CP violation in supersymmetric models with Hermitian Yukawa couplings and $A$-terms

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**Abstract**

We analyse the CP violation in supersymmetric models with Hermitian Yukawa and trilinear couplings. We show that Hermitian Yukawa matrices can be implemented in supersymmetric models with $SU(3)$ flavor symmetry. An important feature of this class of models is that the supersymmetric contributions to the EDM of the neutron and mercury atom are suppressed. In this framework, $\varepsilon_K$ can be saturated by a small non–universality of the soft scalar masses through the gluino contribution. We perform a detailed analysis for the supersymmetric contributions to $\varepsilon'/\varepsilon$. Although, the gluino contribution is found to be negligible due to a severe cancellation between LR and RL mass insertions, the chargino contribution can be significant and accommodate the experimental results. Additionally, we find that the SUSY contributions to $\varepsilon'/\varepsilon$ from the effective $\bar{s}dZ$ vertex and the $\Delta I = 3/2$ operators are insignificant. We point out that the standard model gives the leading contribution to the CP asymmetry in $B \to \psi K_S$ decay, while the dominant chargino contribution to this asymmetry is $\lesssim 0.2$. Thus, no constraint is set on the non–universality of this model by the recent BaBar and Belle measurements.
1 Introduction

CP violation is one of the outstanding problems in high energy physics. Although the standard model (SM) is able (till now) to accommodate the experimentally observed CP violation, there are strong hints of additional sources of CP violation beyond the phase in the Cabibbo-Kobayashi-Maskawa matrix ($\delta_{\text{CKM}}$). The strongest motivation for this suggestion is that the strength of CP violation in SM is not sufficient to explain the cosmological baryon asymmetry of our universe [1].

In supersymmetric (SUSY) extensions of the SM, there are new CP violating phases which arise from the complexity of the soft SUSY breaking terms and the SUSY preserving $\mu$-parameter. These new phases have significant implications and can modify the SM predictions in CP violating phenomena. In particular, they would give large contributions to the electric dipole moment (EDM) of the neutron and mercury atom [2], to CP violating parameters ($\varepsilon$ and $\varepsilon'$) of $K - \bar{K}$ system [3–5], and to the CP asymmetries in the $B - \bar{B}$ system [6–11]. These phases can be classified into two categories. The first category includes flavor-independent phases such as the phases of the $\mu$-parameter, $B$-parameter, gaugino masses and the overall phase of the trilinear couplings. The other category includes the flavor-dependent phases, i.e. the phases of the off–diagonal elements of $A_{ij}$ and phases in the squark mass matrix $m_{ij}^2$. Two of the flavor-independent phases can be eliminated by the $U(1)_R$ and $U(1)_{PQ}$ transformations.

However, the non–observation of EDMs imposes a stringent constraint on flavor–independent SUSY phases, the so–called SUSY CP problem. Four solutions to this problem have been indicated so far. In the first, the CP is an approximate symmetry and therefore the CP violating phases are very small ($\sim 10^{-2}$). However, the large CP asymmetry in $B \rightarrow \psi K_s$ decay observed by BaBar [12] and Belle [13] implies that CP is significantly violated and disfavors this possibility. In the second, the sfermion of the first two generations are very heavy while the third generation remains light. However, it was shown [2] that in order to satisfy the EDM of the mercury atom the sfermion masses have to be of order 10 TeV, which leads to a large hierarchy between SUSY and electroweak scales. A third possibility is that the full computation of SUSY contributions to the EDMs involves accidental cancellations among various contributions which may allow for regions of parameter space with large phases. However, it has been found [2] that such EDM cancellation can not occur simultaneously for the electron, neutron and mercury. Finally, the SUSY CP violation can have a flavor off-diagonal character just as in the SM [14]. In this latter possibility, the origin of the CP violation is assumed to be closely related to the origin of flavor structures rather than the origin of SUSY breaking. Thus the flavor blind quantities as the $\mu$-term and gaugino masses are real.
The class of models with flavor–dependent CP violation are based on two major assumptions. First, the SUSY breaking does not break CP, which can happen in some explicit string models [15]. Second, the flavor structure of the model is Hermitian, i.e., the Yukawa matrices and the $A$–terms have to be Hermitian, in order to ensure that the diagonal elements of the $A$–terms are real in any basis and do not induce unacceptably large EDMs.

