Basic Parameters and Some Precision Tests of the Standard Model(*)

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Abstract

We present a review of the masses (except for neutrino masses) and interaction strengths in the standard model. Special emphasis is put on quantities that have been determined with significantly improved precision in the last few years. In particular, a number of determinations of $\alpha_s$ and the electromagnetic coupling on the $Z$, $\alpha_{\text{QED}}(M_Z^2)$ are presented and their implications for the Higgs mass discussed; the best prediction that results for this last quantity being

$$M_H = 102^{+54}_{-36} \text{GeV}/c^2.$$  

Besides this, we also discuss a few extra precision tests of the standard model: the electron magnetic moment and dipole moment, and the muon magnetic moment.

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1. Introduction

The standard model (SM) of particle physics, based on the gauge interactions described by the group $SU(3) \times SU_L(2) \times U(1)$ supplemented by the Higgs-Weinberg mechanism for spontaneously breaking the symmetry, represents an incredibly precise and successful description of the microscopic Universe. Up to the highest energies explored by our accelerators, a few hundreds of GeV (per constituent, in the case of the Tevatron or HERA) and to an accuracy that reaches the 12 exact digits, for some electromagnetic processes, the standard model provides answers to essentially all the non-gravitational observed phenomena. What is more, there is no established observation that contradicts the predictions of the SM: the scares that, from time to time, lead to hundreds of papers being written on possible new physics signals, have invariably evaporated, and arguments (for example, of naturalness) about new physics lurking just around the bend are, at best, wishful thinking.

In this talk I will concentrate on the determination of some basic parameters; to be precise, masses (except neutrino masses, already widely discussed at this Meeting) and couplings. I will also discuss a few of the precision tests of the standard model, with special emphasis on those to which I have had the occasion to contribute through some of the corresponding theoretical evaluations.

2. Masses and charges of electron, $\mu$ and $\tau$

2.1. Charge and mass of the electron. The electron magnetic moment

When, in 1897, J. J. Thomson discovered the electron, he also tried to find its charge and mass. He could, however, not measure accurately enough the charge deposited in the anode. So, he only was able, by measuring the deviation of the electron in Crooke’s tube, to determine the ratio $e/m$. This ratio was useful, for example, to identify the $\beta$ rays in radioactive decays of electrons (Fig. 1). Nowadays, one measures independently the charge of the electron and, once this known, one derives its mass from the ratio $e/m$, measured by the bending of the trajectories of electrons in magnetic fields, just as Thomson and Chadwick did.

\[
\alpha = \frac{1}{137.0359770} \quad [\text{Josephson effect}]
\]
\[
\alpha = \frac{1}{137.0360037} \quad [\text{Quantum Hall effect}].
\]

\[1 \text{ Of course, I am aware that this is not strictly true, as there are questions which, while not contradicting the standard model, have also not been given a satisfactory answer in its context. I am referring to the existence of the Higgs particle: the observational evidence for its existence is not compelling; to the the question of the neutrino masses (it is unclear how to accommodate the nonzero masses in the model), and to the unexplained smallness of the strong CP violating phase, } \theta_{\text{QCD}}.\]
Note that these numbers differ by more than three standard deviations. The more precise and independent determination of $\alpha$ from the electron magnetic moment, to be discussed presently, favours the value obtained from the quantum Hall effect.

The more precise value for $\alpha$, as anticipated, is an indirect one. Consider the anomalous magnetic moment of the electron, $a(e)$. To fourth order in electromagnetic interactions, we can write

$$a(e) = \frac{1}{2} \left( \frac{\alpha}{\pi} \right) + c_2 \left( \frac{\alpha}{\pi} \right)^2 + c_3 \left( \frac{\alpha}{\pi} \right)^2 + c_4 \left( \frac{\alpha}{\pi} \right)^2;$$

$$c_2 = -0.328478965, \quad c_3 \simeq 1.181241456, \quad c_4 \simeq -1.610 \pm 0.038.$$  

(Actually, $c_2$ and $c_3$ are known exactly, the analytical evaluation of last completed very recently; only for $c_4$ there exists only numerical evaluations). Including also the very small $\left(4.393 \pm 0.027\right) \times 10^{-12}$ $\mu$ and $\tau$ loop and weak and strong corrections one has, using (2.1), and comparing with experiment,

$$a(e) = (1.159652140 \pm 27) \times 10^{-12} \quad [\text{Theory}],$$

$$a(e) = (1.159652188 \pm 3) \times 10^{-12} \quad [\text{Experimental}].$$

The agreement is astonishing: eleven significant digits! Indeed, one should remember that the magnetic moment is given by the formula

$$\mu_e = [1 + a(e)] \mu_B$$

where $\mu_B$ is the Bohr magneton, given by the Dirac equation: theory predicts the whole of $\mu_e$, not merely the anomaly.

