A pedagogic model for Deeply Virtual Compton Scattering with quark-hadron duality

Frank E. Close\(^1\)*, Qiang Zhao\(^2\)†

1) Department of Theoretical Physics, University of Oxford, Keble Rd., Oxford, OX1 3NP, United Kingdom
2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom
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Abstract

We show how quark-hadron duality can emerge for the valence spin averaged structure functions, and for the non-forward distributions of Deeply Virtual Compton Scattering. Novel factorisations of the non-forward amplitudes are proposed. Some implications for large angle scattering and deviations from the quark counting rules are illustrated.

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I. INTRODUCTION

There has been much recent interest in two related, though distinct, processes: (i) Deeply Virtual Compton Scattering (DVCS), measured in non-forward Compton scattering \(\gamma^*(q^2)p \rightarrow \gamma p\) [1–4]; (ii) Bloom-Gilman duality [5,6] for the imaginary part of forward Compton scattering, where the electroproduction of \(N^*\)'s at lower energies and momentum transfers empirically averages smoothly around the scaling curve \(F_2(W^2, Q^2)\) measured at large momentum transfers for both proton and neutron targets [7].

In this paper we shall develop a pedagogic model of the structure functions for non-diffractive inelastic scattering that satisfies duality. Then we shall investigate its implications for the non-forward distribution amplitudes of DVCS [1,2] in certain kinematic regimes, compare with existing models and abstract some general features in hope of gaining insight into the physical significance of measurements planned at Jefferson Lab and HERA.

Our point of departure is the work of Refs. [8,9]. Reference [8] analysed duality in the context of a large-\(N_c\)-based relativistic quark model with an empirically inspired linear potential; it focussed on the dynamics required for a confined struck quark to behave as though it were free, and illustrated how scaling ensured. In a complementary approach,

\*e-mail: F.Close1@physics.ox.ac.uk

†e-mail: Qiang.Zhao@surrey.ac.uk
Ref. [9] investigated the circumstances whereby the structure function for inelastic scattering $F(x)$ - whose magnitude in leading twist is in proportion to the (incoherent) sum of the squares of the (quark and antiquark) constituent charges - can in general match with the excitation of individual resonances which is driven by the coherently summed square of constituent charges. In the present paper we shall combine these to show how the (non-diffractive, “valence”) scaling function emerges for a two body (meson or quark-diquark baryon) state, whose constituents have arbitrary charges.

This $F(x)$ is intimately related to the imaginary part of the forward Compton amplitude, data for which lead to the probability distributions for the partons. Analogously, data on Deeply Virtual Compton Scattering, abstracted from $ep \rightarrow ep\gamma$, may be used to determine non-forward parton distribution amplitudes $F(x,\xi,t)$. Hence we study the implications of the model for DVCS, and discuss the structure and physics of $F(x,\xi,t)$.

The text is arranged as follows. In Sec. II, a simple model for a two-body system with arbitrarily charged scalar constituents is introduced; structure functions and sum rules for the forward and non-forward Compton scattering are derived. In Sec. III, we show how the scaling functions for the forward and non-forward Compton scattering arise and note relations between DVCS and the forward amplitudes which may be more general than the specific model. Functionally the generalised parton distribution function for the DVCS is discussed for both Ji and Radyushkin’s frames in Sec. IV. A phenomenological investigation of the non-forward parton distribution in DVCS based on the functionally generalised form is made in Sec. V. Summary remarks are given in Sec. VI.

II. A SIMPLE MODEL

The original model of Refs. [8,9] ignored spin entirely and considered the inelastic scattering of a “scalar electron” via exchange of a “scalar photon” from a composite system of (two) spinless constituents. The differential cross section was then written as

$$\frac{d\sigma}{dE'd\Omega_f} = \frac{g^4}{16\pi^2} \frac{E_f}{E_i} Q^4 F,$$

where the scalar coupling $g$ had dimensions of mass, and the factor multiplying the scalar structure function $F$ corresponded to the Mott cross section.

In this paper we shall consider the more physical situation where a spin 1/2 electron scatters by exchanging a vector photon. The cross section then has the standard form in terms of two structure functions $W_1$ and $W_2$, which depend on $\nu$ and $Q^2$

$$\frac{d\sigma}{dE'd\Omega_f} = \frac{\sigma_{\text{Mott}}[W_2(\nu,Q^2) + 2W_1(\nu,Q^2)\tan^2\frac{\theta_e}{2}]}{\epsilon(1+\tau)[(1-\epsilon)W_1 + \epsilon(1+\tau)W_2]} = \frac{\sigma_{\text{Mott}}}{\epsilon(1+\tau)}[(1-\epsilon)W_1 + \epsilon(1+\tau)W_2],$$

where $\tau \equiv \nu^2/Q^2$, and $\epsilon \equiv \rho_{LL}/\rho_{TT}$ is the ratio of longitudinal and transverse density matrix elements. As before we consider a composite system of two spinless constituents. While this is still not the real world, the model contains many of the important physical features: there are analogues of resonances, the inelastic structure function exhibits scale invariance and “quark-hadron” duality [8].
Ref. [8] demonstrated the duality for the case of a single particle in a potential (effectively bound to an infinitely massive electrically neutral partner). Ref. [9] extended some of those ideas by considering a composite state made of two equal mass scalars, i.e., “quarks” $q_1$ and $q_2$ with charges $e_1$ and $e_2$ respectively at positions $\vec{r}_1$ and $\vec{r}_2$. It was shown how for the imaginary part of the forward Compton amplitude (hence the inelastic structure function), the excitation of resonance states of opposite parity interferes destructively in all but leading twist. In this paper we demonstrate how the model leads to scaling behaviour for the structure functions and then extend it to the imaginary part of the non-forward Compton amplitude (non-forward distribution function [1,2]).

A. Forward Compton Scattering

The ground state wavefunction for the composite state is $\psi_0(\vec{r})$, where $\vec{r}_{1,2} = \vec{R} \pm \vec{r}/2$ defines the centre of mass and internal spatial degrees of freedom. A photon of momentum $\vec{q}$ is absorbed with an amplitude proportional to

$$\sum_i e_i \exp(i \vec{q} \cdot \vec{r}_i),$$

which excites a “resonant” state with angular momentum $L$, described by the wavefunction $\psi_L(\vec{r})$.

In the previous work which ignored spin entirely, the inelastic structure function in Eq. (1) was given by a sum of squares of transition form factors weighted by appropriate kinematic factors,

$$F(\nu, \vec{q}) = \sum_{N=0}^{\infty} \frac{1}{4E_0E_N} |F_{0,N}(\vec{q})|^2 \delta(E_N - E_0 - \nu),$$

where $\vec{q} \equiv \vec{p}_i - \vec{p}_f$, the form factors $F_{0,N}$ represent transitions from the ground state to states characterised by principal quantum number $N(\equiv L + 2k$, where $k$ is the radial and $L$ the orbital quantum numbers).