In Ref. [14], it was shown that within these assumptions, the supersymmetric CP problem can naturally be resolved and a correlation between the CP asymmetry of the $B \rightarrow X_s\gamma$ decay and the EDMs is predicted. However, it was also found that SUSY contribution to $\varepsilon_K$ is, in general, very small and also that the dominant gluino contribution to $\varepsilon'/\varepsilon$ is negligible due to a cancellation between the contributions involving $(\delta_{12})_{LR}$ and $(\delta_{12})_{RL}$ mass insertions. In fact, the SM prediction for $\varepsilon_K$ can be fitted with the current experimental data, however, due to the large uncertainties in the theoretical estimate of $\varepsilon'/\varepsilon$ it is rather unclear if the SM result is consistent with the observed measurements by KTeV [16] and NA31 [17]. Thus, it is necessary systematically to analyse the different SUSY contributions to $\varepsilon'/\varepsilon$ to show if it is possible to saturate the observed value in SUSY models with Hermitian Yukawa couplings and $A$–terms.

In this paper, we study more explicitly CP violation in the K and B system due to flavor dependent phases in SUSY models with the Hermiticity assumption. We show that, in order to saturate $\varepsilon_K$ in a viable region of parameter space non–universality between the squark soft masses is required. This non–universality is also essential to enhance the chargino contribution to $\varepsilon'/\varepsilon$. We demonstrate that the SUSY flavor off–diagonal phases have significant implications on the direct CP asymmetry in the $B \rightarrow X_s\gamma$ decay, while their effect on the CP asymmetry in the $B \rightarrow \psi K_S$ decay are negligible.

The paper is organized in the following way. In section 2 we start by emphasizing the possibility of obtaining Hermitian Yukawa couplings in SUSY models with $SU(3)$ flavor symmetry, and then show that the EDMs in this class of models are one or two order of magnitude below the experimental constraints without any fine–tuning. Section 3 is devoted to the study of CP violation in the Kaon system. In section 4 we consider the CP violation in the $B$–sector and show that in this framework the large CP asymmetry in the $B \rightarrow \psi K_S$ decay is given by the SM contribution while the SUSY contribution is very small. In contrast, the SUSY contribution to the CP asymmetry in the $B \rightarrow X_s\gamma$ decay can be as large as $\pm10\%$. Finally, the conclusions are presented in section 5.


2 Hermiticity and EDM suppression

As discussed in the introduction, an elegant solution for suppressing the EDMs in SUSY models is to have Hermitian flavor structures, \( Y^a = Y^a \dagger \), and \( A^a = A^a \dagger \). It is known that Hermitian Yukawa matrices can be implemented in models with left–right symmetry [18] and horizontal flavor symmetry [19]. In Ref.[14], it was assumed that Hermitian Yukawa appeared due to a horizontal symmetry \( U(3)_H \) which gets broken spontaneously by the VEVs of the real adjoint fields, \( T^a \), hence the Yukawa couplings are given as

\[
Y_{ij} = g_{H} M^2 \langle T^a \rangle (\lambda^a)_{ij},
\]

where \( g_{H} \) is a coupling constant of order one, \( M \) is a mass scale much higher than the electroweak scale, \( \lambda^a \), for \( a = 1, \ldots, 8 \) are Gell-Mann matrices and \( \lambda^0 \) is proportional to the unit matrix. However, the real fields \( T^a \) may only arise from non–supersymmetric sector. In fact, if \( T^a \) are the scalar components of chiral multiplets, they are intrinsically complex and Hermitian Yukawas arise only if the VEVs of \( T^a \) are real.

As we will show in the following subsection, it is possible to obtain Hermitian Yukawa couplings in SUSY models with \( SU(3) \) flavor symmetry broken by complex VEVs of scalar fields \( \phi_a \) and \( \bar{\phi}^a \) in the triplet and antitriplet representation of \( SU(3) \) respectively. The local \( SU(3) \) flavor symmetry provides a dynamical origin for the observed fermion masses and a natural explanation for three quark-lepton families. A considerable amount of work has been done concerning the implications of this symmetry on solving the fermion flavor problem [20].