What is perhaps even more impressive is that most of the theoretical error comes from the experimental error in $\alpha$, Eq. (2.1). This suggests that we reverse the argument and deduce $\alpha$ from $a(e)$. If we do this, we get

$$\alpha = \frac{1}{137.0359996} \quad [\text{Deduced from } a(e)],$$

(2.3)

almost six times more precise than the values obtained with the traditional methods based on the Hall and Josephson effects.

Because of radiative corrections, the intensity of the interactions depend on the energy at which they are measured. The value we have recorded for $\alpha$ is that measured at zero energy. In many applications one needs the value of $\alpha$ at the rest energy of the $Z$ particle, $\alpha(M_Z^2)$. The more recent figure for this quantity is $\alpha(M_Z^2) = 1128.965$ \cite{17}.

(2.4)

Once $\alpha$ known one can obtain the mass of the electron from the ratio $e/m_e$. According to the Particle Data Group (PDG)\cite{3} one has

$$m_e = 0.51099905[15] \quad \text{MeV} / c^2.$$  

(2.5)

2.2. Charge and masses of $\mu$, $\tau$

The identity of the charges of the electron, $\mu$ ad $\tau$ follow from charge conservation, implied by the masslessness of the photon. This is because of the existence of the decays

$$\mu \rightarrow \bar{\nu} e, \quad \tau \rightarrow \bar{\nu} l, \quad l = e, \mu$$

and the zero electric charge of the neutrinos,\footnote{That the charge of the neutrinos must be zero or at least extremely small follows from the fact that solar $\nu$’s arrive on Earth. These neutrinos are produced at the center of the Sun and, if they had the minutest charge, they could not have escaped traversing the whole of the Sun.} very well established.

The masslessness of the photon, in turn, is linked to the range of electromagnetic interactions. Since these have been measured at some 200,000 Km from the source (the magnetic field of Jupiter), we have a bound on the photon mass of

$$m_\gamma \leq 10^{-32} \quad \text{MeV} / c^2.$$
In view of this, we are justified in assuming the charges of $\mu$, $\tau$ to be identical to that of the electron.

As for the masses of these particles, that of the $\tau$ follows from measurements of the location of the threshold for $\tau^+\tau^-$ production at the Beijing collider. The PDG\cite{pdg} give the figure

$$m_\tau = 1777.03^{+0.30}_{-0.26} \text{ MeV }/c^2.$$  \hfill (2.6)

The $\mu$ particle lives long enough (a milliwhiff of a second) that it leaves clear tracks, and hence its mass can be obtained (since its charge is known) as for the electron, measuring the curvature of these tracks. This way one gets,\cite{pdg}

$$m_\mu = 105.658389 \pm 0.000034 \text{ MeV }/c^2.$$  \hfill (2.7)

However, and as for the electron charge, an indirect method exists that gives a more precise value. Consider the hyperfine splitting, $\Delta\nu$, in the hydrogen-like bound state ($\mu^+e^-$). It can be measured experimentally, and evaluated theoretically. The (very complicated) theoretical calculation is discussed in the contribution of J. Sapiristein and D. Yennie in ref. 4. One has,

$$\Delta\nu = \frac{m_\tau^3}{m_e m_\mu} \left[ 1 + a(\mu) \right] \left\{ 1 + a(e) + (\log 2 - 1)\alpha^2 - (\log \alpha - 2\log 2 + \frac{281}{120}) \frac{8\log \alpha}{3\pi} \alpha^3 + D\alpha^3 \right. $$

$$- \frac{3\alpha m_e m_\mu \log(m_\mu/m_e)}{m_\mu^2 - m_e^2} + \frac{m_e \alpha^2}{m_\mu} \left[ \frac{2\log \frac{m_\tau}{2\gamma} - 6\log 2 + \frac{11}{6} \right] $$

$$+ \left( \frac{\alpha^2}{\pi} \right) \frac{m_e}{m_\mu} \left[ -2 \log^2 \frac{m_\mu}{m_e} + \frac{13}{12} \log \frac{m_\mu}{m_e} + \frac{21}{2} \zeta(3) \right] + \frac{\pi^2}{6} + \frac{35}{9} + (1.9 \pm 0.3) \right\}. $$

Here $m_\tau$ is the reduced mass, $m_\tau^{-1} = m_e^{-1} + m_\mu^{-1}$, $\gamma = m_e\alpha$ and $D = (15.38 \pm 0.29)/\pi + D_1$, with $D_1$ an as yet uncalculated constant related to diagrams involving two virtual photons; see the quoted article of Sapiristein and Yennie. If we took the value of $m_\mu$ from direct measurements, (2.7), we would find\cite{pdg}

$$\Delta\nu = 4463.303.11 \pm 2.6 \text{ [Theory]}$$

$$\Delta\nu = 4463.302.88 \pm 0.16 \text{ [Experiment].} $$  \hfill (2.8)