Focussing on the internal coordinate $\vec{r}$, the transition amplitude is proportional to

$$(e_1 + e_2)[\exp(i \vec{q} \cdot \vec{r}/2) + \exp(-i \vec{q} \cdot \vec{r}/2)] + (e_1 - e_2)[\exp(i \vec{q} \cdot \vec{r}/2) - \exp(-i \vec{q} \cdot \vec{r}/2)].$$

The expansion, $\exp(i \vec{q} \cdot \vec{r}) = \sum_L i^L P_L(\cos \theta) j_L(qr/2)(2L+1)$, projects out the even and odd partial waves such that the form factor is proportional to

$$F_{0,N}(\vec{q}) \sim \int drr^2 \psi^*_L(r) \psi_0(r) j_L(qr/2)$$

$$\times [(e_1 + e_2)\delta_{L=even} + (e_1 - e_2)\delta_{L=odd}].$$

The resulting structure function, summed over resonance excitations, will receive contributions from $L = \text{even}$ ($\text{odd}$) in proportion to $(e_1 \pm e_2)^2$. Since $N \equiv L + 2k$, $L = \text{even}$ ($\text{odd}$) will imply also that $N = \text{even} \equiv 2n$ or $N = \text{odd} = 2n + 1$ and we can expose these separate contributions to the structure function (we factor out the charges to make them explicit too),

$$F(\nu, \vec{q}) = \sum_{N(\nu)} \frac{1}{4E_0E_N} [F^2_{0,2n}(\vec{q})(e_1 + e_2)^2 + F^2_{0,2n+1}(\vec{q})(e_1 - e_2)^2] \delta(E_N - E_0 - \nu),$$

where $\vec{q} \equiv \vec{p}_i - \vec{p}_f$.
or equivalently:

\[ \mathcal{F}(\nu, \vec{q}) = \sum_{N(n)} \frac{1}{4E_0E_N} [(F_{0,2n}^2(\vec{q}) + F_{0,2n+1}^2(\vec{q}))(e_1^2 + e_2^2) + 2e_1e_2(F_{0,2n}^2(\vec{q}) - F_{0,2n+1}^2(\vec{q}))] \times \delta(E_N - E_0 - \nu). \] (6)

This simple example exposes the physics rather clearly. The excitation amplitudes to resonance states contain both diagonal \((e_1^2 + e_2^2)\) and higher twist terms \((\pm 2e_1e_2)\) in the flavour basis. The former set added constructively for any \(L\) and the sum over the complete set of states can now logically give the deep inelastic curve [6]; the latter enter with opposite phases for even and odd \(L\) and destructively interfere. The critical feature that this exposes is that at least one complete set of resonances of each symmetry-type has been summed over.

In general, the strength of the structure function will be proportional to \(e_1^2 + e_2^2\) only if the excitation of even and odd \(L\) states sum to equal strengths. Ref. [9,8] derived the circumstances under which this can occur.

Consider scalar quarks confined in a linear potential, and described by the Klein-Gordon equation such that \(V^2(\vec{r}) = \beta^4 r^2\), where \(\beta^4\) is a generalized, relativistic string constant (compare also Ref. [10]). The energy eigenvalues are then \(E = \pm E_N\) where \(E_N = \sqrt{2\beta^2(N + 3/2) + m^2}\) and \(m\) is the mass of the interacting constituent (“quark”). This choice gives a spectrum that is in accord with that observed empirically [10–12] since the energy eigenvalues follow from the similarity to the Schrödinger equation for a non-relativistic harmonic oscillator potential; the wave functions are algebraically as for the non-relativistic case and thereby enable analytic solutions.

The contribution to \(\mathcal{F}(q)\) from the \((\equiv L + 2k)\) set of degenerate levels is

\[ F_{0,N}^2(\vec{q}) = \frac{1}{N!} (\frac{q^2}{2\beta^2})^N \exp(\frac{-q^2}{2\beta^2}), \] (7)

from which one can immediately see that \(\sum_{N=0}^{\infty} F_{0,N}^2(\vec{q}) = 1\). It is interesting to note that any individual contribution, \(F_{0,N}(\vec{q})\), reaches its maximum value when \(q^2 = 2\beta^2 N\), at which point \(F_{0,N}^2 = F_{0,N+1}^2\). This coincidence is true for all juxtaposed partial waves at their peaks, which gives a rapid approach to the equality of \(\sum_{n=0}^{\infty} F_{0,2n}^2(\vec{q})\) and \(\sum_{n=0}^{\infty} F_{0,2n+1}^2(\vec{q})\).

We turn now to the more physical case of real spinning electron and photon. For scattering from a system of spinless constituents, the leading contributor at large \(Q^2\) [13] is the longitudinal response function

\[ R_L = \frac{\nu}{4M^2x^2}[\nu W_2 - 2MxW_1]. \] (8)

The resonance sum for \(R_L\) is analogous to that in Eq. (3) for \(\mathcal{F}\) but with an extra factor of \((E_0 + E_N)^2\). Hence [14]

\[ R_L(\nu, \vec{q}) = \sum_{N=0}^{\text{max}} \frac{1}{4E_0E_N} |f_{0,N}(\vec{q})|^2 [(E_0 + E_N)^2 \delta(\nu + E_0 - E_N)], \] (9)

where

\[ |f_{0,N}(\vec{q})|^2 \equiv (e_1^2 + e_2^2)[F_{0,2n}^2(\vec{q}) + F_{0,2n+1}^2(\vec{q})] + 2e_1e_2[F_{0,2n}^2(\vec{q}) - F_{0,2n+1}^2(\vec{q})], \] (10)
one has \( \nu_{\text{max}} < |\vec{q}| \) and the sum over \( N \) denotes equivalent sum over \( n \) for \( N = 2n \) and \( N = 2n + 1 \). Furthermore, recall that the energy eigenvalues for the Klein-Gordon equation are \( E = \pm E_N \) where \( E_N = \sqrt{2\beta^2(N + 3/2) + m^2} \). It will be helpful now to take \( E_N \geq 0 \) and to rewrite \( R_L \) as

\[
R_L(\nu, \vec{q}) = \sum_{N=0}^{\infty} \frac{1}{4E_0E_N} |f_{0,N}(\vec{q})|^2 
\times [(E_0 + E_N)^2\delta(\nu + E_0 - E_N) - (E_0 - E_N)^2\delta(\nu + E_0 + E_N)].
\] (11)

This immediately gives the sum rule

\[
S(\vec{q}) \equiv \int_{-\infty}^{+\infty} d\nu R_L(\vec{q}, \nu) = \sum_{N=0}^{+\infty} |f_{0,N}(\vec{q})|^2,
\]

and hence

\[
\int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}) = [(e_1^2 + e_2^2) + 2e_1e_2e^{-(2\vec{q})^2/4\beta^2}],
\] (12)

which generalizes as

\[
S(\vec{q}) \equiv \int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}) = [(e_1^2 + e_2^2) + 2e_1e_2F_{00}(2\vec{q})].
\] (13)

That this is the correct generalisation rather than \( F_{00}^4(\vec{q}) \) as in Ref. [9], will be obvious from the results of the next Section [Eq. (16)]. It also agrees with the physical picture in Fig. 1, where absorption and emission by different constituents leads to a momentum mismatch in the internal wavefunction of \( \vec{q} + \vec{k} \rightarrow 2\vec{q} \). Ref. [8] has shown how such a model satisfies scaling and duality in the particular case when \( e_2 = 0 \). Our results enable this to be generalised to constituents with arbitrary charges. We shall show this in Sec. III. First we generalise the above to non-forward Compton scattering amplitudes as preparation for our derivation of the scaling form of DVCS.

**B. Non-forward Compton Scattering**

We have calculated the imaginary part of the forward Compton scattering amplitude as a sum over the intermediate coherent “resonance” states. This is experimentally accessible by measuring \( \frac{d\sigma}{dE \ dA} (eA \rightarrow eX) \) from the target \( A \). We now consider the generalisation to the imaginary part of the non-forward amplitude: \( \gamma(q)A \rightarrow \gamma(k)A \), which can be measured directly through the single spin asymmetry for \( eA \rightarrow eA\gamma \) (see Sec. IV D in Ref. [1]). Clearly, when we specialise to the case \( \vec{k} \rightarrow \vec{q}; t \rightarrow 0 \), we must recover our results above.

As before, focus on the internal coordinate \( \vec{r} \), and separate the transition amplitude into contributions from \( N \equiv 2n = \text{even} \) and \( N \equiv 2n + 1 = \text{odd} \) excited states.

