Calculation Of Secondary Particles In Atmosphere And Hadronic Interactions

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Calculation of secondary particles produced by the interaction of cosmic rays with the nuclei of Earth’s atmosphere pose important requirements to particle production models. Here we summarize the important features of hadronic simulations, stressing the importance of the so called “microscopic” approach, making explicit reference to the case of the FLUKA code. Some benchmarks are also presented.

1. Introduction

Reliable calculations of flux of secondary particles in atmosphere, produced by the interaction of primary cosmic rays, are essential for the correct interpretation of the large amount of experimental data produced by experiments in the field of astroparticle physics. The increasing accuracy of modern experiments demands also an improved quality of the calculation tools. Different ingredients are required to produce a useful calculation model. Essentially they can be reduced to three important classes: the primary cosmic ray spectrum, the modelization of the environment (atmosphere, geomagnetic field, etc.) and a model of particle production in the hadronic shower following the collisions of primary c.r.’s with the atmosphere nuclei. The uncertainty on the primary spectra is dominated by the systematics of the experiments devoted to their measurement: in the light of recent measurements by AMS\textsuperscript{[1]} and BESS\textsuperscript{[2]}, such an uncertainty is about ±5% below 100 GeV/nucleon, increasing to ±10% at 10 TeV/nucleon. As far as the environmental description is concerned, the large amount of geophysical data now available allows, in principle, to achieve a high level of accuracy. On the other hand, the knowledge of the features of particle production in hadronic interactions is still affected by important uncertainties (≥ ±15%). Since we have not yet a calculable theory for the non-perturbative QCD regime, we remain with many different attempts of building interaction models, which are tuned by comparison to existing experimental data. Sometimes, these models can give satisfactory outputs only in restricted fields of applications. In this work we discuss in some more detail the situation of these attempts to describe hadronic interactions, trying to evidentiate the advantages of the so called “microscopic” models, \textit{i.e.} those which try to embed as much as possible of the current theoretical ideas in terms of elementary constituents and of their fundamental interactions. We shall make explicit reference to the set of models contained in the FLUKA MonteCarlo code\textsuperscript{[3]}.

2. Requirements for Interaction Models

Cosmic ray physics is particularly demanding from the point of view of particle production models, since it is in general necessary to consider a wide range of primary energy and projectiles. If we take for example the case of atmospheric neutrinos, even if we limit ourselves to the class of “contained” or “partially contained” events in Super–Kamiokande\textsuperscript{[4]}, namely events with $E_\nu$ in the range from 0.2 to few tens of GeV’s, primary cosmic rays from 1 GeV up to at least 1 TeV per particle must be considered. Furthermore, primary cosmic rays are composed of protons and nuclei, from He to Fe (higher mass can be neglected with good approximation). This is quite a different situation with respect to the standard case of particle physics, where, in general, almost mono-energetic beams are considered. In addition, while in particle physics the attention is mainly devoted to energy deposition, here instead the details of
single interactions are fundamental to obtain a flux prediction. As previously stated, it is not possible to rely on a unique model capable of giving the same quality of results at all energies, and for all kinematics regimes. Therefore particular attention is necessary in order to assure the right continuity across the transition region between the different regimes.

Last, but not least, there is the necessity of dealing with different nuclear species. So far, this problem has been mainly solved by recurring to the so called “superposition” model, where a nucleus of mass number A and energy $E_0$ is considered to be equivalent to A nucleons, each one having energy $E_0/A$.

The question if this is a totally acceptable approximation is still an argument of discussion.

