Vector Decays Involving Light Scalar Mesons

There is increasing interest in the possibility of light scalar mesons (all of mass < 1 GeV). In particular, the 

Scalar Dominance Model for

Radiative Decays Involving Light Scalar Mesons

where \( F \sim 10^{-5} \). The authors have also examined the case of the axial coupling to the charged pseudoscalars, again for both the axial and \( Q = \text{diag}(2, -1, 1) \). The models yield good agreement with the experimental data, with the exception of the case of the axial coupling to the charged pseudoscalars, which is treated as a separate problem. The authors conclude that the model provides a good description of the radiative decays involving light scalar mesons.
The new feature of the present work is the inclusion of strong trilinear scalar-vector-vector terms in the effective Lagrangian:

\[
\mathcal{L}_{SVV} = \beta_A \epsilon_{abc} \epsilon^{\alpha \beta \gamma} \left[ F_{\mu
u}(p) \right]_{\alpha} \left[ F_{\mu\nu}(p) \right]_{\beta} \left[ F_{\mu\nu}(p) \right]_{\gamma}^D N_c,
+ \beta_B \epsilon_{abc} \left[ \frac{\mathcal{N}}{\mathcal{F}} \right]_{\alpha} \left[ F_{\mu\nu}(p) \right]_{\beta} \left[ F_{\mu\nu}(p) \right]_{\gamma}^D N_c,
+ \beta_C \epsilon_{abc} \left[ \frac{\mathcal{N}}{\mathcal{F}} \right]_{\alpha} \left[ F_{\mu\nu}(p) \right]_{\beta} \left[ F_{\mu\nu}(p) \right]_{\gamma}^D N_c,
+ \beta_D \epsilon_{abc} \left[ \frac{\mathcal{N}}{\mathcal{F}} \right]_{\alpha} \left[ F_{\mu\nu}(p) \right]_{\beta} \left[ F_{\mu\nu}(p) \right]_{\gamma}^D N_c.
\]

(3)

Chiral invariance is evident from (1) and the four flavor-invariants are needed for generality. (A term \( \sim \text{Tr}(F^3) \) is linearly dependent on the four shown). Actually, the \( \beta_D \) term will not contribute in our model so there are only three relevant parameters \( \beta_A, \beta_B \) and \( \beta_C \). Equation (3) is analogous to the PVVV interaction which was originally introduced as a \( \pi \omega \) coupling a long time ago [12]. It is intended to be a leading point-like [13] description of the production mechanism. With (2) one can now compute the amplitudes for \( S \to \gamma \gamma \) and \( V \to S \gamma \) according to the diagrams of Fig. 1.

![Feynman diagrams](image)

FIG. 1: Feynman diagrams for (a) \( S \to \gamma \gamma \) and (b) \( V \to S \gamma \).

The decay matrix element for \( S \to \gamma \gamma \) is written as \( (e^2/g^2) X_S \times (k_1 \cdot k_2 \epsilon_{\alpha} \cdot \epsilon_{\beta} - k_1 \cdot \epsilon_{\beta} \epsilon_{\alpha} \cdot k_2 \cdot \epsilon_{\gamma}) \) where \( \epsilon_{\mu} \) stands for the photon polarization vector. It is related to the width by

\[
\Gamma(S \to \gamma \gamma) = \frac{\alpha^2 \pi}{4} m_S^3 \left| X_S \right|^2,
\]

(4)

and \( X_S \) takes on the specific forms:

\[
X_{\sigma} = \frac{4}{9} \beta_A \left( \sqrt{2} s - 4 c \right) + \frac{8}{3} \beta_B \left( c - \sqrt{2} s \right),
X_{f_0} = -\frac{4}{9} \beta_A \left( \sqrt{2} c + 4 s \right) + \frac{8}{3} \beta_B \left( \sqrt{2} c + s \right),
X_{a_0} = 4 \frac{\sqrt{2}}{3} \beta_A.
\]

(5)

Here \( \alpha = e^2/(4\pi), s = \sin \theta_S \) and \( c = \cos \theta_S \) where the scalar mixing angle, \( \theta_S \) is defined from

\[
\left( \begin{array}{c} \sigma \\ f_0 \end{array} \right) = \left( \begin{array}{cc} c & -s \\ s & c \end{array} \right) \left( \begin{array}{c} N_3^0 \\ N_3^1 \end{array} \right)/\sqrt{2}.
\]

(6)

Furthermore, ideal mixing for the vectors, with \( \rho^0 = (\rho_1 - \rho_2^2)/\sqrt{2}, \omega = (\rho_1 + \rho_2^2)/\sqrt{2}, \phi = \rho_3^2 \), was assumed for simplicity.