The generalization is:

\[
R_L(\nu, \vec{q}, \vec{k}, t) = \sum_{n(N)} \frac{1}{4E_0E_N} (E_0 \pm E_N)^2\delta(\nu + E_0 \mp E_N)
\times \left( \sum_{L=0}^{N} \left[ \mathcal{F}_{0,2n}^{(L)}(\vec{q}) \mathcal{F}_{0,2n}^{(L)}(\vec{k})(e_1 + e_2)^2 + \mathcal{F}_{0,2n+1}^{(L)}(\vec{q}) \mathcal{F}_{0,2n+1}^{(L)}(\vec{k})(e_1 - e_2)^2 \right] d_{00}^L(\vec{k} \cdot \vec{q}) \right),
\] (14)
It is important to realise that such oscillations at 90° where due to the energy gap (15) at 90°. Hence the sum over all states gives

\[ \left( \sum_{N=even} + \sum_{N=odd} \right) F_{0N}(\vec{q}) F_{0N}(\vec{k}) = \exp\left(-\frac{(\vec{q} - \vec{k})^2}{4\beta^2}\right) \equiv F_{00}(|\vec{q} - \vec{k}|) , \]

while that over opposite phases gives

\[ \left( \sum_{N=even} - \sum_{N=odd} \right) F_{0N}(\vec{q}) F_{0N}(\vec{k}) = \exp\left(-\frac{(\vec{q} + \vec{k})^2}{4\beta^2}\right) \equiv F_{00}(|\vec{q} + \vec{k}|) . \]

The generalisation to the non-forward case of the sum rule Eq. (13) is

\[ S(\vec{q}, \vec{k}) \equiv \int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}, \vec{k}, t) = F_{00}(|\vec{q} - \vec{k}|) \left[ (e_1^2 + e_2^2) + 2e_1e_2 \frac{F_{00}(|\vec{q} + \vec{k}|)}{F_{00}(|\vec{q} - \vec{k}|)} \right] . \]

If we restrict attention to \( t << Q^2 \), in which case \( |\vec{q} - \vec{k}| << |\vec{q} + \vec{k}| \), we have the sum rule

\[ \int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}, \vec{k}, t) = F_{00}(t)(e_1^2 + e_2^2) \equiv F_{00}(t) \int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}) . \]

This is the essence of the Ji-Radyushkin sum rule [1,2,15].

For Compton scattering at 90°, for the case \( q^2 \equiv k^2 \), we have

\[ \int_{-\infty}^{+\infty} d\nu R_L(\nu, \vec{q}) \rightarrow (e_1 + e_2)^2 e^{-(q^2/2\beta^2)} \rightarrow (e_1 + e_2)^2 F_{dl}(\vec{q}) , \]

which illustrates how the counting rules of elastic scattering at large angles [16] can emerge from duality. An essential part of such a derivation is the assumed mass degeneracy between states of a given \( N \) but of different \( L \). In more realistic descriptions of the spectrum, there will be spin-dependent mass shifts within a given \( N \) level (e.g. induced by gluon exchange and described perturbatively with a Fermi-Breit Hamiltonian). These will spoil the exact \( L \)-degeneracy within any fixed \( N \) and lead to oscillations about a smooth \( s^{-n} \) dependence [16] at 90°, where \( s \) is the c.m. energy squared. Such oscillations are seen in the data [17].

It is important to realise that such oscillations at 90° will be a signal primarily for mass splitting \( \text{within} \) a given \( N \) level. This is distinct from the more usual measure of duality, as discussed in most of this paper, which involves the energy gap \( \text{between} \) multiplets of different \( N \) and different parity (and would be present even in the absence of gluon exchange mass splittings).

We now establish how scaling obtains in this picture.
III. FORWARD COMPTON SCATTERING: SCALING OF $F_2(X,Q^2)$

It is not immediately obvious that the above model scales. Nor do the detailed algebraic
manipulations that follow easily reveal why this property emerges. One can begin to see
how the scale invariance arises by focussing on the shape of the curve for an excited state
$N$, and in particular the position of its maximum. This maximum is at $x_{bj} \sim 1/A$ for $A$
constituents, for any excited state $N$ as we now show.

The scattering is on energy shell whereby $\nu_N = E_N - E_0$. The three momentum is
dispersed, giving a maximum magnitude for the contribution to the structure function when
$\tilde{q}_N^2 = 2\beta^2 N$. The energy eigenvalues $E_N = \sqrt{2\beta^2 (N + 3/2) + m^2}$ satisfy $E_N^2 - E_0^2 = 2\beta^2 N$ and so at the maximum we have $E_N^2 - E_0^2 \equiv \tilde{q}_N^2$, whereby $\tilde{q}_N^2 - \nu_N^2 \equiv Q_N^2 = (E_N - E_0)2E_0$. Hence the peak occurs when

$$Q_N^2 = 2E_0\nu_N.$$  

Thus using the kinematic variable $x_{bj} \equiv Q^2/2M\nu$, where $M$ is the mass of the ground state
hadron $\equiv \sum_i E_0^{(i)}$ (i.e., summed over all constituents), then for excitation of the $N$-th level, the peak occurs at

$$x_{bj}^{(N)} \equiv \frac{Q_N^2}{2M\nu_N} = \frac{E_0}{M},$$

which suggests that the peak occurs at a common value of $x_{bj}$ for all $N$. Consequently
we find the physically sensible result that the peaking of the structure function occurs at $E_0/M \sim 1/A$ where $A$ is the number of active constituents. To obtain the explicit form of the distribution we replace the sum over discrete levels by an integral, and use Stirlings
formula (for details see Sec. V of Ref. [8], where we generalize their results to the case of constituents with arbitrary charge).

Focus on Eq. (11) and the leading piece at high $Q^2$, i.e., the $(e_1^2 + e_2^2)$ term. Write this, following Ref. [8] as

$$R_L(\tilde{q}, \nu) = (e_1^2 + e_2^2) \sum_{N=0}^{\infty} \Delta N \frac{1}{4E_0E_N} \frac{1}{N!} \left(\frac{\tilde{q}^2}{2\beta^2}\right)^N \exp\left(-\frac{\tilde{q}^2}{2\beta^2}\right)$$

$$\times \left[(E_N + E_0)^2 \delta(E_N - E_0 - \nu)\right.$$  

$$- (E_0 - E_N)^2 \delta(E_N + E_0 + \nu)\right],$$ (17)

where $\Delta N = 1$.

Some essential steps in manipulating this are to rewrite the $\delta$-functions,e.g.

$$\delta(E_N - (E_0 + \nu)) \rightarrow \frac{E_N}{\beta^2} \delta \left[(N + \frac{E_0^2}{2\beta^2}) - \frac{(\nu + E_0)^2}{2\beta^2}\right],$$

then to use Stirling’s formula whereby

$$\frac{1}{N!} \rightarrow \frac{1}{\sqrt{2\pi N}} \frac{1}{N^N} e^N,$$

and to take a continuum limit $\sum_{\Delta N} \rightarrow \int d\nu$. Then tedious algebra gives
\[ R_L(\bar{q}, \nu) = \frac{(\nu + 2E_0)^2}{4\beta E_0 \sqrt{\pi} \nu} \exp \left[ \frac{\nu^2 + 2E_0\nu}{2\beta^2} \ln \left( \frac{Q^2 + \nu^2}{\nu^2 + 2E_0\nu} \right) - \frac{Q^2 + \nu^2}{2\beta^2} + \frac{\nu^2 + 2E_0\nu}{2\beta^2} \right] (e_1^2 + e_2^2), \]  

(18)

and a similar term with \( \nu \to -\nu \).

