that this asymmetry is large and in agreement with the indirect determination of the angle \( \beta \) leads to two important conclusions on the nature of CP violation:

- CP is not an approximate symmetry of nature (as could have been if CP violation in kaon decays were caused by some non-standard interactions).
- the Kobayashi-Maskawa mechanism of CP violation is most likely the dominant source of CP violation at the electroweak scale.

New flavour-changing interactions certainly could have affected \( B\bar{B} \) mixing and could have been revealed first by the direct \( \sin(2\beta) \) measurement. If the CKM matrix were the only source of flavour-changing processes also in an extension of the Standard Model, then the new interactions that modify \( B\bar{B} \) mixing also affect \( K\bar{K} \) mixing. One

Figure 2. Summary of unitarity triangle constraints (excluding the direct measurement of \( \sin(2\beta) \) which is overlaid) \([38]\).
then finds (if one adds an additional assumption that there are no new operators in the low-energy effective weak Hamiltonian) that only small modifications of the $B\bar{B}$ mixing phase (still related to the phase of $V_{td}$ in such models of “minimal flavour violation”) can have been possible, in particular $\sin(2\beta) > 0.42$ with a preferred range from 0.5 to 0.8 [41–44]. Vice versa the observation of a very small or very large $\sin(2\beta)$ would have implied a new mechanism of flavour violation with (probably) new CP phases. As discussed below in a more general context, this would have led to a CP problem.

### 4. MORE CP VIOLATION IN B MESON DECAYS

The search for CP violation will continue in many ways ($B$ decays, $D$ decays, $K$ decays, electric dipole moments), but the Kobayashi-Maskawa mechanism predicts large effects only in $B$ decays and very rare $K$ decays. The primary focus of the coming years will be to verify relations between different observables predicted in the Kobayashi-Maskawa scenario and to search for small deviations.

An example of this type is $\bar{B}_d \to \phi K$ due to the penguin $b \to s\bar{s} s\bar{s}$ transition at the quark level. In the Standard Model the time-dependent CP asymmetry of this decay is also proportional to $\sin(2\beta)$ to reasonable (though not as good) precision. However, new interactions are more likely to affect the loop-induced penguin transition than the tree decay $b \to c\bar{c}s$ and may be revealed if the time-dependent asymmetry in $\bar{B}_d \to \phi K$ turns out to be different from that in $\bar{B}_d \to J/\psi K$. However, if the difference is small, its interpretation requires that one controls the strong interference effects connected with the presence of a small up-quark penguin amplitude with a different weak phase. This difficulty is of a very general nature in $B$ decays.

#### 4.1. CP violation in decay

The need to control strong interference effects is closely related to the possibility of observing CP violation in the decay amplitude. The decay amplitude has to have at least two components with different weak phases,

$$A(B \to f) = A_1e^{i\delta_{S1}}e^{i\delta_{W1}} + A_2e^{i\delta_{S2}}e^{i\delta_{W2}}. \quad (18)$$

If the strong interaction phases are also different, the partial width of the decay differs from that of its CP-conjugate, $\Gamma(B \to f) \neq \Gamma(\bar{B} \to f)$. Many rare $B$ decays are expected to exhibit CP violation in decay due to the interference of a tree and a sizeable or even dominant penguin amplitude. The weak phase difference can only be determined, however, if the strong interaction amplitudes are known. This is also necessary for mixing-induced CP asymmetries, if the decay amplitude is not dominated by a single term.

There exist two complementary approaches to obtain the strong interaction amplitudes. The first employs a general parameterization of the decay amplitudes of a set of related decays, implementing SU(2)-isospin relations. The remaining strong interaction parameters are then determined from data (often also using SU(3) flavour symmetry and “little” further assumptions on the magnitudes of some amplitudes). Very often this needs difficult measurements. The second approach attempts to calculate the strong interaction amplitudes directly from QCD with factorization methods also used in high-energy strong interaction processes. This approach makes essential use of the fact that the $b$ quark mass is large. There is currently no theoretical framework that also covers $1/m_b$-corrections systematically, so there is an intrinsic limitation to the accuracy that one can expect from this approach. Nonetheless, the additional information on the dynamics of the decay provided by this approach is important as long as data is sparse and will continue to be useful later on.