Similarly, the decay matrix element for \( V \to S \gamma \) is written as \( (e^2/g^2) C_V \times \left[ p \cdot k \gamma_{\epsilon} - p \cdot k \cdot \epsilon \cdot \epsilon \right] \). It is related to the width by

\[
\Gamma(V \to S \gamma) = \frac{\alpha}{3} \left| k_\gamma \right|^2 \left| \frac{C_V}{g} \right|^2,
\]

(7)

where \( k_\gamma = (m_{\gamma}^2 - m_{S}^2)/(2m_{V}) \) is the photon momentum in the V rest frame. For the energetically allowed \( V \to S \gamma \) processes we have

\[
C_V^{(a)} = \frac{2 \sqrt{2}}{3} \beta_A c - \frac{4}{3} \beta_B \left( \sqrt{2} c + s \right) + \frac{\sqrt{2}}{3} \beta_C \left( c - \sqrt{2} s \right),
C_V^{(b)} = -\frac{2 \sqrt{2}}{3} \beta_A s + \frac{2 \sqrt{2}}{3} \beta_B \left( c - \sqrt{2} s \right) - \frac{2}{3} \beta_C \left( c + \sqrt{2} s \right),
C_V^{(c)} = 2 \sqrt{2} \beta_A c + 2 \sqrt{2} \beta_B \left( c - \sqrt{2} s \right).
\]

(8)

In addition, the same model predicts amplitudes for the energetically allowed \( S \to V \gamma \) processes: \( f_0 \to \omega \gamma, f_0 \to \rho^0 \gamma, a_0 \to \omega \gamma, a_0 \to \rho^0 \gamma \) and, if \( \kappa^0 \) is sufficiently heavy \( \kappa^0 \to K^* \gamma \). The corresponding width is

\[
\Gamma(S \to V \gamma) = \frac{\alpha}{3} \left| k_\gamma \right|^2 \left| \frac{D_V^V}{g} \right|^2,
\]

(9)

where \( k_\gamma = (m_{\gamma}^2 - m_{S}^2)/(2m_{S}) \) and

\[
D_V^{f_0} = \frac{2}{3} \beta_A \left( \sqrt{2} c + \sqrt{2} s \right) + \frac{2}{3} \beta_B \left( 2 c + \sqrt{2} s \right),
D_V^{f_0} = \frac{2}{3} \beta_C \left( \sqrt{2} c - \sqrt{2} s \right),
D_V^{a_0} = -2 \sqrt{2} \beta_A s + 2 \beta_B \left( 2 c + \sqrt{2} s \right),
D_V^{a_0} = 2 \beta_C ,
D_V^{a_0} = \frac{4}{3} \beta_A,
D_V^{a_0} = -\frac{2}{3} \beta_A.
\]

(10)

All the different decay amplitudes are described by the parameters \( \beta_A, \beta_B \) and \( \beta_C \). The reason \( \beta_B \) does not appear at all and \( \beta_C \) does not appear for \( S \to \gamma \gamma \) is that, noting Eq. (2), the Tr\( (F^2) \) factor is seen to give zero when coupled to an external photon line. Because the \( \sigma \) and \( \kappa \) are so broad, the simple two body final state approximation in decays like \( \omega, \phi \to \sigma \gamma \to \pi^\pm \pi^\mp \gamma \) is not
accurate. It is better to consider these decays as having three body final states with the terms in Eq. (3) giving the vertices and to take into account large width corrections in the scalar propagators as well as non resonant background.

These formulas can be used for different choices of the quark structure of the scalar nonet $N^0_8$ (e.g. the usual $qQ$ scenario or the “dual” scenario $Q_8, ar{Q}_8$ where $Q_8 = e^{i \phi_Q \bar{Q}}$). The characteristic mixing angle $\theta_S$ is expected to differ, depending on the scheme. In the literature, besides conventional $qQ$ models, $qq\bar{q}$ models [14], meson-meson “molecule” models [15] and unitarized meson-meson [16] models have been investigated. Recently models featuring mixing between a $qq\bar{q}$ nonet and a heavier $qq\bar{q}$ nonet have been proposed [17]; in this case two sets of interactions like Eq. (3) should be included.

Now we shall illustrate the procedure for a single putative scalar nonet [7] with a mixing angle, $\theta_S \simeq \pm 20^\circ$ (characteristic of $qq\bar{q}$ type scalars).