As \( Q^2 \to \infty \) for fixed \( x_{bj} \equiv \frac{Q^2}{2M\nu} \),

\[ R_L(\nu, \bar{q}) \to F_L(x_{bj}, Q^2) = \frac{Q^2}{8\beta \sqrt{\pi M x_{bj} E_0}} e^{\frac{\nu^2}{\beta^2}(\frac{\nu}{\mu} - x_{bj})^2} (e_1^2 + e_2^2) \times \left[ \theta \left( \frac{Q^2}{2M x_{bj}} \right) - \theta \left( -\frac{Q^2}{2M x_{bj}} \right) \right]. \]  

(19)

For spinless partons as here, the longitudinal response function dominates at high \( Q^2 \), i.e., in Eq. (8), the transverse structure function \( W_1 \) vanishes at \( Q^2 \to \infty \). The implication for the structure function \( W_2 \) is that

\[ |\nu|W_2(x_{bj}, Q^2) = F_2(x_{bj}) = \frac{x_{bj}^2 M^2}{\beta \sqrt{\pi E_0}} e^{-\frac{x_{bj}^2}{\beta^2}(\frac{\nu}{\mu} - x_{bj})^2} (e_1^2 + e_2^2). \]  

(20)

As discussed in Section II A, the higher twist term vanishes in the limit of \( N(n) \to \infty \), where the parity even and odd partial waves are summed to the same strengths, i.e., \( \sum_{n=0}^{\infty} F_{0,2n}(\bar{q}) = \sum_{n=0}^{\infty} F_{0,2n+1}(\bar{q}) \).

In the quark parton model, an explicit assumption is that the quarks are on mass shell throughout the scattering. It thus leads to the equality of the light cone momentum fraction \( x \) and the kinematical quantity \( x_{bj} \), whereby

\[ F_2(x_{bj}) \sim \sum_i e_i x_{bj} q(x_{bj}) \delta(x - x_{bj}), \]  

(21)

where \( q(x_{bj}) \) is the parton distribution function. The analogue in the present model is that the photon energy matches the energy gap between the ground and excited states. Equation (20) immediately gives the analogue of the familiar quark parton model (QPM) sum rule:

\[ \int_{-\infty}^{+\infty} \frac{F_2(x_{bj})}{x_{bj}} dx_{bj} = (e_1^2 + e_2^2) \int_{-\infty}^{+\infty} q(x_{bj}) dx_{bj} = e_1^2 + e_2^2, \]  

(22)

which can be compared to Eq. (13).

The momentum sum rule for the structure function \( F_2(x_{bj}) \) can be also investigated,

\[ \int_{-\infty}^{+\infty} F_2(x_{bj}) dx_{bj} = \frac{1}{M}(E_0 + \frac{\beta^2}{2E_0})(e_1^2 + e_2^2). \]  

(23)

This result sheds light on the “dynamics” of the duality picture, namely, the relation between resonance phenomena and naive parton model. On the one hand, it shows that the results of the most naive quark parton model, which is valid at high \( Q^2 \) and where the partons can be regarded as quasi-free particles with \( q(x) \sim \delta(x - 1/A) \), can be only realised at \( \beta \to 0 \). In this case, the sum rule gives,
\[ \int_{-\infty}^{+\infty} F_2(x_{b_j}) dx_{b_j} = \frac{E_0}{M}, \]  

which is the momentum sum rule predicted by the QPM. It gives the energy fraction carried by the constituents.

The structure function \( F_2(x_{b_j}) \sim x_{b_j} q(x_{b_j}) \) exhibits an intuitive connection between resonance excitations and the naive parton model. With \( \beta = 0.4 \) GeV for the potential and \( m = 0.33 \) GeV for the constituent mass, we plot \( x_{b_j} q(x_{b_j}) \) versus \( x_{b_j} \) in Fig. 3 (solid curve); this has a broad and flattened behavior and is dominated by the resonance excitations. Obviously, if \( \beta \to 0 \), we obtain the Delta function peaking at \( x_{b_j} \to 1/2 \) for the equal mass constituents, which recovers the naive QPM prediction \cite{13}. To show this, we also present the calculation (dashed curve) for a weak potential with \( \beta = 0.1 \) GeV in contrast to the solid curve, where the dashed curve peaking at \( x_{b_j} = E_0/M \) is clear. Thus we see how the physics of the naive parton model is recovered as \( \beta \to 0 \): in this limit the constituents are free, all excitation levels are degenerate and the structure function collapses to an effective expression, \( (e_1^2 + e_2^2) \delta(x_{b_j} - 1/A) \). The physical distribution is smeared around \( x_{b_j} = 1/A \); the position of the peak is related to the number of active participants that share momenta, and the width of the peak is driven by their momentum spread which is in turn related to their confinement and the energy gap for resonance excitation.

Equation (23) suggests that the resonance phenomena violate the naive QPM result. Recalling that the groundstate energy \( E_0 = \sqrt{3\beta^2 + m^2} \), where \( 3\beta^2 \sim p_T^2 + p_z^2 \) denotes the Fermi motion momentum of a constituent, the second term in Eq. (23) can be understood as a kinetic energy correction to the naive QPM result associated with the longitudinal component of the Fermi momentum.

Certainly, this model has many features that prevent it being a serious dynamical description of the real world. In particular, although we have generated a pedagogic picture of how the sum over all the resonances leads to a scaling curve which peaks in a physically sensible region, the quantitative details of how the scaling function behaves away from this region are not sensibly produced by the model. For example, the model allows a kinematic range \(-\infty < x < +\infty\), whereas the physical region for deep inelastic scattering is limited to \( 0 \leq x \leq 1 \).

With such caveats in mind, we now investigate the (unmeasured) non-forward DVCS amplitude in this model, with the goal of abstracting its general features.

**IV. NONFORWARD DVCS: SCALING OF \( F_2(X, \xi, T) \)**

The extensive discussions of non-forward Compton scattering in the literature have used two frames. One (which we shall refer to as the Radyushkin’s kinematics) has the momenta of the initial state as for forward Compton and throws all of the momentum transfer \( \Delta \) into the final state. Thus \( \gamma(q) + p(P) \to \gamma(k = q - \Delta) + p(P + \Delta) \) where \( t = -\Delta^2 \). In this frame, the initial \( \gamma p \) system defines the longitudinal \( z \) direction and the final state particles have in general transverse components. An alternative choice is the symmetric frame of Ji, where the transfer \( \Delta \) is shared by initial and final states. Thus \( \gamma(q) + p(P - \Delta/2) \to \gamma(k = q - \Delta) + p(P + \Delta/2) \). In this frame, both the initial and final \( \gamma p \) systems have non-zero transverse momenta.
First we define the four vectors as follows

\[ P \equiv [\bar{M}; 0; 0], \]
\[ q \equiv [\frac{Q^2}{M\eta} - \eta\bar{M}; 0; -\frac{Q^2}{M\eta} - \eta\bar{M}] / 2, \]
\[ \Delta \equiv [0; -\Delta_T; -\xi M]. \] (25)

For Radyushkin’s kinematics one has the initial target momentum \( p \equiv P \) and hence \( \bar{M} \equiv M \). Furthermore \( x_{bj} \equiv \xi \sim \xi \). For Ji’s kinematics on the other hand, one has \( p \equiv (P - \frac{\Delta}{2}) \) and \( M^2 \equiv M^2 + \frac{\Delta^2}{4} \). In this frame \( x_{bj} \equiv \frac{2q}{2q + \Delta} \).

Comparison with our previous calculation is most immediate if we use Radyushkin’s frame which has the initial \( \gamma p \) system defining the longitudinal \( z \) direction. Results are identical in Ji’s frame, as we shall illustrate later.