3. Different approaches: parametrized vs “microscopic” models

In building a suitable model for particle production, we can identify two main different attitudes: parametrized codes and “microscopic” models. In the first case, one relies upon analytical formulas derived from some general phenomenological features of particle production, and with parameters that are obtained from fits to experimental data. An example of these fundamental properties of hadronic interactions, is Feynmann scaling, which is known to be a rather good (although not completely exact) approximation, especially in the forward (“fragmentation”) kinematic region which is dominant in secondary production by cosmic rays. This property can be expressed as follows. The number of pions of energy $E_\pi$ produced in an interaction by a primary proton of energy $E_0$ is well represented expression:

$$\frac{dn_\pi}{dE_\pi}(E_\pi, E_0) \simeq \frac{1}{E_\pi} F \left( \frac{E_\pi}{E_0} \right)$$

(1)

where the function $F(x) = x = E_\pi / E_0 \sim x_{\text{Feynman}}$, is approximately independent from the primary energy $E_0$, and decreases monotonically from a finite value for $x \rightarrow 0$, to zero for $x \rightarrow 1$. The shape of the curve can be easily expressed by means of a combination of elementary functions with just a few parameters that can be extracted from experimental data sets. A noticeable example of this kind of approach is at the basis of the work which has been carried on by the Bartol group for many years, producing many valuable results, and in particular the prediction for atmospheric neutrino fluxes[5]. For this purpose, they constructed the TARGET numerical model, a module which can be easily inserted in any cascade program. TARGET considers hadron interactions on light nuclei, like Oxygen and Nitrogen, subdividing the available energy between leading nucleons and other produced hadrons on the basis of an assumed elasticity function. Pions, kaons, etc. are then produced according to parametric formulas reproducing the scaling properties described above. Experimental data at different energies fix the parameters and guide the evolution of multiplicities as a function of energy. At low and intermediate energies, resonance production is considered. Care has been taken to assure event by event energy conservation. The advantage of this kind of approach, mainly used in the framework of a 1-dimensional description, is that it can lead to the comprehension of some important and general properties of particle production in terms of analytical expressions. The price to pay is the lack of generality and the spoiling of correlations among reaction products.

The second line of approach is instead the use of models which try to describe interactions in terms of the properties of elementary constituents. In principle one would like to derive all features of “soft” interactions (low-$p_T$ interactions) from the QCD Lagrangian, as it is done for hard processes. Unfortunately the large value taken by the running coupling constant prevents the use of perturbation theory. Indeed, in QCD, the color field acting among quarks is carried by the vector bosons of the strong interaction, the gluons, which are “colored” themselves. Therefore the characteristic feature of gluons (and QCD) is their strong self-interaction. If we imagine that quarks are held together by color lines of force, the gluon-gluon interaction will pull them together into the form of a tube or a string. Since quarks are confined, the energy required to “stretch” such a string is increasingly large until it suffices to materialize a quark-antiquark couple from the vacuum and the string breaks into two shorter ones, with still quarks at both ends. Therefore it is not unnatural that because of quark confinement, theories based on interacting strings emerged as a powerful tool in understanding QCD at the soft hadronic scale (the non-perturbative regime). Different implementations of this idea exist, having obtained remarkable success in describing the features of hadronic interactions. Some of these codes have already found applications in astroparticle physics. We can quote a few major examples:

$^1$an upgrade of the TARGET model, also in view of 3-D applications, has been presented in [6]
1. the atmospheric neutrino flux by M. Honda et al.\[35\], which has been obtained by a combinations of microscopic codes embedded into an original shower code;

2. the CORSIKA shower code\[31\], which offer the choice among different microscopic models.

3. the already mentioned general purpose MonteCarlo code FLUKA, which is now applied also to cosmic ray physics by us and other authors, and that will be later described in more detail.

In our opinion, in the microscopic approach each step has sound physical basis and allows to reach a deep understanding of the phenomena and a high reliability of predictions. The performances are optimized comparing with particle production data at single interaction level. The final predictions are obtained with a minimal set of free parameters, fixed for all energies and target/projectile combinations. Results in complex cases as well as scaling laws and properties come out naturally from the underlying physical models. The basic conservation laws are fulfilled “a priori”. A microscopic model can reach a very high level of detail, at least in principle, and therefore is a good choice when aiming at precision calculations. The price to pay is the loss of simplicity and flexibility: there are no more simple analytical guidelines which allow to understand the basic properties. Furthermore microscopic codes are more demanding than parametrizations in terms of computing power. Parametrized models (if parametrizations are performed at the level of single interactions) are instead useful as a first, fast and flexible approach.