#### 4.2. The angle $\gamma$

In the Standard Model the angle $\beta$ is obtained accurately from the time-dependent CP asymmetry in $B_d \to J/\psi K$. It remains to determine directly the angle $\gamma$, the phase of $V_{ub}^*$. The preferred methods rely on decays with interference of $b \to c\bar{u} D$ (no phase) and $b \to ucD$ (phase $\gamma$) transitions and their conjugates ($D = d, s$). These decays receive no penguin contributions and are arguably insensitive to new flavour-
changing interactions. \( \gamma \) can be extracted from either of the following decay classes, \( B_d(t) \to D^{\pm} \pi^\mp \) [45], \( B_s(t) \to D K_S \) [46], \( B^\pm \to K^\pm D_{CP} \) [47], \( B_s(t) \to D_s^0 K^\mp \) [48], since every one of them provides sufficiently many observables to eliminate all strong interaction parameters, thus providing nice illustrations of the first of the two approaches mentioned above. None of these strategies is simple to carry out experimentally, however, since they imply that each of the modes is of order \( 10^{-5} \) and have already been measured with an error of \( \pm (10 - 20) \% \), including first measurements of direct CP asymmetries (all compatible with zero). The drawback of these and related modes is that the amplitudes contain more strong interaction parameters than there are observables. SU(3) symmetry and the structure of the weak effective Hamiltonian allow one to construct a number of interesting bounds on \( \gamma \) [49,50], but a full understanding of these modes requires a calculation of the penguin-to-tree amplitude ratio including its strong rescattering phase. In the following I describe very briefly one such method.

4.3. QCD factorization

In the heavy quark limit the \( b \) quark decays into very energetic quarks (and gluons), which must recombine to form two mesons. Methods from the heavy quark expansion and soft-collinear factorization (“colour-transparency”) can be argued to imply a factorized form of the amplitude of a decay into two light mesons [51,52]. Schematically,

\[
A(\bar{B} \to M_1 M_2) = F^{B \to M_1}(0) \int_0^1 du T^{I}(u) \Phi_{M_2}(u) + \int d\xi du T^{I}(\xi, u) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u),
\]

where \( F^{B \to M_1} \) is a form factor, \( \Phi_X \) denote light-cone distribution amplitudes and \( T^{I,I} \) are perturbative hard-scattering kernels, which also contain the strong rescattering phases. This result is valid up to \( 1/m_b \) corrections, some of which can be large. The extent to which the QCD factorization formalism can be of quantitative use is not yet fully known. The approach has been successful in explaining the universality of strong-interaction effects in class-I \( B \to D^+ \) light meson decays and understanding the non-universality in the corresponding class-II decays [52]. It also appears to account naturally for the magnitude of the \( \pi K \) branching fractions, sometimes considered as unexpectedly large, but there is currently no test that would allow one to conclude that the computation of strong interaction phases which are either of order \( \alpha_s \) or \( 1/m_b \) is reliable in the case of penguin-dominated final states [53]. Such tests will be possible soon and the non-observation of direct CP violation at the current level of sensitivity already supports the idea that strong rescattering effects are suppressed.

Figure 3 shows the result of a global fit of \((\bar{\rho}, \bar{\eta})\) to CP-averaged \( B \to \pi K, \pi\pi \) branching fractions with a QCD factorization computation.
used as an input [53]. The result is consistent with the standard fit based on meson mixing and $V_{ub}$, but shows a preference for larger $\gamma$ or smaller $V_{ub}$. If the estimate of the theory uncertainty (included in the curves in the Figure) is correct, non-leptonic decays together with $|V_{ub}|$ from semileptonic decays already imply the existence of a CP-violating phase of $V_{ub}$.

4.4. Resumé

The $B$ factories are already providing data on dozens of rare $B$ decay modes. QCD calculations – though probably not very precise – will be necessary to interpret these data beyond “simple” quantities like the mixing-induced CP asymmetry in $B_d \to J/\psi K$. The immediate future should be very interesting since the measurements of direct CP asymmetries at the few percent level and the mixing-induced CP asymmetry in $B_d \to \pi^+\pi^-$ decay provide tests of the theoretical framework and further information on CP violation. Subsequent second generation $B$ physics experiments will probably supply enough data to rely more and more on measurements and symmetries. Altogether the Kobayashi-Maskawa mechanism will be decisively and precisely tested, but on the way one can expect many discussions on hadronic physics, imagined and, perhaps, true new physics signals.

Lattice calculations will continue to play an important role by making more precise the standard unitarity triangle fit. They could also provide some of the non-perturbative quantities (form factors, light-cone-distribution amplitudes) that are needed in factorization-based calculations of non-leptonic decay amplitudes. It will be much harder for lattice QCD to make an impact on non-leptonic, exclusive decays directly, since inelastic rescattering dominates final-state interactions and there is currently no method that would allow one to compute this on the lattice.