The analogy of Eq. (17) for nonforward scattering becomes (see also Appendix 1),

\[ R_L(\nu, \vec{q}, \vec{k}, t) = \sum_{N=0}^{\infty} \Delta N \frac{1}{4E_0E_N} \frac{1}{N!} (\frac{\vec{q} \cdot \vec{k}}{2\beta^2})^N \exp\left(-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}\right) \left(e_1^2 + e_2^2\right) \]
\[ \times \left[(E_N + E_0)^2\delta(E_N - E_0 - \nu) - (E_0 - E_N)^2\delta(E_N + E_0 + \nu)\right]. \] (26)

First replace \( k \rightarrow q - \Delta \) throughout, so we consider \( R_L(\nu, q, \Delta, t) \). Then after tedious algebraic manipulations, analogous to those used in Sec. III, we find a scaling behaviour as follows:

\[ R_L(\nu, q, \Delta, t) \rightarrow F_L(x_{bj}, \Delta, t, Q^2) = \sum_{N=0}^{\infty} \Delta N \frac{1}{4E_0E_N} \frac{1}{N!} (\frac{\vec{q} \cdot \vec{k}}{2\beta^2})^N \exp\left(-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}\right) \left(e_1^2 + e_2^2\right) \]
\[ \times \left[(E_N + E_0)^2\delta(E_N - E_0 - \nu) - (E_0 - E_N)^2\delta(E_N + E_0 + \nu)\right]. \] (27)

Note that in this model \( F_{00}(t) \equiv e^{-\frac{\Delta^2}{4\beta^2}} \) and so we see the explicit presence of the elastic form factor multiplying a skewed distribution function where, in effect, the \( x_{bj} \) has been shifted relative to the forward Compton case, in a \( t \)-dependent manner: \( x_{bj} \rightarrow x_{bj}(1 - \frac{\vec{q} \cdot \vec{\Delta}}{Q^2}) \).

The above was all in Radyushkin’s frame. If instead we had calculated in Ji’s frame, all the steps follow analogously leading to an identical expression to the above except that the argument of the exponent is modified:

\[ \left(\frac{E_0}{M} - x_{bj} + \frac{\vec{q} \cdot \Delta}{2q \cdot p}\right)^2 \rightarrow \left(\frac{E_0}{M} - x_{bj} + \frac{\vec{q} \cdot \Delta}{2q \cdot (p - \frac{\Delta}{2})}\right)^2. \]

Although superficially these appear to differ, when expressed in terms of observables they are identical.

To see most immediately what is happening, we first ignore corrections of \( O(\Delta^2/Q^2) \). Then the argument of the exponent becomes in Radyushkin’s kinematics:

\[ \left(\frac{E_0}{M} - x_{bj} + \frac{\xi_{Rad}}{2}\right)^2, \]

while for Ji’s kinematics it is:
\[
\left[ \frac{E_0}{M} - x_{bj} + \left( \frac{\xi J_i}{2 + \xi J_i} \right) \right]^2.
\]

However, for the imaginary part, which is all that we are considering here, there are the identities [18]:

\[
\left( \frac{\xi J_i}{2 + \xi J_i} \right) \equiv \xi_{Rad}/2 \equiv x_{bj}/2,
\]

and so in either frame and in the particular kinematics \( t \ll Q^2 \), we have

\[
F_L \sim e^{-\left( \frac{E_0}{M} - x_{bj} \right)^2} F_{00}(t), \tag{28}
\]

which leads to the amusing factorization:

\[
F_L(x, \xi, t) \rightarrow q\left( \frac{x}{2} \right) F_{el}(t), \tag{29}
\]

where \( x \equiv x_{bj} \). This is in contrast to a commonly used ansatz \( F(x, \xi, t) \rightarrow q(x) F_{el}(t) \) [4]. We shall return to this later at Eqs. (35) and (37). This specific example reinforces the result of a recent study [19] which suggested that the factorization form \( q(x) F_{el}(t) \) is not a general result in DVCS.

We now show how this generalises when \( \Delta^2/Q^2 \) corrections are included. In this case we need to look more carefully at

\[
\frac{q \cdot \Delta}{q \cdot p} = \xi \frac{q_3}{q_0} = -\xi \left( 1 + \eta^2 \frac{M^2}{Q^2} \right) / \left( 1 - \eta^2 \frac{M^2}{Q^2} \right) = -x_{bj} \left( 1 + \eta^2 \frac{M^2}{Q^2} \right) \frac{\xi}{\eta}.
\]

The most general form is to write these in terms of invariants:

\[
\bar{q} \cdot \bar{\Delta} \equiv -q \cdot \Delta \equiv (k^2 + Q^2 - \Delta^2)/2.
\]

The symmetry of the ensuing expressions is clearest if we define \( K^2 \equiv -k^2 \). A particular example is forward Compton scattering for which \( K^2 \equiv Q^2 > 0 \). Although in practice we will be interested in \( K^2 = 0 \), we keep this variable in the formula to give

\[
\frac{\bar{q} \cdot \bar{\Delta}}{2q \cdot p} \equiv \frac{x_{bj}}{2} \left( 1 - \frac{K^2 + \Delta^2}{Q^2} \right),
\]

which also confirms that \( \eta = \xi \) for large \( Q^2 \). We therefore define

\[
x_{bj}^{in} \equiv \frac{Q^2}{2p \cdot q}; \quad x_{bj}^{fin} \equiv \frac{K^2 + \Delta^2}{2p \cdot q}.
\]

Hence for the forward case, since \( t = \Delta^2 \rightarrow 0 \), \( x_{bj}^{fin} \equiv x_{bj}^{in} \) will be recovered when \( K^2 \equiv Q^2 \).

Then Eq. (27) can be manipulated into the following form
\[ F_t(x_{bj}^{in}, x_{bj}^{fin}, t, Q^2) = (e_1^2 + e_2^2) \frac{\sqrt{Q^2(K^2 + t)}}{8\beta \sqrt{\pi} M \sqrt{x_{bj}^{in} x_{bj}^{fin} E_0}} \times \left[ e^{-\frac{M^2}{2\beta^2}(E_0 - x_{bj}^{in})^2} e^{\frac{M^2(x_{bj}^{in} - x_{bj}^{fin})^2}{4\beta^2}} e^{-\frac{M^2(E_0 - x_{bj}^{fin})^2}{4\beta^2}} \right] e^{-\left(\frac{x^2}{4\beta^2}\right)} , \] (32)

and by steps analogous to section III, the structure function can be expressed as

\[ F_2(x_{bj}^{in}, x_{bj}^{fin}, t) = (e_1^2 + e_2^2)x_{bj}^{in} x_{bj}^{fin} \frac{M^2}{\beta \sqrt{\pi} E_0} \times \left[ e^{-\frac{M^2}{2\beta^2}(E_0 - x_{bj}^{in})^2} e^{\frac{M^2(x_{bj}^{in} - x_{bj}^{fin})^2}{4\beta^2}} e^{-\frac{M^2(E_0 - x_{bj}^{fin})^2}{4\beta^2}} \right] e^{-\left(\frac{x^2}{4\beta^2}\right)} , \] (33)

which is to be compared with Eq. (20) for the forward case.

Thus Eq. (33) can be written as

\[ F_2(x_{bj}^{in}, x_{bj}^{fin}, t) \sim \sqrt{F_2(x_{bj}^{in}) \times F_2(x_{bj}^{fin}) \times F_{el}(\Delta_T)} \] (34)

The factor \( F_{el}(\Delta_T) \) is dependent on \( x_{bj}^{in}, x_{bj}^{fin} \) and \( t \). When \( x_{bj}^{in} - x_{bj}^{fin} (\equiv \xi) \rightarrow 0 \) this form reduces to the parton density distribution modulated by the transverse momentum transfer (compare Ref. [20]). However, we can restore the \( F_{el}(t) \) by recognising the full import of Eqs. (32) and (33).

Comparison with the forward scattering Eq. (20) shows that the first and third exponentials in the square bracket of Eq. (33) are effectively the amplitudes for finding a parton at \( x_{bj}^{in} \) in the initial hadron and at \( x_{bj}^{fin} \) in the final hadron. From Eq. (33) the term outside the square bracket, \( e^{-\left(\frac{\xi^2}{4\beta^2}\right)} \), is seen to be the elastic form factor for the hadron, \( F_{el}(t) \equiv F_{00}(t) \), which arises because the initial and final hadron are effectively scattered elastically with invariant four momentum transfer squared \( t \equiv -\Delta^2 \). The central term in the square bracket, \( e^{\frac{M^2}{2\beta^2}(E_0 - x_{bj}^{in})^2} \), appears to be a specific property of the sum over intermediate states in the dual model and merits some discussion.