2.1 Hermitian Yukawa from \( SU(3) \) flavor symmetry

Here, we show that Hermitian Yukawa couplings can be motivated by supersymmetric models with flavor symmetry \( SU(3)_F \). We consider a SUSY model with the gauge group \( G_{SM} \times SU(3)_F \), where \( G_{SM} \) refers to the standard model gauge group. Under \( SU(3)_F \), the matter content of the MSSM is assigned the following quantum numbers:

\[
\{Q_a, L_a\} \equiv 3 \quad \text{and} \quad \{u^c_a, d^c_a, e^c_a\} \equiv \bar{3},
\]

while the MSSM Higgs are singlets under the \( SU(3)_F \) and have the charges \( \{H_u, H_d\} \equiv 1 \). The extra Higgs fields that are used to break \( SU(3)_F \) are \( \phi_a \equiv 3 \) and \( \bar{\phi}^a \equiv 3, a = 1, 2, 3 \).

In this class of models, the lowest dimensional \( SU(3)_F \) invariant operators in the superpotential, which are responsible for generating the fermion masses, are given by

\[
W_{Yuk} = h_u Q_a u^c_b H_u \bar{\phi}^a \phi_b M^2 + h_d Q_a d^c_b H_d \bar{\phi}^a \phi_b M^2 + h_l L_a e^c_b H_d \bar{\phi}^a \phi_b M^2.
\]

Thus for \( \langle \bar{\phi} \rangle = \langle \phi \rangle^* \), i.e., \( \langle \phi_a \rangle = v_a e^{i \varphi_a} \) and \( \langle \bar{\phi}_b \rangle = v_b e^{-i \varphi_b} \), the Yukawa couplings are
given by
\[ Y_{ab}^u = h_u \frac{v_a v_b}{M^2} e^{i(\phi_a - \phi_b)} . \]  
(S3)

Similar expressions hold for \( Y^d \) and \( Y^l \). Eq.(3) clearly displays the usual form for Hermitian Yukawa couplings. Now let us discuss briefly the minimization of the scalar potential of the \( \phi \) fields. The most general renormalizable superpotential involving these Higgs fields has the form
\[ W = \mu \phi_a \bar{\phi}^a + \lambda \phi_a \bar{\phi}^a S + W'(S) , \]  
(S4)

where the \( S \) field is a singlet under both \( G_{SM} \) and \( SU(3)_F \). The requirement that the \( \phi \)'s and \( S \) fields do not contribute to SUSY breaking implies that \( F_{\phi} = F_{S} = 0 \). The scalar potential is given by
\[ V = \left| \mu \phi^a + \lambda \phi^a S \right|^2 + \left| \mu \phi^a + \lambda \phi^a S \right|^2 + \left| \lambda \phi_a \bar{\phi}^a + \frac{\partial W'}{\partial S} \right|^2 + V_D + V_{SB} , \]  
(S5)

where
\[ V_D = \frac{g_3^2 H}{2} \sum_b \left( \phi^b T^b_{\phi_a} + \bar{\phi}^a T^b \bar{\phi}_a \right)^2 . \]  
(S6)

Note that \( \phi^a \) is an anti-triplet under \( SU(3)_F \) as well as \( \bar{\phi}^a \) so the above potential is \( SU(3)_F \) invariant. In the above equation, \( T^b \) are the generators of the \( SU(3)_F \) group, and the sum extends over all these generators. Finally we assume the following soft SUSY breaking terms
\[ V_{SB} = m_{\phi_a}^2 |\phi_a|^2 + m_{\bar{\phi}_a}^2 |\bar{\phi}_a|^2 + \left[ A_{\phi_a} \phi_a \bar{\phi}^a S + B_{\phi_a} \phi_a \bar{\phi}^a + h.c. \right] . \]  
(S7)

The minimization of the scalar potential with respect to \( \phi_a \) and \( \bar{\phi}_a \) depends on the soft SUSY breaking terms and, for a particular choice of these parameters, one can obtain the following VEV’s for \( \phi_a \) and \( \bar{\phi}_b \)
\[ \langle \phi_a \rangle = v_a e^{i\phi_a} , \quad \langle \bar{\phi}_b \rangle = v_b e^{-i\phi_b} , \]  
(S8)

as required in order to get Hermitian Yukawa textures.