Instead, models tuned on “integral data”, like calorimeter resolutions, thick target yields etc., can be very inaccurate at the level of single interactions, as shown in ref. \[7\] for the case of GEANT-GHEISHA: such a model cannot be used to obtain a reliable calculation of particle fluxes. In our opinion this might be a problem for the low energy (<80 GeV) calculations performed with CORSIKA, when GHEISHA is selected under that energy threshold.

In the following section we shall concentrate on the example of the FLUKA MonteCarlo code.

4. The FLUKA model

The modern FLUKA\[3\] is an interaction and transport MonteCarlo code able to treat with a high degree of detail the following problems:

- Hadron-hadron and hadron-nucleus interactions 0-100 TeV
- Electromagnetic and $\mu$ interactions 1 keV-100 TeV
- Charged particle transport - ionization energy loss
- Neutron multigroup transport and interactions 0-20 MeV
- Nucleus-nucleus and hadron-nucleus interactions 0-10000 TeV/n: under development

Here we shall review the two hadronic models which are used inside FLUKA to describe nonelastic interactions:

- The “low-intermediate” energy one, PEANUT, which covers the energy range up to 5 GeV
- The high energy one which can be used up to several tens of TeV, based on the color strings concepts sketched in the previous section.

The nuclear physics embedded in the two models is very much the same. The main differences are a coarser nuclear description (and no preequilibrium stage) and the Gribov-Glauber cascade for the high energy one.

4.1. The PEANUT Model

Hadron-nucleus non-elastic interactions are often described in the framework of the IntraNuclear Cascade (INC) models. This kind of model was developed at the very beginning of the history of energetic nuclear interaction modelling, but it is still valid and in some energy range it is the only available choice. Classical INC codes were based on a more or less accurate treatment of hadron multiple collision processes in nuclei, the target being assumed to be a cold Fermi gas of nucleons in their potential well. The hadron-nucleon cross sections used in the calculations are free hadron–nucleon cross sections. Usually, the only
quantum mechanical concept incorporated was the Pauli principle. Possible hadrons were often limited to pions and nucleons, pions being also produce or absorbed via isobar (mainly $\Delta_{33}$) formation, decay, and capture. Most of the historical weaknesses of INC codes have been mitigated or even completely solved in some of the most recent developments [3,8], thanks to the inclusion of a so called “preequilibrium” stage, and to further quantistic effects including coherence and multibody effects.

All these improvements are considered in the PEANUT (PreEquilibrium Approach to NUclear Thermalization) model of FLUKA. Here the reaction mechanism is modelled in by explicit intranuclear cascade smoothly joined to statistical (exciton) preequilibrium emission [9] and followed by evaporation (or fission or Fermi break-up) and gamma deexcitation. In both stages, INC and exciton, the nucleus is modelled as a sphere with density given by a symmetrized Woods-Saxon [10] shape with parameters according to the droplet model [11] for $A>16$, and by a harmonic oscillator shell model for light isotopes (see [12]). The effects of the nuclear and Coulomb potentials outside the nuclear boundary are included. Proton and neutron densities are generally different. Binding Energies are obtained from mass tables. Relativistic kinematics is applied at all stages, with accurate conservation of energy and momentum including those of the residual nucleus. Further details and validations can be found in [3].

For energies in excess of few hundreds MeV the inelastic channels (pion production channels) start to play a major role. The isobar model easily accommodates multiple pion production, for example allowing the presence of more than one resonance in the intermediate state (double pion production opens already at 600 MeV in nucleon-nucleon reactions, and at about 350 MeV in pion-nucleon ones). Resonances which appear in the intermediate states can be treated as real particles, that is, they can be transported and then transformed into secondaries according to their lifetimes and decay branching ratios.