The error in the theoretical evaluation is due almost exclusively to that of $m_\mu$ from direct measurements. Hence, as with the anomalous magnetic moment of the electron and its charge, we may reverse the argument and obtain, from the experimental hyperfine splitting, $\Delta\nu$, and the theoretic calculations, a figure for the muon mass seven times more precise than the value obtained with traditional methods:

$$m_\mu = 105.658357 \pm 0.000005 \text{ MeV }/c^2.$$  \hfill (2.9)

3. The strong interaction coupling, $\alpha_s$, and quark masses

3.1. The strong interaction coupling

There are, in the recent years, a number of theoretical calculations that have been carried to NNLO (next to next to leading order) in $\alpha_s$. This has permitted an evaluation of $\alpha_s$ to an error of 1%; the consistency of the calculations furnishes a set of tests of QCD at the level of a few percent. The processes in question are 1. Hadronic decays of the $\tau$. 2. Hadronic decays of the $Z$. 3. Hadronic annihilations of $e^+e^-$. 4. Deep inelastic collisions of electrons, muons and neutrinos with nucleons (DIS). In addition we have sum rules, in particular the Bjorken and Gross–Llewelyn Smith sum rules.

The ratio of the decays of $\tau$ into hadrons and leptons (Fig. 2), and the like ratios for $Z$ decays and $e^+e^-$ annihilations

$$R = \frac{\sigma(e^+e^- \rightarrow \text{ hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

- 3 -
yielded some of the first precise measurements of $\alpha_s$, because they were the first to be evaluated to NNLO. For example, one has, with $s^{1/2}$ the c.m. energy, and neglecting quark masses,\[5\]

$$R(s) = 3 \sum_{f=1}^{n_f} Q_f^2 \left\{ 1 + \frac{\alpha_s(s)}{\pi} + r_2 \left( \frac{\alpha_s(s)}{\pi} \right)^2 + r_3 \left( \frac{\alpha_s(s)}{\pi} \right)^3 \right\} + O(\alpha_s^4),$$

(3.1a)

where

$$r_2 = \frac{365}{22} - 11\zeta(3) + \left[ \frac{3}{4} \zeta(3) - \frac{11}{12} \right] n_f \simeq 2.0 - 0.12 n_f,$$

$$r_3 = -6.637 - 1.200 n_f - 0.005 n_f^2 - 1.240 \left( \sum_{f=1}^{n_f} Q_f \right)^2 \left( \sum_{f=1}^{n_f} Q_f^2 \right)^{-1}. \quad (3.1b)$$

$Q_f$ are the quark charges, and $n_f$ the number of active flavours. The three loop expression for the running constant has to be used here:\[3\]

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 L} \left\{ 1 - \frac{\beta_1 \log L}{\beta_0^2 L} + \frac{\beta_2^2 \log^2 L - \beta_2^4 \log L + \beta_2 \beta_0 - \beta_0^2}{\beta_0^4 L^2} \right\}.$$

$L = \log Q^2/A^2$.

The values of the $\beta_i$ are given after Eq. (3.7).

\[3\] Actually, $\alpha_s$ is known to four loops; cf. ref. 6.
DIS (Fig. 3) furnished the first NLO evaluations of $\alpha_s$, and now provide the more precise NNLO results, because their energy range is so large. Moreover, they also give an important check of QCD in that the theoretical evaluations successfully predict the experimental scaling violations.

The results are summarized in Table 1. There, DIS means deep inelastic scattering, Bj stands for the Bjorken, and GLS for the Gross–Llewellyn Smith sum rules. The values given for $Z \to \text{hadrons}$ assume the Higgs mass constrained by $100 \leq M_H \leq 200$ GeV, with the central value for $M_H = 115$ GeV. We have presented two results here for $e/\mu p$ DIS, the one for the smaller $Q^2$ range (I, probably the more reliable) and the extended range one, labeled II. DIS provides the more precise determination of $\alpha_s$, which is not surprising: the extended $Q^2$ range allows a sizable variation to the logarithmic dependence of $\alpha_s$ on $Q^2$.