The parameters $\beta_A$ and $\beta_B$ may be estimated from the $S \rightarrow \gamma \gamma$ processes. Substituting $\Gamma_{\text{ew}}(a_0 \rightarrow \gamma \gamma) = (0.28 \pm 0.09) \text{keV}$ (obtained using [19] $B(a_0 \rightarrow K \bar{K})/B(a_0 \rightarrow \eta \pi) = 0.177 \pm 0.024$) into Eqs. (4) and (5) yields $\beta_A = (0.72 \pm 0.12) \text{GeV}$ (assumed positive in sign). Of course, this value is independent of the value of $\theta_S$. Then, $\Gamma_{\exp}(f_1 \rightarrow \gamma \gamma) = 0.39 \pm 0.13 \text{keV}$ yields either $\beta_B = (0.61 \pm 0.10) \text{GeV}^{-1}$ or $\beta_B = (-0.62 \pm 0.10) \text{GeV}^{-1}$. In turn we normally predict $\Gamma(\sigma \rightarrow \gamma \gamma)$ to be either $(0.024 \pm 0.023) \text{keV}$ or $(0.38 \pm 0.09) \text{keV}$ respectively.

Next consider the $\phi$ radiative decays. Assuming $\phi \rightarrow \eta \pi \gamma$ is dominated by $\phi \rightarrow a_0 \gamma$, $\Gamma_{\exp}(\phi \rightarrow a_0 \gamma) = (0.47 \pm 0.07) \text{keV}$ and Eq. (8) determines $\beta_C$ as either $(7.7 \pm 0.5) \text{GeV}^{-1}$ or $(-4.8 \pm 0.5) \text{GeV}^{-1}$. Note that $|\beta_A|$ and $|\beta_B|$ are almost an order of magnitude smaller than $|\beta_C|$. Thus, the $\phi$ radiative decay rates are mainly determined by $|\beta_C|$. Knowing $\beta_A$, $\beta_B$ and $\beta_C$ we can predict $\Gamma(\phi \rightarrow f_2 \gamma)$ using Eq. (8). There are four possibilities due to the two possibilities each for $\beta_B$ and $\beta_C$. The largest number, $\Gamma(\phi \rightarrow f_2 \gamma) = (0.21 \pm 0.03) \text{keV}$ corresponds [18] to the choice $\beta_B = (-0.62 \pm 0.10) \text{GeV}^{-1}$ and $\beta_C = (7.7 \pm 0.5) \text{GeV}^{-1}$.

Unfortunately this is still considerably smaller than the listed value [10]: $\Gamma_{\exp}(\phi \rightarrow f_2 \gamma) = (1.51 \pm 0.41) \text{keV}$ [20]. Recently Close and Kirk [21] proposed that the ratio $\Gamma(\phi \rightarrow f_2 \gamma)/\Gamma(\phi \rightarrow a_0 \gamma)$ could be boosted by considering the effects of the isospin violating $a_0^0(980) \rightarrow f_1(980)$ mixing. We will now see that these effects are small in our model. One may simply introduce the mixing by a term in the effective Lagrangian: $\mathcal{L}_{\phi f} = A_{\phi f} a_0^0 f_0$. A recent calculation [22] for the purpose of finding the effect of the scalar mesons in the $\eta \rightarrow 3\pi$ process obtained the value $A_{\phi f} = -4.66 \times 10^{-3} \text{GeV}^2$. It is convenient to treat this term as a perturbation. Then the amplitude for $\phi \rightarrow f_2 \gamma$ includes a term which is a constant term of the $\phi \rightarrow a_0^0 \gamma$ amplitude given in Eq. (8) multiplied by $A_{\phi f}$ and by the $a_0$ propagator. The $\phi \rightarrow a_0^0 \gamma$ amplitude has a similar correction. In terms of the amplitudes in Eq.(8) the desired ratio is then,

$$\frac{\text{amp}(\phi \rightarrow f_2 \gamma)}{\text{amp}(\phi \rightarrow a_0^0 \gamma)} = \frac{C'_1 + A_{\phi f} C'_0 / D_0 (m_0^2)}{C'_0 + A_{\phi f} C'_0 / D_0 (m_0^2)}$$