4.2. The Dual Parton Model for high energy

A theory of interacting strings can be managed by means of the Reggeon-Pomeron calculus in the framework of perturbative Reggeon Field Theory [13], an expansion already developed before the establishment of QCD. Regge theory makes use explicitly of the constraints of analyticity and duality. On the basis of these concepts, calculable models can be constructed and one of the most successful attempts in this field is the so called “Dual Parton Model” (DPM), originally developed in Orsay in 1979 [14]. It provides the theoretical framework to describe hadron-nucleon interaction from several GeV onwards. In DPM a hadron is a low-lying excitation of an open string with quarks, antiquarks or diquarks sitting at its ends. In particular mesons are described as strings with their valence quark and antiquark at the ends. (Anti)baryons are treated like open strings with a (anti)quark and a (anti)diquark at the ends, made up with their valence quarks.

At sufficiently high energies, the leading term in high energy scattering corresponds to a “Pomeron” (IP) exchange (a closed string exchange with the quantum numbers of vacuum), which has a cylinder topology. By means of the optical theorem, connecting the forward elastic scattering amplitude to the total inelastic cross section, it can be shown that from the Pomeron topology it follows that two hadronic chains are left as the sources of particle production (unitarity cut of the Pomeron). While the partons (quarks or diquarks) out of which chains are stretched carry a net color, the chains themselves are built in such a way to carry no net color, or to be more exact to constitute color singlets like all naturally occuring hadrons. In practice, as a consequence of color exchange in the interaction, each colliding hadron splits into two colored system, one carrying color charge $c$ and the other $\bar{c}$. These two systems carry together the whole momentum of the hadron. The system with color charge $c$ ($\bar{c}$) of one hadron combines with the system of complementary color of the other hadron, in such a way to form two color neutral chains. These chains appear as two back-to-back jets in their own centre-of-mass systems. The exact way of building up these chains depends on the nature of the projectile-target combination (baryon-baryon, meson-baryon, antibaryon-baryon, meson-meson). Let us take as example the case of nucleon-nucleon (baryon-baryon) scattering. In this case, indicating with $q_p^v$ the valence quarks of the projectile, and with $q_t^v$ those of the target, and assuming that the quarks sitting at one end of the baryon strings carry momentum fraction $x_p^v$ and $x_t^v$ respectively, the resulting chains are $q_t^v = q_p^v q_t^v$ and $q_p^v - q_p^v q_t^v$, as shown in fig. 1.

Energy and momentum in the centre-of-mass system of the collision, as well as the invariant mass squared of the two chains, can be obtained from:

$$E_{ch1}^* \approx \frac{\sqrt{s}}{2}(1 - x_p^v + x_t^v)$$
Figure 1. Leading two-chain diagram in DPM for $p - p$ scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities.

\[ E_{ch2}^* \approx \frac{\sqrt{s}}{2} (1 - x_t^u + x_t^v) \]

\[ p_{ch1}^* \approx \frac{\sqrt{s}}{2} (1 - x_t^v - x_t^u) = -p_{ch2}^* \]

\[ s_{ch1} \approx s(1 - x_t^v)x_t^u \]

\[ s_{ch2} \approx s(1 - x_t^u)x_t^v \]  \hspace{1cm} (2)

The single Pomeron exchange diagram is the dominant contribution, however higher order contributions with multi-Pomeron exchanges become important at energies in excess of 1 TeV in the laboratory. They correspond to more complicated topologies, and DPM provides a way for evaluating the weight of each, keeping into account the unitarity constraint. Every extra Pomeron exchanged gives rise to two extra chains which are built using two $q\bar{q}$ couples excited from the projectile and target hadron sea respectively. The inclusion of these higher order diagrams is usually referred to as multiple soft collisions.