<table>
<thead>
<tr>
<th>Process</th>
<th>Average $Q^2$ or $Q^2$ range [GeV]^2</th>
<th>$\alpha_s(M_Z^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIS; $\nu$, Bj</td>
<td>1.58</td>
<td>0.121 $\pm$ 0.005</td>
</tr>
<tr>
<td>DIS; $\nu$, GLS</td>
<td>3</td>
<td>0.112 $\pm$ 0.010</td>
</tr>
<tr>
<td>$\tau$ decays</td>
<td>$(1.777)^2$</td>
<td>0.1181 $\pm$ 0.0031</td>
</tr>
<tr>
<td>$e^+e^- \to \text{hadrons}$</td>
<td>100 $\to$ 1600</td>
<td>0.128 $\pm$ 0.025</td>
</tr>
<tr>
<td>$Z \to \text{hadrons}; \Gamma_Z$</td>
<td>$(91.2)^2$</td>
<td>0.1230 $\pm$ 0.0038</td>
</tr>
<tr>
<td>$Z \to \text{hadrons}; \text{GrandLEP}$</td>
<td>$(91.2)^2$</td>
<td>0.1185 $\pm$ 0.0030</td>
</tr>
<tr>
<td>$Z \to \text{jets}$</td>
<td>$(91.2)^2$</td>
<td>0.117</td>
</tr>
<tr>
<td>DIS($\nu N; xF_3$)</td>
<td>8 $\to$ 120</td>
<td>0.1153 $\pm$ 0.0041</td>
</tr>
<tr>
<td>DIS ($ep$, I)</td>
<td>3.5 $\to$ 230</td>
<td>0.1166 $\pm$ 0.0013</td>
</tr>
<tr>
<td>DIS ($ep$, II)</td>
<td>3.5 $\to$ 5000</td>
<td>0.1163 $\pm$ 0.0014</td>
</tr>
</tbody>
</table>

Table 1

As is seen, the more precise evaluations use DIS for $ep$ scattering. These in turn employ the calculations for the so-called Wilson coefficients and anomalous dimensions. Of these, only the ones relating to a few moments (six-seven) are known. This number increased in the last year; before, only four were known. Using only these four, the value obtained would be

$$\alpha_s(M_Z^2) = 0.1172 \pm 0.0024 \quad ep, \text{DIS, only four moments}.$$
The consistency with the values quoted in the Table is a proof of the stability of the DIS calculations. All determination are compatible with one another, within errors. The world average is

\[ \alpha_s(M^2_Z) = 0.1173 \pm 0.0011. \]  

(3.2)

3.2. The mass of the \( t \) quark

The quark \( t \) was predicted theoretically long before it was found experimentally. In 1977, Veltman showed, from an analysis of radiative corrections, that the measurements of the parameters of electroweak interactions could be made compatible only if \( m_t \) was less than some 300 GeV/c\(^2\). Then the measurements of those parameters were refined, especially after the beginning of operations of LEP, and the error in the theoretical prediction improved to 10%, which was the value just prior to experimental discovery. It, of course, has gone on improving, as we will show presently.

The reason why one gets so sensitive results is that the \( \rho \)-parameter (essentially, the ratio of the \( Z \), \( W \) masses) depends quadratically on \( m_t \).

\[ M_W/M_Z = \left\{ 1 + \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \right\} \cos \theta_W. \]

\( \theta_W \) is called the weak mixing angle, and measures the mixing of electromagnetic and neutral weak currents. \( G_F \) is the Fermi constant, given below.

Presently, we have two evaluations of the mass of the quark \( t \). The mass deduced from observables other than the \( t \) quark itself is, for the pole mass,

\[ m_t = 168.2^{+9.6}_{-7.4} \text{ GeV} / c^2 \quad \text{[Radiative corrections]} \quad (3.3) \]

while the direct experimental value, obtained after seven years of measurements, is

\[ m_t = 174.3 \pm 5.1 \text{ GeV} / c^2 \quad \text{[Tevatron]}. \quad (3.4) \]

The agreement is more impressive if we realize that part of the theoretical error is due to the uncertainty in the mass of the Higgs particle.

We have taken both values from the PDG.\(^3\) Note that the mass obtained with the direct measurement can only be identified with the pole mass if one neglects the decay of the \( t \), so (3.4) should have an extra error, likely small, of order \( m_t \alpha_s \alpha_W \sim 0.6 \text{ GeV}. \)

3.3. The masses of the quarks \( b, c \)

The more precise evaluations of the masses of these quarks are those obtained from quarkonia states. Here the precision is of \( O(\alpha_s^4) \) and the leading nonperturbative correction, involving the gluon condensate \( \langle \alpha_s G^2 \rangle_{\text{vac}} \), is also incorporated. For the \( b \) quark mass, the corrections due to the finite mass of the \( c \) quark, and the leading correction of order \( O(\alpha_s^5 \log \alpha_s) \) can also be included. The subleading nonperturbative corrections can be estimated, and are (relatively) small. Thus, we have a very precise evaluation of the mass of this quark. For the \( c \) quark, however, the nominally subleading corrections are so large that it is not even clear to what extent the inclusion of the perturbative corrections of \( O(\alpha_s^5) \) improve the results.