5. CP VIOLATION IN EXTENSIONS OF THE STANDARD MODEL

The emerging success of the Kobayashi-Maskawa mechanism of CP violation is sometimes accompanied by a sentiment of disappointment that the Standard Model has not finally given way to a more fundamental theory. However, returning to the perspective of the year 1973, when the mechanism was conceived, one can hardly feel this way. After all, the Kobayashi-Maskawa mechanism predicted a new generation of particles on the basis of the tiny and obscure effect of CP violation in $K\bar{K}$ mixing. It then predicted relations between CP-violating quantities in $K$, $D$, $B$-physics which a priori might be very different. The fact that it has taken nearly 30 years to assemble the experimental tools to test this framework does not diminish the spectacular fact that once again Nature has realized a structure that was concepted from pure reasoning.

Nevertheless several arguments make it plausible that the Kobayashi-Maskawa mechanism is not the final word on CP violation. The strong and cosmological CP problem (baryogenesis) continue to call for an explanation, possibly related to high energy scales. There may be an aesthetic appeal to realizing the full Poincaré group as a symmetry of the Lagrangian, in which case CP and P symmetry breaking must be spontaneous. One of the strongest arguments is, however, that the electroweak hierarchy problem seems to require an extension of the Standard Model at the TeV scale. Generic extensions have more sources of CP violation than the CKM matrix. These have not (yet) been seen, suggesting that there is some unknown principle that singles out the CKM matrix as the dominant source of flavour and CP violation. In the following I give a rather colloquial overview of CP violation in generic TeV scale extensions of the Standard Model. This is perhaps an academic catalogue, but it illustrates how restrictive the Kobayashi-Maskawa framework is.

5.1. Extended Higgs sector

Extending the Higgs sector by just a second doublet opens many new possibilities. The Higgs potential may now contain complex couplings, leading to Higgs bosons without definite CP parity, to CP violation in charged Higgs interactions, flavour-changing neutral currents, and CP violation in flavour-conserving interactions such as $t\bar{t}H$ and electric dipole moments. The Lagrangian
could also be CP-conserving with CP violation occurring spontaneously through a relative phase of the two Higgs vacuum expectations values [2].

Both scenarios without further restrictions already cause too much CP violation and flavour-changing neutral currents, so that either the Higgs bosons must be very heavy or some special structure imposed. For example, discrete symmetries may imply that up-type and down-type quarks couple to only one Higgs doublet, a restriction known as “natural flavour conservation” [54,55], since it forbids flavour-changing neutral currents (and also makes spontaneous CP violation impossible with only two doublets). With flavour conservation imposed, CP and flavour violation occurs through the CKM matrix, but in addition to the usual charged currents also in charged Higgs interactions. This is usually considered in the context of supersymmetry, since extended Higgs models suffer from the same hierarchy problem as the Standard Model.

5.2. Extended gauge sector (left-right symmetry)

Left-right-symmetric theories with gauge group SU(2)_L × SU(2)_R × U(1)_{B−L} are attractive [56], because parity and CP symmetry can be broken spontaneously. The minimal model requires already an elaborate Higgs sector (with triplets in addition to doublets) and suffers from the hierarchy problem. CP violation in the quark sector now occurs through left- and right-handed charged currents with their respective CKM matrices. But since all CP violation arises through a single phase in a Higgs vacuum expectation value there is now a conflict between suppressing flavour-changing currents and having this phase large enough to generate the CP violating phenomena already observed. The minimal left-right symmetric model with spontaneous CP violation is therefore no longer viable [57,58].

5.3. Extended fermion sector

The Standard model can be extended by an extra d-type quark with electric charge −1/3 [59]. This quark should be a weak singlet in order not to conflict electroweak precision tests. After electroweak symmetry breaking, the down-quark mass matrix must be diagonalized by a unitary 4 × 4 matrix. The motivation for such an extension of the Standard Model may be less clear, in particular as there is no symmetry principle that would make the extra singlet quark naturally light. However, this theory provides an example in which the unitarity triangle does no longer close to a triangle, but is extended to a quadrangle:

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + U_{db} + U_{d\alpha} = 0 \]
\[ \approx 8 \times 10^{-3} < 10^{-3} \]

The unitarity triangle “deficit” \( U_{db} \) also determines the strength of tree-level flavour-changing \( Z \) boson couplings and is currently constrained by \( B\bar{B} \) mixing and rare decays to about a tenth of the length of a side of the triangle. (The corresponding coupling \( U_{d\alpha} \) is constrained much more tightly in the kaon system.) This model could in principle still give large modifications of \( B\bar{B} \) mixing and non-leptonic \( B \) decays, including CP asymmetries [60].