Furthermore, since \( \bar{\phi}^a \phi_a \) is a singlet under both the \( G_{SM} \) and \( SU(3)_F \), it can couple to \( H_u \) and \( H_d \) to generate the \( \mu \)-term. In this case we can have the following leading term in the superpotential:
\[ W_\mu \sim \frac{\bar{\phi}^a \phi_a}{M} H_u H_d . \]  
(S9)

Thus, the \( \mu \)-term will be given, after the \( SU(3)_F \) is spontaneously broken, by \( \mu = v_a^2 / M \) which is real. However the \( SU(3)_F \) symmetry, like any other flavor symmetry and left–right symmetry, can not guarantee the reality of the gaugino masses and we have to make the additional assumption that the SUSY breaking dynamics conserves CP. This seems natural if CP breaking is associated with the origin of the flavor structure [14].
2.2 EDM–free SUSY CP violating phases

We have shown that Hermitian Yukawa matrices and a real $\mu$–term can arise naturally in models with $SU(3)_F$ symmetry and the assumption that CP violation and SUSY breaking have different origins leads to $\text{arg}(M_i) = 0$ where $M_i$ are the gaugino masses. In this case the $A$–terms can also be Hermitian and the EDM problem is naturally avoided [14, 18].

In supergravity models, the trilinear parameters are given in terms of the Kähler potential and the Yukawa couplings

$$A_{\alpha\beta\gamma} = F^m[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(\hat{K}_\alpha \hat{K}_\beta \hat{K}_\gamma)] \,, \quad (10)$$

where the Latin indices refer to the hidden sector fields that break SUSY and the Greek indices refer to the observable fields. According to our previous assumption, the $F^m$ is real. Also $\hat{K}_\alpha$ and $\hat{K}_m$ are always real, thus the $A$–terms are Hermitian if the derivatives of the Kähler potential are either generation–independent or the same for the left and right fields of the same generation, i.e., if $\hat{K}_{Q_L_i} \hat{K}_{U_R_i}^{\dagger} = \hat{K}_{Q_L_j} \hat{K}_{U_R_i}^{\dagger}$. These conditions are usually satisfied in string models. It is interesting to note that although the SUSY breaking does not bring in new source of CP violating, the trilinear soft parameters involve off–diagonal CP violating phases of $O(1)$. This stems from the contribution of the term $\partial_m \ln Y_{\alpha\beta\gamma}$, which has been found to be significant and sometimes even dominant in string models [21]. In what follows, we will show that these phases are unconstrained by the EDMs and will study their phenomenological implications in the $K$ and $B$ systems.

The relevant quantities appearing in the soft Lagrangian are $(Y^A)_{ij} = (Y_q)_{ij} (A_q)_{ij}$ (indices not summed) which are also Hermitian at the GUT scale. For the sake of definiteness, we consider the following Hermitian Yukawa matrices at the GUT scale

$$Y^u = \lambda_u \begin{pmatrix} 5.94 \times 10^{-4} & 10^{-3} i & -2.03 \times 10^{-2} \\ -10^{-3} i & 5.07 \times 10^{-3} & 2.03 \times 10^{-5} i \\ -2.03 \times 10^{-2} & -2.03 \times 10^{-5} i & 1 \end{pmatrix} \,, \quad (11)$$

$$Y^d = \lambda_d \begin{pmatrix} 6.84 \times 10^{-3} & (1.05 + 0.947 i) \times 10^{-2} & -0.023 \\ (1.05 - 0.947 i) \times 10^{-2} & 0.0489 & 0.0368 i \\ -0.023 & -0.0368 i & 1 \end{pmatrix} \,, \quad (12)$$