As Fig. 4 schematically illustrates, we explicitly included coherent intermediate “resonant” hadron states. Although we recover the leading twist of the quasi-free parton model, nonetheless confinement is present and imposing itself throughout, in particular in the intermediate hadron state for which a parton has entered with \( x_{bj}^{in} \) and departed with \( x_{bj}^{fin} \). If we considered the limit \( \beta \rightarrow 0 \), recovering the most naive form of a quasi-free independent parton model, with no memory of confinement and no non-trivial excitation spectrum in the intermediate state, this term will vanish (unless \( x_{bj}^{in} = x_{bj}^{fin} \)), as will the entire amplitude. As mentioned in section III, \( \beta \neq 0 \) leads to a physical excitation gap, which both smears the distributions and leads to a finite overlap when \( x_{bj}^{in} \neq x_{bj}^{fin} \).

It is \( \beta \neq 0 \) that enables duality via this intermediate state overlap. Specifically, the basic photon-parton (Q) scattering \( \gamma(q)Q \rightarrow \gamma(k)Q \) has a non-zero cross section, as does the hadronic process \( \gamma A \rightarrow A^* \rightarrow \gamma(k)A \) via a specific intermediate state \( A^* \). However, the duality is non-trivial. If confinement is “hidden” such that \( \beta \rightarrow 0 \), then although \( \gamma(q)Q \rightarrow \gamma(k)Q \) exists, its embedding in the hadronic initial and final states will vanish due to the misalignment of momenta of the struck parton relative to that of the spectator(s).
In this case the duality is still realised because all the $A^*$ states become degenerate and destructively interfere unless $x_{b_j}^{in} = x_{b_j}^{fin}$.

Thus, the structure in Eq. (32) suggests that the non-forward structure function may generalize to a factorisation between:

(i) the hadron-parton distribution amplitudes;
(ii) a “quark-in, quark-out” term associated with the intermediate coherent state;
(iii) the invariant momentum transfer $t$.

We can restore the appearance of $F_{00}(t)$ by recognising that contribution (ii) describes the longitudinal part of the momentum transfer being “shared” in a two-step process - excitation and decay of the intermediate coherent state. In effect it is an indication of so-called $\xi$-dependence discussed in the literature [15,18,21,22], arising from the “memory” of coherent confinement in the (sum over) intermediate states. Thus Eq. (33) may be generalised to,

$$F_2(x_{b_j}^{in}, x_{b_j}^{fin}, t) \equiv \sqrt{F_2(x_{b_j}^{in})} \times \frac{1}{F_{00}(\xi^2)} \times F_2(x_{b_j}^{fin}) \times F_{00}(t)$$

$$\sim \sqrt{F_2(x_{b_j}^{in})} \times F_2(x_{b_j}^{fin}) \times F_{00}(\Delta_T).$$

(35)

Equation (35) has the advantage of separating out the form factors governed by the external hadron system [$F_{00}(t)$] and internal ones. As discussed in the previous paragraph, such a form also explains the onset of the quark-hadron duality in DVCS.

An alternative way to write Eq. (32) is

$$F_L(x_{b_j}^{in}, x_{b_j}^{fin}, t, Q^2) = (e_1^2 + e_2^2) \frac{\sqrt{Q^2(K^2 + t)}}{8\beta^2 \sqrt{\pi} M \sqrt{x_{b_j}^{in} x_{b_j}^{fin} E_0}}$$

$$\times \left[ e^{\frac{-M^2}{\beta^2}} \left( \frac{E_0}{2} \frac{x_{b_j}^{in} + x_{b_j}^{fin}}{2} \right) \right] e^{-\frac{Q^2}{4M^2}},$$

(36)

which can be generalised to

$$F_L(x_{b_j}^{in}, x_{b_j}^{fin}, t, Q^2) \equiv F_L(x, t, Q^2) \times F_{00}(t) \times \left[ 1 - \left( \frac{\xi}{x} \right)^2 \right],$$

(37)

where we have defined:

$$x \equiv (x_{b_j}^{in} + x_{b_j}^{fin})/2; \ \xi \equiv (x_{b_j}^{in} - x_{b_j}^{fin})/2.$$

(38)

Accordingly, in this case the structure function can be written as

$$F_2(x, \xi, t) = (e_1^2 + e_2^2) \frac{(x - \xi)(x + \xi)}{x^2} F_2(x) F_{el}(t),$$

(39)

where $F_2(x)$ is the structure function for the forward scattering.

The reduction of Eq. (39) to forward Compton scattering is obvious. If there were no intermediate coherent state, we expect that the physics would be symmetric in $x_{b_j}^{in}$ and $x_{b_j}^{fin}$, through which the forward Compton scattering ($x_{b_j}^{in} = x_{b_j}^{fin}$) corresponds to the kinematics,
\[ x = x_{bj}^{in} = x_{bj}^{fin} \] and \( \xi \to 0 \). We conjecture that if the intermediate coherent state were absent, as in the incoherent parton model description, a plausible factorisation between the overall \( F_{el}(t) \) and parton effective probability distribution, symmetric in \( x_{bj}^{in} \) and \( x_{bj}^{fin} \) would be

\[ F_2(x_{bj}^{in},x_{bj}^{fin},t) \equiv F_2(x,\xi = 0,t) = F_2(x) \times F_{el}(t) \tag{40} \]

Note that \( x = (x_{bj}^{in} + x_{bj}^{fin})/2 \) and so this contains Eq. (29) as a special case in the approximations that were used there, whereby \( x_{bj}^{fin} \to 0 \) and, as remarked earlier, differing from a commonly used ansatz.

In the Gaussian wavefunction model, the equivalence between Eq. (35) and (39) is exact. In Eq. (40) the non-forward amplitude effectively becomes an “average” of the forward distributions, modulated by the kinematic factor \( [1 - (\xi/x)^2] \) where \( \xi \to 0 \) in the forward case. It remains to be investigated whether this analytic equivalence between Eqs. (35) and (39) is an artifact of the Gaussian distributions and the restriction to \( F_L \) with spinless constituents. The latter in particular needs study as the imaginary part of DVCS is measurable in electron scattering from a polarised target, and with spinless constituents this is manifestly a non-leading effect.

\section*{V. PHENOMENOLOGY}

In this paper we have constructed a model for composite systems consisting of two spinless particles. For phenomenology we make the most direct generalisation of Eqs. (35) and (39) to a three quark system for more realistic comparison with non-diffractive data.

In Fig. 5, we test Eq. (35) using a quark counting rule parametrisation for a three quark system, i.e., \( F_2(x) = xq(x) = x(1-x)^3 \) (for which \( \langle xq(x) \rangle / \langle q(x) \rangle = 1/3 \)). We adopt a dipole elastic form factor \( F_{el}(t) = 1/(1 - t/\lambda)^2 \), where \( \lambda = 0.7 \text{ (GeV/c)}^2 \) is the empirical energy scale, and choose \( |t| = 1 \text{ (GeV/c)}^2 \) in Fig. 5.