(11)

where $D_0 (m_0^2) = -m_0^2 + m_1^2 - im_0 \Gamma_f$ and $D_f (m_f^2) = -m_0^2 + m_f^2 - im_f \Gamma_f$. In this approach the propagators are diagonal in the isospin basis. The numerical values of these resonance widths and masses are, according to the Review of Particle Physics [19] $m_{0/a} = (984 \pm 1) \text{MeV}$, $\Gamma_{0/a} = (50 - 100) \text{MeV}$, $m_{f/a} = (980 \pm 10) \text{MeV}$ and $\Gamma_f = 40 - 100 \text{MeV}$. For definiteness, from column 1 of Table II in Ref. [23] we take $m_{f/a} = 987 \text{MeV}$ and $\Gamma_f = 65 \text{MeV}$ while in Eq. (4.2) of Ref. [24] we take $\Gamma_{0/a} = 70 \text{MeV}$. In fact the main conclusion does not depend on these precise values. It is easy to see that the mixing factors are approximately given by

$$\frac{A_{\phi f}}{D_0 (m_0^2)} \approx \frac{A_{\phi f}}{D_f (m_f^2)} \approx \frac{i A_{\phi f}}{m_0 \Gamma_f} \simeq -0.07i$$

(12)

Noting that $C'_0 / C'_0 \approx 0.75$ in the present model, the ratio in Eq.(11) is roughly $0.75 - 0.07i$ and (1 - 0.05$i$). Clearly, the correction to $\Gamma(\phi \rightarrow f_2 \gamma)/\Gamma(\phi \rightarrow a_0 \gamma)$ due to $a_0 \rightarrow f_2$ mixing only amounts to a few per cent, nowhere near the huge effect suggested in [21]. It may be remarked that Eq.(11) is practically accurate to all orders in $A_{\phi f}$, corresponding to iterating any number of $a_0 \rightarrow f_2$ transitions. Then, after summing a geometric series, the numerator picks up a correction factor $[1 - A_{\phi f}^2/(D_0 (m_0^2) D_f (m_f^2))]^{-1}$ and the denominator, the similar factor $[1 - A_{\phi f}^2/(D_0 (m_0^2) D_f (m_f^2))]^{-1}$.

Vector meson dominance, together with the assumptions of $SU(3)$ flavor symmetry and a single nonet of scalar mesons makes many more predictions. These are listed in Table I for two of the allowed $(\beta_A, \beta_B, \beta_C)$ parameter sets, neglecting $a_0 \rightarrow f_1$ mixing. It will be interesting to see if future experiments confirm the pattern of predicted widths.

We have given a leading order correlation of many radiative decays involving scalars, based on flavor symmetry and vector meson dominance. Clearly further improvements can be made. Elsewhere, we will study flavor symmetry breaking effects, higher derivitative interaction terms, treatment of the $\Sigma R$ final states as $P P \gamma$, and the case of mixed $q\bar{q}$ and $q\bar{q}g\bar{q}$ scalar nonets.

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\[
\begin{align*}
\beta_A &= 0.72 \pm 0.12, \\
\beta_B &= 0.61 \pm 0.10 - 0.62 \pm 0.10, \\
\beta_C &= 7.7 \pm 0.52, \\
\frac{f_0}{a_0} &= 0.26 \pm 0.06, \\
\Gamma(\sigma \to \gamma \gamma) &= 0.024 \pm 0.023, \\
\Gamma(\phi \to \sigma \gamma) &= 137 \pm 19, \\
\Gamma(\omega \to \sigma \gamma) &= 16 \pm 3, \\
\Gamma(\rho \to \sigma \gamma) &= 0.23 \pm 0.47, \\
\Gamma(\phi \to \omega \gamma) &= 126 \pm 20, \\
\Gamma(\phi \to \rho \gamma) &= 19 \pm 5, \\
\Gamma(a_0 \to \omega \gamma) &= 641 \pm 87, \\
\Gamma(a_0 \to \rho \gamma) &= 3.0 \pm 1.0.
\end{align*}
\]

TABLE I: Fitted values of \(\beta_A, \beta_B \) and \(\beta_C\) together with the predicted values of the ratio \(\Gamma(\phi \to f_0 \gamma) / \Gamma(\phi \to a_0 \gamma)\) and the decay widths of \(V \to S + \gamma \) and \(S \to V + \gamma\). Units of \(\beta_A, \beta_B \) and \(\beta_C\) are GeV\(^{-1}\) and those of the decay widths are keV.

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[11] In M. Harada and K. Yamawaki, Phys. Rev. Lett. 87, 132001 (2001), it was shown that this is also satisfied even at the quantum level in three flavor QCD.


[13] In N. N. Achasov, hep-ph/0201209, it is shown that some non pointlike effects are required to fit the experimental spectrum shape for \(\phi \to \pi^0 \pi^0 \gamma\). These may be introduced straightforwardly as higher derivative terms in our effective Lagrangian.


[18] We took \(m_{f_0} = 987\) MeV here; taking 980 MeV would almost double the result.