where $\lambda_u = m_t/v \sin \beta$ and $\lambda_d = m_b/v \cos \beta$. These matrices reproduce, at low energy, the quark masses and the CKM matrix. The renormalization group (RG) evolution of Yukawa couplings and the $A$ terms slightly violate the Hermiticity. Therefore, the resulting $Y_q^A$ at the electroweak scale has very small non–zero phases in the diagonal elements (due to the large suppression from the off–diagonal entries of the Yukawa). However, what matters is the relevant phases appearing in the squark mass insertions in the super-CKM basis, i.e.,
the basis where the Yukawa matrices are diagonalized by a unitary transformation of the quark superfields \( \hat{U}_{L,R} \) and \( \hat{D}_{L,R} \) (Note that since the Yukawas are Hermitian matrices they are diagonalized by one unitary transformation, i.e. \( V_L^T V_R = V_R^T V_L \)):

\[
\begin{align*}
\hat{U}_{L,R} & \rightarrow V^u \hat{U}_{L,R}, \quad \hat{D}_{L,R} \rightarrow V^d \hat{D}_{L,R}, \\
Y^u & \rightarrow V^u Y^u V^{u^*} \equiv \text{diag}(h_u, h_c, h_t), \\
Y^d & \rightarrow V^d Y^d V^{d^*} \equiv \text{diag}(h_d, h_s, h_b). \quad (13)
\end{align*}
\]

Accordingly the trilinear terms \( Y^A_q \) transform as \( Y^A_q \rightarrow V^q T Y^A_q V^q \). Thus the \( Y^A_q \) stay Hermitian to a very good degree in the super-CKM basis. Therefore, the imaginary parts of the flavor conserving mass insertions

\[
(\delta_{ii}^{d(u)})_{LR} = \frac{1}{m_q^2} \left[ (V^q Y^A_q V^{q^*})_{ii} v_1(2) - i \mu Y^A_q v_2(1) \right], \quad (14)
\]

that appear in the EDM calculations are suppressed. In the above formula the \( m_q \) refers to the average squark mass and \( v_i = \langle H_0^i \rangle / \sqrt{2} \).

The effective Hamiltonian for the EDM of a fermion \( f \) containing dimension-5 and 6 operators is given by [22]

\[
H_{\text{EDM}}^{\text{eff}} = \sum_i C_i(\mu) \mathcal{O}_i + \text{h.c.}, \quad (15)
\]

where \( \mathcal{O}_i \) are given by

\[
\begin{align*}
\mathcal{O}_1 &= -\frac{i}{2} \bar{f} \sigma_{\mu\nu} \gamma_5 f F_{\mu\nu}, \\
\mathcal{O}_2 &= -\frac{i}{2} \bar{f} \sigma_{\mu\nu} \gamma_5 f G^{a}_{\mu\nu}, \\
\mathcal{O}_3 &= -\frac{1}{6} \bar{f}_{abc} G^{a}_{\mu\rho} G^{b}_{\nu\sigma} G^{c}_{\lambda\lambda} \epsilon^{\mu\nu\rho\sigma}. \quad (16)
\end{align*}
\]

\( \mathcal{O}_{1,2} \) refer to the electric and chromoelectric dipole moment operators and \( \mathcal{O}_3 \) to the Weinberg three gluino operator. All these operators can contribute to the quark EDM while only \( \mathcal{O}_1 \) contributes to the electron EDM, i.e., the Wilson coefficients \( C_2^e \) and \( C_3^e \) of the electron are identically zero. The supersymmetric contributions to the Wilson coefficients of the quark result from the 1-loop gluino, chargino, and neutralino exchange diagrams and also the 2-loop gluino–quark–squark diagram. As emphasized in Ref.[2], the most stringent constraint on the SUSY CP violating phases comes from the recent experimental bounds on the EDMs of the neutron and mercury atom. Therefore we will not discuss the electron EDM here.

The EDMs of quarks, using the naive dimension analysis, are given by

\[
d_q = \eta_1 C_1 + \frac{e}{4\pi} \eta_2 C_2 + \frac{e\Lambda}{4\pi} \eta_3 C_3, \quad (17)
\]

where the QCD correction factors are \( \eta_1 = 1.53, \eta_2 \simeq \eta_3 \simeq 3.4, \) and \( \Lambda \simeq 1.19 \text{ GeV} \) is the chiral symmetry breaking scale. The dominant 1-loop gluino contributions to the Wilson
coefficients of the down and up quarks are given by

\[ C^{d(u)}_1 = -\frac{2}{3} \frac{\alpha_s}{\pi} Q^{d(u)} \frac{\tilde{m}_\mathbb{g}}{m^2} \text{Im}(\delta^{d(u)}_{11})_{LR} M_1(x), \]

\[ C^{d(u)}_2 = -\frac{1}{4} \frac{g_s \alpha_s}{\pi} \frac{\tilde{m}_\mathbb{g}}{m^2} \text{Im}(\delta^{d(u)}_{11})_{LR} M_2(x). \]