The mass of the heavy \( b, c \) quarks may then be obtained by solving the implicit equation that gives, in terms of it, the mass of the ground state (for which one sets \( n = 1, l = 0 \) below) of the corresponding quarkonium state,

\[ E_{nl} = 2m - m \frac{C_F \alpha_s^2}{4n^2} + \sum V \delta^{(1)}_V E_{nl} + \delta^{(2)}_V E_{nl} + \delta_{NP} E_{nl}. \quad (3.5) \]

We define the analogue of the Bohr radius,

\[ a(\mu^2) = \frac{2}{m C_F \alpha_s(\mu^2)}, \]

\(^4\) I am grateful to P. Langacker for discussions and information on this.
Moreover, the sum over $V$ in (3.5) runs over the following pieces:

$$
\delta_{V_{l, n, c}}^{(1)} E_{nl} = - \frac{2}{n^3 m^3 a^4} \left[ \frac{1}{2l + 1} - \frac{3}{8n} \right] + \frac{C_F \alpha_s}{m^2} \frac{2l + 1 - 4n}{n^4(2l + 1)a^4}; \tag{3.7a}
$$

$$
\delta_{V_{l, n, c}}^{(1)} E_{nl} = - \frac{\beta_0 C_F \alpha_s^2(\mu^2)}{2\pi n^2 a} \left[ \log \frac{na\mu}{2} + \psi(n + l + 1) \right]; \tag{3.7b}
$$

$$
\delta_{V_{l, n, c}}^{(1)} E_{nl} = - \frac{C_F C_2}{\pi n^2 a} \left[ \log \frac{na\mu}{2} + \psi(n + l + 1) \right]; \tag{3.7c}
$$

$$
\delta_{V_{l, n, c}}^{(1)} E_{nl} = - \frac{C_F \beta_0^2 \alpha_s^3}{4\pi n^2 a} \left( \log^2 \frac{na\mu}{2} + 2\psi(n + l + 1) \log \frac{na\mu}{2} \right.
\left. + \psi(n + l + 1)^2 + \psi'(n + l + 1) \right.
\left. + \theta(n - l - 2) \frac{2\Gamma(n - l)}{\Gamma(n + l + 1)} \sum_{j=0}^{n-l-2} \frac{\Gamma(2l + 2 + j)}{j!(n - l - j)!} \right); \tag{3.7d}
$$

$$
\delta_{V_{l, n, c}}^{(1)} E_{nl} = \frac{C_F a_2 \alpha_s^2}{m} \frac{1}{n^3(2l + 1)a^2}; \tag{3.7e}
$$

and

$$
\delta_{V_{l, n, c}}^{(2)} E_{nl} \equiv - \frac{C_F \beta_0^2 \alpha_s^4}{4m^2 \pi^2} \left\{ N_0^{(n, l)} + N_1^{(n, l)} \log \frac{na\mu}{2} + \frac{1}{4} \log^2 \frac{na\mu}{2} \right\}. \tag{3.7f}
$$

Here,

$$
N_0^{(1, 0)} = - \frac{\gamma_E}{2} \approx 0.1406608
$$

$$
N_1^{(2, 0)} = \frac{1}{4} - \frac{2\gamma_E}{4} \approx -0.0068078
$$

$$
N_1^{(3, 1)} = \frac{5}{12} - \frac{6\gamma_E}{12} \approx 0.111856
$$

$$
N_0^{(1, 0)} = \frac{3 + 3\gamma_E^2 - \pi^2 + 6\zeta(3)}{12} \approx 0.111856
$$

$$
N_0^{(2, 0)} = - \frac{5}{16} - \frac{7\gamma_E}{16} - \frac{\pi^2}{12} + \zeta(3) \approx 0.0068043
$$

$$
N_0^{(3, 1)} = \frac{5\gamma_E}{12} - \frac{7\gamma_E}{4} - \frac{11\pi^2}{36} + \zeta(3) \approx 0.0068043.
$$

For the vector states ($Y, T', Y'$; $J/\psi, \psi', \ldots$) one has to add the hyperfine shift, at tree level (setting $s$, the spin, equal to unity):

$$
\delta_{V_{s, n, c}}^{(1)} E_{nl} = \delta_{s1} \delta_{l0} \frac{8C_F \alpha_s}{3n^3 m^2 a^2}. \tag{3.7g}
$$

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5. The one loop static correction can be found in ref. 10; the velocity dependent one in ref. 11. Two loop corrections are from ref. 12, and the rest of the $O(\alpha_s^4)$ contributions are from ref. 13.
In these formulas,
\begin{align*}
\beta_0 &= 11 - \frac{2}{3} n_f; \\
\beta_1 &= 102 - \frac{38}{3} n_f; \\
\beta_2 &= \frac{2847}{2} - \frac{2033}{18} n_f + \frac{345}{9} n_f^2; \\
\epsilon_2^{(L)} &= a_1 \beta_0 + \frac{1}{3} \beta_1 + \frac{1}{2} \gamma_E \beta_0; \\
C_F &= 4/3, \quad C_A = 3, \quad T_F = 1/2,
\end{align*}
and \( \mu \) is a reference momentum, usually taken to be \( \mu = 2/na \).