Two more ingredients are required to completely settle the problem. The former is the momentum distribution for the $x$ variables of valence and sea quarks. Despite the exact form of the momentum distribution function, $P(x_1, \ldots, x_n)$, is not known, general considerations based on Regge arguments allow to predict the asymptotic behavior of this distribution whenever each of its arguments goes to zero. The behavior turns out to be singular in all cases, but for the diquarks. A reasonable assumption, always made in practice, is therefore to approximate the true unknown distribution function with the product of all these asymptotic behaviors, treating all the rest as a normalization constant.

The latter ingredient is a hadronization model, which must take care of transforming each chain into a sequence of physical hadrons, stable ones or resonances. The basic assumption is that of chain universality, which assumes that once the chain ends and the invariant mass of the chain are given, the hadronization properties are the same regardless of the physical process which originated the chain. Therefore the knowledge coming from hard processes and $e^+e^-$ collisions about hadronization can be used to fulfill this task. There are many more or less phenomenological models which have been developed to describe hadronization (examples can be found in [15,16]). In principle hadronization properties too can be derived from Regge formalism [17].
It is possible to extend DPM to hadron-nucleus collisions too [14], making use of the so-called Glauber-Gribov approach. Furthermore DPM provides a theoretical framework for describing hadron diffractive scattering both in hadron-hadron and hadron-nucleus collisions. General information on diffraction in DPM can be found in [18] and details as well as practical implementations in the DPM framework in [19].

At very high energies, those of interest for high energy cosmic ray studies (10–10⁵ TeV in the lab), hard processes cannot be longer ignored. They are calculable by means of perturbative QCD and can be included in DPM through proper unitarization schemes which consistently treat soft and hard processes together. The interested reader can find more information as well as practical implementations and results in [14,20].

DPM exhibited remarkable successes in predicting experimental observables. The quoted references include a vast amount of material showing the capabilities of the model when compared with experimental data. However, it must be stressed that other models are available, but most of them are not an approach based on string formation and decay. For example, the Quark Gluon String Model [21] has been developed more or less in parallel with DPM. This model shares most of the basic features of DPM, while differing for some details in the way chains are created and in the momentum distribution functions.

5. Benchmarks of the FLUKA Model

The predictions of FLUKA have been checked with a large set of experimental data collected in accelerator experiments. Here we shall limit ourselves to show only a few examples, among the most important in view of the application of the code to cosmic ray applications.

Two sets of data are of particular relevance to check the quality of a model to be used for the calculation of atmospheric neutrino fluxes. These concern p-Be collisions and are reported in fig. 2: in ref.[22] the central rapidity region has been mainly explored, while in ref.[23] the forward region has been investigated. In both cases the agreement of FLUKA predictions is quite good.

Measurements of \(\pi^\pm\) and \(K^\pm\) production rates by 400 GeV/c protons on Be targets were performed by Atherton et al. [24] for secondary particle momenta above 60 GeV/c and up to 500 MeV/c of transverse momentum. Recently the NA56/SPY (Secondary Particle Yields) experiment [25] was devoted to directly measure these yields in the momentum region below 60 GeV/c. The SPY experiment measured the production at different angles \(\theta\) and momenta \(P \leq 135\) GeV/c down to 7 GeV/c for pions, kaons, protons and their antiparticles, using a 450 GeV/c proton beam impinging on Be targets. These data were extremely valuable to improve the hadronization model of FLUKA so as to arrive at the present version.

FLUKA is in agreement with the Atherton and the SPY measurements at the level of \(\sim 20\%\) in the whole momentum range of all secondaries, with the exception of a few points mostly for negative kaons. The case of pions is reported in fig.3. Also the \(\theta\) dependence of the measured yields is reasonably described by FLUKA. The measured \(K^\pm/\pi^\pm\) ratios are reproduced to better than \(20\%\) below 120 GeV/c.

This example was of particular relevance, since other attempts, using for instance the hadronic interfaces of GEANT (and in particular GEANT-GHEISHA) yielded a much worse agreement, as shown in ref.[26].