Here \( m_{\mathbb{g}} \) is the gluino mass and the function \( M_1(x) \) is defined by

\[ M_1(x) = \frac{1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x)}{2(1 - x)^4}, \]

\[ M_2(x) = \frac{22 - 20x - 2x^2 + 9\ln(x) + 16x \ln(x) - x^2 \ln(x)}{3(1 - x)^4}, \]

with \( x = \frac{m^2_{\mathbb{g}}}{m^2_{\tilde{q}}} \). In the quark model, the EDM of the neutron is given by \( d_n = \frac{1}{3}(4d_d-d_u) \) and the current experimental bound [23]

\[ d_n < 6.3 \times 10^{-26} \text{ e cm (90\% C.L.)} \]  

leads to the constraint \( \text{Im}(\delta^{d(u)}_{11})_{LR} \lesssim 10^{-6} - 10^{-7} \). However, it turns out that the recent experimental limit on the EDM of the mercury atom [24]

\[ d_{Hg} < 2.1 \times 10^{-28} \text{ e cm}, \]

implies stronger bounds on these mass insertions (more than an order of magnitude more stringent than those imposed by the EDM of the neutron) and in addition to \( \text{Im}(\delta^{d(u)}_{22})_{LR} \lesssim 10^{-5} - 10^{-6} \) [2], due considerable contributions from the strange quark to the mercury EDM. Recall that the mercury EDM is sensitive to the chromoelectric EDM of quarks \( C^q_2 \) and the limit in Eq.(23) can be translated into \( |C^d_2 - C^u_2 - 0.012C^s_2|/g_s < 7 \times 10^{-27} \text{ cm} \) [25].

We start our analysis by revisiting the EDM constraints on the flavor off diagonal phases of SUSY models with Hermitian Yukawa as in Eq.(11) and the following Hermitian \( A \)-terms:

\[ A_d = A_u = \begin{pmatrix} A_{11} & A_{12} e^{i\varphi_{12}} & A_{13} e^{i\varphi_{13}} \\ A_{12} e^{-i\varphi_{12}} & A_{22} & A_{23} e^{i\varphi_{23}} \\ A_{13} e^{-i\varphi_{13}} & A_{23} e^{-i\varphi_{23}} & A_{33} \end{pmatrix}. \]

We also assume that the soft scalar masses and gaugino masses \( M_a \) are given by

\[ M_a = m_{1/2}, \quad a = 1, 2, 3, \]

\[ m^2_Q = m^2_{H_1} = m^2_{H_2} = m^2_0, \]

\[ m^2_U = m^2_D = m^2_0 \text{ diag}\{1, \delta_1, \delta_2\}, \]
where the parameters $\delta_i$ and $A_{ij}$ can vary in the ranges $[0, 1]$ and $[-3, 3]$ respectively. Note that in most string inspired models, the squark mass matrices are diagonal but not necessary universal. The non–universality of the squark masses is not constrained by the EDMs. However, this non–universality (specially between the first two generations of the squark doublets) is severely constrained by $\Delta M_K$ and $\varepsilon_K$.

In Fig. 1 we display scatter plots for the neutron and mercury EDMs versus the phase $\varphi_{12}$ for $\tan \beta = 5$, $m_0 = m_{1/2} = 200$ GeV, $A_{ij}$ are scanned in the range $[-3, 3]$, and the phases $\varphi_{13}$ and $\varphi_{23}$ are randomly selected in the range $[0, \pi]$. As stated above, the parameters $\delta_i$ are irrelevant for the EDM calculations and we set them here to one. Finally, since $\mu$ is real the EDM results display very little dependence on $\tan \beta$.