In addition to this one has to consider the nonperturbative (NP) contributions to the energy levels. The dominant ones are associated with the gluon condensate and are\[^{[14]}\]
\begin{equation}
\delta_{NP} E_{nl} = m\epsilon_{nl} (\alpha_s G^2) \left( \frac{na}{2} \right)^4 = m \frac{\epsilon_{nl} n^6 \pi (\alpha_s G^2)}{(mC_F \alpha_s)^4}; \tag{3.8}
\end{equation}
\begin{align*}
\epsilon_{10} &= \frac{1872}{1275}, \quad \epsilon_{20} = \frac{4102}{4929}, \quad \epsilon_{21} = \frac{9292}{9945}.
\end{align*}

We will not give explicit formulas for the subleading nonperturbative effects, the \( O(\alpha_s^5 \log \alpha_s) \) corrections\[^{[15]}\] or the finite \( c \) mass contribution\[^{[16]}\] (for the \( b \) quark). The result is, taking them into account,\[^{[16a]}\]
\begin{equation}
m_b = 5022 \pm 58 \text{ MeV}/c^2; \quad \bar{m}_b(\bar{m}_b^2) = 4285 \pm 36 \text{ MeV}/c^2. \tag{3.9}
\end{equation}
The relation between the \( \overline{\text{MS}} \) mass and the pole mass is evaluated taking into account one, two and three loop corrections,\[^{[17]}\] so, while \( m_b \) is exact to \( O(\alpha_s^5 \log \alpha_s) \), \( \bar{m}_b(\bar{m}_b^2) \) is “only” correct to \( O(\alpha_s^3) \).

For the \( c \) quark we have
\begin{align*}
m_c &= 1866^{+215}_{-133} \text{ MeV}/c^2; \quad O(\alpha_s^5) \\
m_c &= 1570 \pm 100 \text{ MeV}/c^2; \quad O(\alpha_s^5) + O(\nu^2). \tag{3.10a}
\end{align*}
Here the first result is that of ref. 13, the second from ref. 11. As stated, it is unclear which is more believable. The reason is that adding two loop corrections shifts the mass beyond the errors that are obtained for the one loop result. The corresponding values of the \( \overline{\text{MS}} \) mass are,
\begin{align*}
\bar{m}_c &= 1542^{+163}_{-104} \text{ MeV}/c^2 \\
\bar{m}_c &= 1306 \pm 90 \text{ MeV}/c^2 \tag{3.10b}
\end{align*}

Other methods for estimating the masses use the decay \( Z \to \bar{b}b + G \), lattice calculations or sum rules.\[^{6}\] While giving results compatible with the former, they are less precise.

We finish this subsection with a few words to clarify a matter which is at times misunderstood. Evaluating the \( E_{nl} \) as we have done here is not a model calculation; no assumptions about “confining potentials” or the like are made. The method of solving the Schrödinger equation with a static potential obtained from a field-theoretic calculation to a fixed order \( N \) (two loops, in our case) and perturbing with relativistic and other corrections is indeed fully equivalent to a field theoretic evaluation, in which one rearranges the perturbative series adding infinite ladders to a kernel, obtained at order \( N \), as in Fig. 4.

\[^{6}\] For a summary see ref. 18.
3.4. The masses of the light quarks: $u, d, s$

Light quark masses are smaller than the QCD parameter $\Lambda$, so bound state evaluations are useless to get them. One may use lattice calculations, or chiral perturbation arguments (for the ratios) and chiral sum rules,$^{[19,20]}$ for the values of specific combinations of masses. Only bounds may be obtained from first principles; to get specific values models are needed.

In these methods one considers the pseudoscalar condensate,

$$
\Psi^{12}_5(t) = i \int d^4x \langle \text{vac} | T \partial^\mu A^{ud}_\mu(x) \partial^\nu A^{ud}_\nu(0) | \text{vac} \rangle,
$$