5.4. Supersymmetry

The minimal supersymmetric standard model [61,62] is arguably the most natural solution to the electroweak hierarchy problem, but it is not particularly natural in its most general form from the point of view of CP violation. The Lagrangian including the most general Lagrangian that breaks supersymmetry softly contains 44 CP-violating constants of nature. (R-parity conservation is assumed.) One of them is the usual CKM phase which appears in charged current and chargino interactions. Three phases appear in flavour-conserving CP observables, 27 in flavour- and CP-violating quark-squark-gluino interactions (squark mass matrices and \( A \)-terms) and 13 in the (s)lepton sector.

The flavour-conserving phases must be small to comply with the non-observation of electric dipole moments. An intriguing feature of supersymmetry is the existence of CP and flavour violation in strong interactions (gluinos, squarks). These interactions can be much stronger than the Standard Model weak interactions and to suppress them to a phenomenologically acceptable
level, one has to assume that either (some of) the masses of superparticles are rather large, or that the squark mass matrices are diagonal in the same basis that also diagonalizes the quark mass matrices (alignment) or that the squarks have degenerate masses, in which case a generalization of the GIM mechanism suppresses flavour-changing couplings \cite{63,64}. Since almost all CP-violating phases of the minimal supersymmetric standard model originate from supersymmetry breaking terms, one must understand supersymmetry breaking to answer the question why CP and flavour violation are so strongly suppressed. There exist mechanisms which can naturally realize one or the other of the conditions listed above (for example supersymmetry breaking through gauge interactions \cite{65}), but none of the mechanisms is somehow singled out.

There is currently much activity aiming at constraining the flavour- and CP-violating couplings from the many pieces of data that become now available. In fact these couplings are so many-fold that the CP-violating effects observed in kaon and $B$ meson decays can all be ascribed to them at the price of making the consistency of the Kobayashi-Maskawa mechanism appear accidental. The hope could be that eventually some pattern of restrictions on these small couplings is seen that could give a hint on the origin of supersymmetry breaking. It is also plausible to assume that strong flavour and CP violation is absent (or too small to observe) in supersymmetry. Neglecting also the flavour-conserving CP-violating effects, the CKM matrix is then the only effect of interest. The presence of additional particles with CKM couplings still implies modifications of meson mixing and rare decays, but these modifications are now much smaller and, in general (but excepting rare radiative decays), precise theoretical results are needed to disentangle them from hadronic uncertainties.

Whatever the outcome of the search for new CP violation may be, it will restrict the options for model building severely. The current data point towards a privileged standing of the CKM matrix. However, a theoretical rationale for this privileged standing is yet to be discovered.

6. CONCLUSIONS

I. The (expected) observation of large CP violation in $B$ decays together with $\epsilon'/\epsilon$ and the consistency of indirect determinations of the unitarity triangle imply that:

- CP is not an approximate symmetry of nature – rather CP violation is rare in the Standard Model because of small flavour mixing.

- the Kobayashi-Maskawa mechanism of CP violation in charged currents is probably the dominant source of CP violation at the electroweak scale.

II. CP and electroweak symmetry breaking provide complementary motivations to search for extensions of the Standard Model, but:

- on the one hand, there exists no favoured candidate model for CP violation beyond the Standard Model – rather there is a CP problem in many conventional extensions.

- on the other hand, baryogenesis requires CP violation beyond the Standard Model, probably decoupled from CP violation observable at accelerators.

III. The study of CP violation is at a turning point with many new experimental capabilities and new theoretical methods to interpret non-leptonic decay data. Perhaps the most important result of the near future, however, will be to find (or not find) the $B_s$ mass difference $\Delta M_{B_s} \approx 17.5 \text{ ps}^{-1}$, confirming once more the Standard Model paradigm (or to put it into serious difficulty).

ACKNOWLEDGEMENT

I wish to thank the organizers, F. Jegerlehner and M. Müller-Preussker in particular, for their support and endless patience.

REFERENCES

18. S. Ryan, talk at this symposium [hep-lat/0110110].
19. G. Martinelli, talk at this symposium [hep-lat/0112013].
37. S. Adler et al. [E787 Collaboration], [hep-ex/0111091].
48. R. Aleksan, I. Dunietz and B. Kayser, Z.