It is important to mention that we have also imposed the constraints which come from the requirement of absence of charge and colour breaking minima as well as the requirement that the scalar potential be bounded from below [26]. These conditions may be automatically satisfied in minimal SUSY models, however in models with non–universal $A$–terms they have to be explicitly checked. In fact, sometimes these constraints are even stronger than the usual bounds set by the flavor changing neutral currents [27].

As can be seen from Fig. 1, the EDMs do not exceed the experimental bounds for most of the parameter space. Generally, they are one or two order of magnitude below the present limit, and the flavor–off diagonal phases of the $A$–terms can be $\mathcal{O}(1)$ without fine–tuning. The points that lead to mercury EDM above the experimental bound correspond to $\varphi_{23} \simeq \pi/2$. This phase induces a considerable contribution to the chromoelectric EDM of the strange quark $C_s^s$. Thus the compatibility with mercury EDM experiment requires that the phase $\varphi_{23}$ should be slightly smaller than $\pi/2$. 
3 CP violation in the Kaon system

We have shown in the previous section that the Hermiticity of the Yukawa couplings and $A$–terms allows the existence of large off–diagonal SUSY CP violating phases while keeping the EDMs sufficiently small. However, the important question to address is whether these “EDM–free” phases can have any implication on other CP violation experiments. In this section, we will concentrate on possible effects in the kaon system.

Recently, it has been pointed out that, in SUSY models with generic non–degenerate $A$–terms (where the phases of the diagonal elements are set to be very small by hand in any basis to satisfy the EDM bounds), it is possible to have large effects in CP violation observables, in particular $\varepsilon_K$ and $\varepsilon'/\varepsilon$ [3–5]. However, as we will show below, in the Hermitian scenario the situation is quite different and it is not straightforward to realize the above mentioned mechanism (which relies on the LR down squark mass insertions) to obtain significant SUSY contribution to $\varepsilon_K$ and $\varepsilon'/\varepsilon$. As shown in Ref.[14], the typical values of $\varepsilon_K$ in this class of models are smaller than the experimental measurement (even with very large off–diagonal elements, $A_{ij} \sim 5m_0$). Moreover, it turns out that $\varepsilon'/\varepsilon$ is also very small ($\sim 10^{-6}$) due to a severe cancellation between the different contributions. In the following, we will show that we have to consider the other SUSY contributions from the LL and RR sectors in order to saturate both $\varepsilon_K$ and $\varepsilon'/\varepsilon$.

3.1 Indirect CP violation

In the kaon system and due to a CP violation in $K^0 - \bar{K}^0$ mixing, the neutral kaon mass eigenstates are superpositions of CP–even ($K_S$) and CP–odd ($K_L$) components. However, the CP–odd $K_L$ decays into two pions through its small CP–even component. This decay, $K_L \rightarrow \pi\pi$, was the first observation of CP violation. The measure for the indirect CP violation is defined as

$$\varepsilon_K = \frac{A(K_L \rightarrow \pi\pi)}{A(K_S \rightarrow \pi\pi)}.$$  \hspace{1cm} (28)

The experimental value for this parameter is $\varepsilon_K \simeq 2.28 \times 10^{-3}$. Generally, $\varepsilon_K$ can be calculated via

$$\varepsilon_K = \frac{1}{\sqrt{2\Delta M_K}} \text{Im}\langle K^0|H_{\text{eff}}^{\Delta S=2}|\bar{K}^0 \rangle.$$  \hspace{1cm} (29)

Here $H_{\text{eff}}^{\Delta S=2}$ is the effective Hamiltonian for the $\Delta S = 2$ transition. It can be expressed via the Operator Product Expansion as

$$H_{\text{eff}}^{\Delta S=2} = \sum_i C_i(\mu)Q_i + h.c.$$  \hspace{1cm} (30)

where $C_i(\mu)$ are the Wilson coefficients and $Q_i$ are the relevant local operators, which are given in Ref.[29]. The main uncertainty in this calculation arises from the matrix
elements of $Q_i$, whereas the Wilson coefficients can be reliably calculated at high energies and evolved down to low energies via the RG running.