The factorisation of Eq. (35) implies that the non-forward parton distribution will have the form:

\[
F_2(x_i,x_f,t) = \sqrt{F_2(x_i)F_2(x_f)} \frac{F_{el}(t)}{1 + (x_i - x_f)^2M^2/\lambda} = \frac{\sqrt{x_i(1-x_i)^3\sqrt{x_f(1-x_f)^3}}}{[1 + (x_i - x_f)^2M^2/\lambda]^2(1 - t/\lambda)^2} . \tag{41}
\]

As \( x_i(x_f) < 1 \), one has \( (x_i - x_f)^2 << 1 \) and the \( F_2(x_i,x_f,t) \) is almost symmetric in \( x_i \) and \( x_f \). Hence for fixed \( x_f \) the figure shows a rise and fall as a function of \( x_i \) similar to that observed for the “usual” forward structure function, and to a bag model study by Ji \textit{et al} \cite{15}. However, there are significant dynamical differences with that model: we find the distribution exhibits a sensitivity to \( x_f \), which is not apparent in \cite{15}. In contrast to Ref. \cite{15}, in which the origins of the \( \xi \) insensitivity are obscure \cite{23}, the systematics of the distribution in the present model, especially its \( x_f \) dependence, are more transparent.

For comparison, we also investigate the factorisation of Eq. (39) using the same quark counting rule parametrization as input. The original form only applies in the limit \( x_{bj}^{in} > x_{bj}^{fin} \).
and so it is not clear whether its generalisation should be taken seriously out of that region. However, if we take the generalized form to apply throughout the entire physical range where the variable \( x \) and \( \xi \) each have \(-1 \leq x \leq 1\) and \( \xi \geq 0 \), we obtain some interesting features. As shown in Fig. 6, with \(|t| = 1 \text{ (GeV/c)}^2\), the distribution function is positive at \( x > \xi \), which is the region dominated by the constituent (quark) distributions. The distribution function exhibits significant sensitivity to \( \xi \). The region \( x >> \xi \), which corresponds to \( Q^2 \sim |t| \), is the place where resonance effects could play a role if \(|t|\) is small. As is to be expected with our quark counting rule parameterization, the largest probabilities of the quark distribution occur at \( \sim 1/3 \). A sign change occurs at \( x = \xi \), where \( Q^2 >> |t| \). In the region \( x < \xi \) where \( K^2 + t > Q^2 \), the partons have negative momentum fraction \( x^{fm}_{bj} = x - \xi < 0 \); this may be interpreted as due to antiquark \( \bar{q} \) which can play a role in this kinematic region. Interestingly, the interpretation of this generalisation is consistent with that in Ji’s frame (see e.g. discussions in Ref. [24]). One can see that with \( \xi \to 0 \), which corresponds to larger \( Q^2 \) with a fixed \( t \), this factorisation succeeds in reproducing the forward Compton scattering. A scaling behavior is also observed. However, notice that a fast crossover occurs at \( x = 0 \) if \( x < \xi \), where the usual quark density becomes infinite. This feature could imply that the physical region of this generalisation form only make sense at \( x > \xi \) [23,24].

VI. CONCLUSION AND DISCUSSION

We have an explicit model, in which both forward and non-forward Compton scattering have been investigated. The model also exhibits scaling. Although this model is far from reality, we may still draw intuition from it as to how the physical DVCS and related processes may behave.

For forward scattering we have extended the work of Ref. [8] to constituents with arbitrary charges and given an explicit dynamical model realising the general picture outlined by Close and Isgur [9]. For the first time we have also determined the implications of such models for the non-forward Compton scattering. We stress that this is at best only a pedagogic picture due to its emphasis on spinless constituents.

The model showed scaling behavior explicitly for the forward scattering and also satisfied well known sum rules. This also has been extended to the non-forward case. We find here too that scaling is predicted and explicit \( \xi \) dependence is also seen.

The behaviour of the Compton amplitude at 90° showed how the effective simple \( s^{-n} \) dependence, previously derived from counting rules [16], arises. For us it is directly driven by the dependence of \( F_{el}(t) \) once our Gaussian forms are generalised to phenomenological form factors. We found that the degeneracy among states with a common \( N(= L + 2k) \) but different \( L \) causes a destructive interference among all but the elastic Born term. Thus the \( s^{-n} \) behaviour, in this interpretation, is effectively dominated by elastic scattering in the direct channel. Away from 90° in the model this interference is no longer exact. Indeed, in the real world where this \( N-L \) degeneracy is broken (e.g. by spin dependent effects arising from one-gluon exchange), oscillations around the overall average smooth \( s^{-n} \) are predicted rather naturally. It is therefore interesting that such effects are qualitatively evident in the data [17]. A quantitative investigation of this will be reported elsewhere.

Our work is restricted to the imaginary part of DVCS. This in turn is measurable in electron scattering with polarized target. This exposes the limit of our spinless model. It is
the longitudinal response functions that are leading in such a model and exhibit interesting 
factorisations (e.g. Eq. (35) and (39)). We expect that such relations will survive for 
investigations with more realistic models. However, it is perhaps interesting to note that 
the factorizations exhibited in the present model are novel and different from some popular 
an
t{	extit{ansatz}} used in the literature [4].

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\section*{Appendix 1}

For a given excited level \( N \), there are degenerate states with \( L = N, N - 2, \cdots 0(1) \) for \( N = \text{even} \)(odd). Whereas in the forward scattering we dealt immediately with \( \sum_N |F_{0N}(q)|^2 \), here we must first sum over the various \( L \) states within a given \( N \), thus

\[
\sum_N \left( \sum_{L=0(1)}^{N} \mathcal{F}_{0N}^{(L)}(\vec{k}) \mathcal{F}_{0N}^{(L)}(\vec{q}) d_{00}(\hat{k} \cdot \hat{q}) \right),
\]

which with Gaussian wavefunctions reduces to

\[
\sum_N F_{0N}(\vec{q})F_{0N}(\vec{k}) = \sum_N \frac{1}{N!} \left( \frac{\vec{q} \cdot \vec{k}}{2\beta^2} \right)^N \exp\left(-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}\right).
\]

As illustration we show the first non-trivial case for \( N = 2 \). Here, for \( L = 2 \)

\[
\mathcal{F}_{02}^{(2)}(\vec{k}) \mathcal{F}_{20}^{(2)}(\vec{q}) = \frac{1}{3} \left( \frac{3(\hat{k} \cdot \hat{q})^2 - 1}{2} \right) \left( \frac{kq}{2\beta^2} \right)^2 e^{-(\vec{q}^2 + \vec{k}^2)/4\beta^2},
\]

and for \( L = 0 \)

\[
\mathcal{F}_{02}^{(0)}(\vec{k}) \mathcal{F}_{20}^{(0)}(\vec{q}) = \frac{1}{6} \left( \frac{kq}{2\beta^2} \right)^2 e^{-(\vec{q}^2 + \vec{k}^2)/4\beta^2}.
\]

Thus

\[
F_{02}(\vec{q})F_{20}(\vec{k}) = \mathcal{F}_{02}^{(2)}(\vec{k}) \mathcal{F}_{20}^{(2)}(\vec{q}) + \mathcal{F}_{02}^{(0)}(\vec{k}) \mathcal{F}_{20}^{(0)}(\vec{q}),
\]

which is the \( N = 2 \) component in Eq. (42).