$3.11a$

$t = -q^2$ and $|\text{vac}\rangle$ is the physical vacuum; the axial current is

$$
A^{ud}_\mu(x) = \bar{q}_u(x) \gamma_\mu \gamma_5 q_d(x),
$$

$3.11b$

(for, e.g., the $ud$ combination). Then one writes the dispersion relation

$$
F^{12}_5(t) = \int_0^\infty ds \frac{1}{(s + t)^3} \frac{2 \text{Im} \Psi^{12}_5(s)}{\pi};
$$

$$
F^{12}_5(t) = \partial^2 \Psi^{12}_5(t) / \partial t^2.
$$

One can then calculate the l.h.s. with QCD, at large $t$; to leading order,$^7$

$$
F^{12}_5(t) = \frac{3}{8\pi^2} \frac{[m_u(t) + m_d(t)]^2}{t}, \quad t \gg \Lambda^2,
$$

and the r.h.s. is evaluated with the pion contribution, for small $s$ and, for large $s$, again with QCD. Then one either writes an inequality, noting that in the remaining intermediate energy region $\text{Im} \Psi^{12}_5(s) \geq 0$, or one approximates $\text{Im} \Psi^{12}_5(s)$ for intermediate $s$ by the $\pi'$ pole. In this way, we get two types of results: the rigorous, model independent bounds

$$
\bar{m}_d(1 \text{ GeV}) + \bar{m}_u(1 \text{ GeV}) \geq 9 \text{ MeV/}c^2, \quad \bar{m}_d - \bar{m}_u \geq 3 \text{ MeV/}c^2, \quad \bar{m}_s(1 \text{ GeV}) \geq 150 \text{ MeV/}c^2
$$

$3.12$

or the (somewhat model dependent) absolute values

$$
\bar{m}_d(1 \text{ GeV}) + \bar{m}_u(1 \text{ GeV}) = 13 \pm 4 \text{ MeV/}c^2,
$$

$$
\bar{m}_d(1 \text{ GeV}) = 200 \pm 50 \text{ MeV/}c^2, \quad \bar{m}_d(1 \text{ GeV}) = 8.9 \pm 4.3 \text{ MeV/}c^2, \quad \bar{m}_u(1 \text{ GeV}) = 4.2 \pm 2 \text{ MeV/}c^2.
$$

$3.13$

From decays into strange particles of the $\tau$ one also obtains a precise determination of the mass of the $s$ quark,$^{[22]}$

$$
m_s(1 \text{ GeV}) = 235^{+32}_{-40} \text{ MeV/}c^2,
$$

$3.14$

in agreement with the former results, but with the advantage that it does not require the use of models.

$^7$ For higher order corrections, see ref. 21.
Lattice results are compatible with these, but less reliable in that the quenched approximation (necessary to obtain meaningful calculations) is not well justified for the corresponding evaluations. Indeed, at times lattice results are incompatible with the values reported above, even with the bounds (3.12). The values of the light quark masses obtained with these other methods are discussed in some detail in the text of the author, ref. 18.

4. Masses and interaction intensity for weak interactions

4.1. $\alpha_W$ and $G_F$

Traditionally, instead of the intensity of the weak interaction, $\alpha_W \simeq 0.034$, one gives the precision value for the Fermi coupling, $G_F$, linked to the former via the $W$ mass by the relation (to first order)

$$G_F = \frac{\pi}{\sqrt{2} M_W^2} \alpha_W.$$ 

According to the PDG$^{[3]}$, $G_F = 1.16639[1] \times 10^{-5}$ GeV$^{-2}$. (4.1)

4.2. $M_W$, $M_Z$ and the mass of the Higgs particle, $M_H$

As for the masses of the $W$ and $Z$ particles the precision measurements of LEP and Tevatron give (PDG, ref. 3)

$$M_W = 80.419 \pm 0.056 \text{ GeV} / c^2,$$
$$M_Z = 91.1882 \pm 0.0022 \text{ GeV} / c^2.$$ (4.2)

From these masses, the value of $\alpha(M_Z)$, and $G_F$, it is possible to deduce the mass of the Higgs particle by consistency. Using the LEPEWWG method one finds,

$$M_H = 102^{+54}_{-36} \text{ GeV} / c^2 \quad \text{[Radiative corrections]}$$ (Grünwald, private communication). A similar (but much less precise) value follows from consistency of determinations of $\alpha_s$ (M. J. Herrero and the author, unpublished). One may take the value of this quantity obtained from processes involving energies much smaller than 91.2 GeV, and extrapolate to predict $\alpha_s(M_Z^2)$. Comparing this with measurements of $\alpha_s(M_Z^2)$ on the $Z$ itself (see the Table in Sect. 3.1) one gets consistency at 1.5 $\sigma$ level only if $M_H \leq 133$ GeV, and at 2 $\sigma$ if $M_H \leq 493$ GeV.

In the months prior to the decommissioning of LEP, evidence was found for this particle with a mass of 115 GeV/$c^2$. The same experiments produced the bound $M_H \geq 114$ GeV. While the evidence is inconclusive, the coincidence with the value found from radiative corrections, (4.3), is encouraging.