6. Example of calculations of particles in atmosphere

In the last years, the FLUKA interaction models have been used to produce new predictions for the atmospheric neutrino fluxes within a full 3D calculation[28]. These fluxes have been also considered by the Super–Kamiokande experiment[4]. In the framework of the same group, a new calculation has been developed, choosing again another microscopic code based on the Dual Parton Model: DPMJET-III[29,30]. It gives results close to those obtained with FLUKA.

In the last two years a considerable amount of work has been devoted to cross check the validity of the calculation model. As far as the FLUKA approach is concerned, at least two remarkable results can be quoted:

1. The reproduction of the features of primary proton flux as a function of geomagnetic latitude as measured by AMS[32], thus showing that the geomagnetic effects and the overall geometrical description of the 3–D setup are well under control. In addition, the same work shows that also the fluxes of secondary \(e^+e^-\) measured at high altitude (eventually the last stage of the chain decay of produced mesons) are reproduced.
2. The good reproduction of the data on muons in atmosphere as measured by the CAPRICE experiment[33], both at ground level and at different floating altitudes[34], when starting from the same primary flux (Bartol fit) used to generate atmospheric neutrinos. See the quoted reference for relevant plots and numbers.

The fluxes of atmospheric muons are strictly related to the neutrino ones, because almost all $\nu$’s are produced either in association, with, or in the decay of $\mu^\pm$. Therefore it is possible to conclude that, for that choice of primary spectrum, the $\nu$ fluxes predicted by FLUKA are probably in the right range. To a large extent the agreement between the original HKKM[35] and Bartol[5] calculations of the $\nu$ fluxes, despite they started from different estimates of the primary flux and different hadronic interaction models, is not casual, but the result of the $\mu$ constraint. Furthermore, the agreement exhibited by the FLUKA simulation for muons of both charges gives confidence on the predictions of FLUKA for the parent mesons of muons (mostly pions).

The shower simulations in atmosphere have been compared also to the most recent hadron spectra at different latitudes and altitudes, obtaining remarkable agreement. As an example, in fig. 4 we compare MonteCarlo results to the hadron flux measured with the KASKADE experiment[36].
Figure 3. Double differential cross section for $\pi^+$ (left) and $\pi^-$ (right) production for 450 GeV/c protons on a 10 cm thick Be target (data from ref. [24] and [25]). Data are given as a function of $\theta_{\text{lab}}$ and for different momentum bins. From top to bottom: 7, 15, 40 and 135 GeV/c, scaled respectively by a factor of 19683, 2187, 81 and 9. Histograms are simulation results.
Figure 4. Hadron flux measured with the KASKADE experiment [36]. Histogram is simulation result.
7. Conclusions

The phenomenological study of hadronic interactions is still a fundamental issue for astroparticle physics. Now that, at least for primary energy lower than 100 GeV, the uncertainties on primary spectrum have been substantially reduced by the quality of new experiments like AMS[1] and BESS[2], the need for a better quality model of hadronic interactions is even more necessary if accuracy of predictions has to be pursued. For this goal the “microscopic” codes are mostly recommended, thanks to their predictive power in a very large kinematic region, constrained by a limited number of parameters. Uncertainties on the modelling of hadronic interactions will remain a fundamental issue, and probably only new data, if experimental systematics can be kept under reasonable control, will help model builders. The HARP experiment[27] at CERN is aiming at this goal. This kind of activity is beneficial not only for particle physics and astrophysics, but also for applied science, since these calculations are necessary to understand radiation fluxes in the Earth’s atmosphere, and this is of great interest for civil aviation and for the design of satellite activities[37]. The FLUKA MonteCarlo model is already being used for this purpose: doses to commercial flight are the subject of a work in progress, together with the development of a specific model for heavy ion transport and interaction: this will be of the utmost importance for dose and damage calculation in space aircrafts.

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