That this result generalises is readily seen in the Cartesian basis where the sum over 
resonances for non-forward Compton scattering in this relativistic model. We start with the 
general form
\[ M = \delta(\vec{q} + \vec{P}_1 - \vec{k} - \vec{P}_2) \sum_{N=0}^{\infty} \langle \Psi_0(\vec{r}) | e^{i\vec{k} \cdot \vec{r}/2} + e^{i\vec{q} \cdot \vec{r}/2} | \Psi_N \rangle \]

\times \langle \Psi_N | e^{i\vec{q} \cdot \vec{r}/2} + e^{i\vec{k} \cdot \vec{r}/2} | \Psi_0(\vec{r}) \rangle, \quad (43)

where \( \Psi_N = \psi_n_x \psi_n_y \psi_n_z \) is the harmonic oscillator wave function, and \( N = n_x + n_y + n_z \) is the main quantum number. The one dimension harmonic oscillator, e.g. the \( z \) component has the expression

\[ |\psi_{n_z}(z)\rangle = \left[ \frac{\beta}{2\sqrt{\pi}2^{n_2}n_z!} \right]^{1/2} e^{-\frac{1}{4}z^2} H_{n_z}(\beta z/2), \quad (44) \]

where \( H_{n_z}(\beta z/2) \) is the Hermite polynomial and the orthogonal relation is,

\[ \int_{-\infty}^{+\infty} H_n(x) H_m(x) e^{-x^2} dx = \sqrt{\pi} 2^n n! \delta_{nm} . \quad (45) \]

For the excitation process we fix the incoming photon momentum to be colinear with that of the target, and refer to it here as the \( z \) direction. Only the \( z \) component \( \psi_{n_z} \) will be excited. We have

\[ \langle \psi_{n_z}(z) | e^{i\vec{q} \cdot \vec{r}/2} | \psi_0(z) \rangle = \frac{1}{\sqrt{n_z!}} \left( \frac{iq}{\sqrt{2\beta}} \right)^{n_z} e^{-q^2/4\beta^2} . \quad (46) \]

The excited intermediate state will emit a photon with momentum \( \vec{k} \) and fall to the ground state. The transition amplitude thus can be expressed as

\[ \langle \psi_0(x) \psi_0(y) | e^{-i(k_n x + k_n y + k_n z)/2} | \psi_0(x) \psi_0(y) \psi_{n_z}(z) \rangle . \quad (47) \]

Following the same strategy, we can explicitly derive,

\[ \langle \psi_0(z) | e^{-ik_z z/2} | \psi_{n_z}(z) \rangle = \frac{1}{\sqrt{n_z!}} \left( -\frac{i k_z}{\sqrt{2\beta}} \right)^{n_z} e^{-k_z^2/4\beta^2}, \quad (48) \]

for the \( z \) component. The \( x \) and \( y \) components are

\[ \langle \psi_0(x) | e^{-ik_x x/2} | \psi_0(x) \rangle = e^{-k_x^2/4\beta^2} , \]

\[ \langle \psi_0(y) | e^{-ik_y y/2} | \psi_0(y) \rangle = e^{-k_y^2/4\beta^2} . \quad (49) \]

Thus, we obtain

\[ \langle \psi_0(x) \psi_0(y) \psi_0(z) | e^{-i\vec{k} \cdot \vec{r}/2} | \psi_0(x) \psi_0(y) \psi_{n_z}(z) \rangle = \frac{1}{\sqrt{n_z!}} \left( -\frac{i k \cos \theta}{\sqrt{2\beta}} \right)^{n_z} e^{-k^2/4\beta^2} , \quad (50) \]

where \( k_z = k \cos \theta \), and \( \theta \) is the angle between the \( \vec{k} \) and \( z \) direction (the direction of \( \vec{q} \)).

The total transition of Eq. (43) can be then expressed as

\[ M = \delta(\vec{q} + \vec{P}_1 - \vec{k} - \vec{P}_2) \sum_{N=0}^{\infty} e^{-(k^2 + q^2)/4\beta^2} \]

\times \left[ (e_1^2 + e_2^2) \frac{1}{N!} \left( \frac{i k q \cos \theta}{2\beta^2} \right)^N + 2e_1 e_2 \frac{1}{N!} \left( -\frac{i k q \cos \theta}{2\beta^2} \right)^N \right] , \quad (51) \]

where the main quantum number \( N = n_z \) in this transition (which in general will include degenerate states with \( N = L, L - 2 \cdots \)). The above deduction gives the origin of Eqs. (42) and (26).
In the previous Sections, the term proportional to $e_1 e_2$ has not been discussed in detail. Here, we shall show that this term would vanish in the limit of $N \to \infty$ for the non-forward Compton scattering process.

The positive energy term proportional to $e_1 e_2$ in the structure function (Eq. (6)) is,

$$2 e_1 e_2 \sum_{N=0}^{\infty} \frac{\sqrt{\nu^2 + Q^2}}{4E_0 \beta^2} \frac{1}{N!} \left( -\frac{\vec{q} \cdot \vec{k}}{2\beta^2} \right)^N e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} \delta(E_N - E_0 - \nu).$$

(Correspondingly, the term proportional to $e_1 e_2$ in the scaling function is,

$$R_L(\nu, \vec{q}, \vec{k}, t) = 2 e_1 e_2 \sum_{N=0}^{\infty} \frac{|\vec{q}| \sqrt{\nu^2 + Q^2}}{4E_0 \beta^2} \frac{1}{(2n)!} \left( -\frac{\vec{q} \cdot \vec{k}}{2\beta^2} \right)^{2n} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} \delta(E_{2n} - E_0 - \nu),$$

where $|\vec{q}|^2 = 2\beta^2 N$. Compared to the term proportional to $(e_1^2 + e_2^2)$, a factor $(-1)^N$ arises from the sum over the resonances. We therefore separately consider the $N = \text{even}$ and odd terms which gives,

$$R_L(\nu, \vec{q}, \vec{k}, t) \equiv R_L^{\text{even}} + R_L^{\text{odd}}$$

$$= 2 e_1 e_2 e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} \left[ \sum_{n=0}^{\infty} \frac{|\vec{q}| \sqrt{\nu^2 + Q^2}}{4E_0 \beta^2} \frac{1}{(2n)!} \left( -\frac{\vec{q} \cdot \vec{k}}{2\beta^2} \right)^{2n} \delta(E_{2n} - E_0 - \nu) \right.
\left. - \sum_{n=0}^{\infty} \frac{|\vec{q}| \sqrt{\nu^2 + Q^2}}{4E_0 \beta^2} \frac{1}{(2n+1)!} \left( -\frac{\vec{q} \cdot \vec{k}}{2\beta^2} \right)^{2n+1} \delta(E_{2n+1} - E_0 - \nu) \right].$$

Following the analogous steps as outlined for the term of $(e_1^2 + e_2^2)$ in Sec. III, the factorial part can be expanded the same way by using the Stirling’s formula. After some tedious, but essentially the same algebra, we obtain,

$$R_L^{\text{even}} + R_L^{\text{odd}}$$

$$= 2 e_1 e_2 e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}} e^{-\frac{\vec{q}^2 + \vec{k}^2}{4\beta^2}}$$

$$\times [\delta(E_{2n} - E_0 - \nu) - \delta(E_{2n+1} - E_0 - \nu)].$$

where the \( \delta \) functions are shown explicitly because we assume that the photon energy \( \nu \) always satisfies the condition that the excited states are \( 2n \) for \( R_L^{\text{even}} \) and \( 2n + 1 \) for \( R_L^{\text{odd}} \). Consequently, taking the limit \( n \to \infty \), \( E_{2n} = E_{2n+1} \) can be satisfied, which thus leads to \( R_L^{\text{even}} + R_L^{\text{odd}} = 0 \). Namely, the twist term vanishes. Similar investigation for the energy negative solution results in the same conclusion.
REFERENCES

FIG. 1. Schematic diagrams of DVCS for a two-scalar-constituent system.

FIG. 2. Schematic diagrams of forward and non-forward DVCS.
FIG. 3. Structure function $F_2(x_{bj}) \sim x_{bj}q(x_{bj})$ without the charge factor. $\beta$ denotes the linear potential strength.

FIG. 4. Schematic diagram for the quark-hadron duality in the non-forward Compton scattering.
FIG. 5. Non-forward structure function following the phenomenology of Eq. (41) with $t = -1$ (GeV/c)$^2$ fixed. As $x_f/x_i \sim -t/Q^2$, $Q^2$ increases with increasing $x_i$.

FIG. 6. Non-forward structure function following factorisation of Eq. (39) with $t = -1$ (GeV/c)$^2$ fixed. Note that $t \to 0$ as $x \to \xi$. 