5. A note on the magnetic moment of the muon, and the dipole moment of the electron

In this last section we present two precision tests of the standard model, beyond those afforded by the consistencies already noted. The first is an extension of that already discussed, the magnetic moment of the muon. Besides its recent popularity, this is interesting because it involves all three interactions. The second is the dipole moment of the electron, that furnishes the best lower bound on new physics.
5.1. The muon anomaly

We have in Subsect. 2.2 mentioned the equality of the $e$, $\mu$ charges. This is, in particular, a consequence of the structure of the standard model, especially of so-called electron-muon universality that specifies that all properties, except the mass, of $e$ and $\mu$ are identical. As a consequence of this it follows that we can evaluate the magnetic moment of the muon like we did for the electron, but taking into account the difference in masses.

![Figure 5. The order $\alpha^2$ hadronic contributions to the muon magnetic moment. The blob represents an arbitrary hadronic state.](image)

In 2001 a very precise measurement of the muon anomaly, $a(\mu)$, was carried over in Brookhaven.\cite{23a} It improved the previous CERN precision\cite{23b} by a factor of about six, and (according to hasty voices) showed signs of discrepancy with experiment. This discrepancy was due to two factors. First, a somewhat overoptimistic and incomplete evaluation of the hadronic part of the photon vacuum polarization (h.v.p.) contribution, (Fig. 5); and secondly, a sign mistake in one of the $O(\alpha^3)$ pieces, the so-called hadronic light by light diagram, Fig. 6.

The various theoretical contributions may be grouped into three sets: purely QED, weak and hadronic. The first and second are\cite{1}

\[
10^{11} \times a(\text{QED}) = 116,584,705 \pm 1.8
\]

\[
10^{11} \times a(\text{Weak}) = 151 \pm 4.
\]

![Figure 6. A typical diagram for the hadronic light by light contribution to $a_\mu$.](image)
The hadronic contributions may in turn be split into four pieces: the $O(\alpha^2)$ hadronic photon vacuum polarization (h.v.p.) piece (Fig. 5), and the same, to order $O(\alpha^3)$; we take both from ref. 24, which gives a much improved calculation with the help of very recent experimental data. We next have the light-by-light contribution (Fig. 6), with the sign mistake redressed;\(^{25}\) and the rest, evaluated by Krause.\(^{26}\) Thus, one has
\[
10^{11} \times a^{(2+3)}(\text{h.v.p.}) = 7002 \pm 66;
\]
\[
10^{11} \times a(\text{Hadronic light by light}) = 92 \pm 20;
\]
\[
10^{11} \times a(\text{Rest}) = -101 \pm 6.
\]

Once these added, the theoretical value is\(^{24}\)
\[
a_\mu = (116591849 \pm 72) \times 10^{-11} \quad [\text{Theory, standard model}],
\]
perfectly compatible (at 1.1 $\sigma$) with the experimental figure,
\[
a_\mu = (116592030 \pm 150) \times 10^{-11} \quad [\text{Experiment}].
\]

We may write the comparison, perhaps more transparently, as
\[
a_\mu(\text{Exp.}) - a_\mu(\text{Theory}) = (1.81 \pm 1.66) \times 10^{-9}.
\]

This thus checks all three sectors of the the standard model (electromagnetic, weak and strong interactions are necessary to get the agreement) to nine digits. This is also the degree of accuracy of the verification of electron-muon universality.

### 5.2. The electron dipole moment

The interaction of a fermion, say the electron, with an electromagnetic field through an electric and a magnetic moments may be described with the phenomenological interactions
\[
-\frac{i}{2} \mu_\psi \bar{\psi} e \sigma_{\mu\nu} F^{\mu\nu}\psi, \quad \frac{i}{2} d_\psi \bar{\psi} e \sigma_{\mu\nu} F^{\mu\nu}\gamma_5\psi.
\]

We have already commented on the implications of the observed and calculated values of $\mu_e$, $\mu_\mu$. What about the dipole moment? Let us write $d_e = (e/m) \delta_e$. The present experimental bound\(^{27}\) on $d_e$, $2 \times 10^{-27} e \cdot cm$, implies
\[
|\delta_e| \leq 5 \times 10^{-16}.
\]

In the standard model, $\delta_e$ is zero, except for corrections of very high order, much smaller than the experimental bound; but this is not the case in other theories: therefore, the smallness of $d_e$ is a stringent test on structures beyond the standard model. Specifically, in the minimal supersymmetric (SUSY) extension of the standard model the bound (5.4) implies a lower bound of the order of 1 TeV for the SUSY partners, if the SUSY CP-violating phases and mixing angles are of the same order of magnitude as the non-SUSY ones.
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References

1.- The values of $\alpha$ and the calculations of $a(\epsilon)$, and (some of) those of $a_\mu$, with references, may be found in the review of V. W. Hughes and T. Kinoshita, Rev. Mod. Phys., 71, S133 (1999).


4.- For a review of this, see the text T. Kinoshita et al., Quantum Electrodynamics, World Scientific, 1990.