The $K^0 - \bar{K}^0$ transition can be generated through the box diagrams with W, Higgs, neutralino, gluino, and chargino exchange. The off-diagonal entry in the kaon mass matrix, $M_{12} = \langle K^0 | H^{\Delta S=2} | \bar{K}^0 \rangle$, is given by

$$M_{12} = M^\text{SM}_{12} + M^H_{12} + M'^{\bar{q}}_{12} + M'^{\tilde{q}}_{12}.$$  \hfill (31)

The SM contribution can be written as [30]

$$M^\text{SM}_{12} = \frac{G_F M^2_W}{12\pi^2} F^2_{K^0} M_{K} \hat{B}_K F^*.$$  \hfill (32)

For the specific Hermitian Yukawa ansatz we are considering, we find the following SM contributions:

$$\varepsilon^\text{SM}_K \approx 1.8 \times 10^{-3}.$$  \hfill (33)

We see that the SM prediction for $\varepsilon_K$ is close to the measured value. However, a precise prediction cannot be made due to the hadronic and CKM uncertainties.

Now let us turn to the supersymmetric contributions. The Higgs and the neutralino contributions are very suppressed and can be neglected. The chargino contribution to the $K^0 - \bar{K}^0$ mixing is given by [31]

$$M^\tilde{\chi}_i = \frac{g^2}{768\pi^2 m_{\tilde{q}}} \frac{1}{3} M_K f^2_{\tilde{q}} \hat{B}_1(\mu) \left( \sum_{a,b} K^*_{a2}(\delta^u_{LL})_{ab} K_{b1} \right) ^2 \sum_{i,j} |V_{i1}|^2 |V_{j1}|^2 H(x_i, x_j),$$  \hfill (34)

where $x_i = (m^{\tilde{q}}_i / m_{\tilde{q}})^2$, $K$ refers the CKM matrix, $a, b$ are the flavor indices, $i, j$ label the chargino mass eigenstates, and $V$ is the matrix that is used for diagonalizing the chargino mass matrix. The loop function $H(x_i, x_j)$ is given in Ref.[31]. However, the chargino contribution can be significant only if there is a large LL mixing in the up- sector, namely $\text{Im}(\delta^u_{LL})_{21} \sim 10^{-3}$ and $\text{Re}(\delta^u_{LL})_{21} \sim 10^{-2}$ [31]. Such mixing can not be accommodated with the universal scalar masses assumption (i.e., $\delta_i = 1$). In this case, the values of the $\text{Im}(\delta^u_{LL})$ are of order $10^{-6}$ which leads to a negligible chargino contribution to $\varepsilon_K$.

With non–universal soft scalar masses ($\delta_i \neq 1$) a possible enhancement in the chargino contribution is expected however, as we will show, this non–universality also leads to a larger enhancement in the gluino contribution. So, the dominant SUSY contribution to the $K^0 - \bar{K}^0$ mixing in this class of models will be provided by the gluino exchange diagrams.

The gluino contribution to the $\Delta S = 2$ effective Hamiltonian is given by [29]

$$H^{\Delta S=2}_\text{eff} = \frac{-\alpha_s^2}{216m_{\tilde{q}}} \left( (\delta^d_{12})^2_{LL} \left( 24 Q_1 x f_6(x) + 66 Q_1 \tilde{f}_6(x) \right) + (\delta^d_{12})^2_{RR} \left( 24 \tilde{Q}_1 x f_6(x) \right) \right)$$

11
\[ +66Q_1\tilde{f}_6(x) + (\delta_{12}^{d})_{LL}(\delta_{12}^{d})_{RR}(504Q_4xf_6(x) - 72Q_4\tilde{f}_6(x) + 24Q_5xf_6(x) \\
+120Q_5\tilde{f}_6(x) + (\delta_{12}^{d})^2_{RL}(204Q_2xf_6(x) - 36Q_3xf_6(x)) + (\delta_{12}^{d})^2_{LR}(204\tilde{Q}_2xf_6(x) \\
-36\tilde{Q}_3xf_6(x)) + (\delta_{12}^{d})_{LR}(\delta_{12}^{d})_{RL} (-132Q_4\tilde{f}_6(x) - 180Q_5\tilde{f}_6(x)) \} , \] (35)

where \( x = m_Q^2/m_{\tilde{q}}^2 \), \( m_{\tilde{q}} \) is the gluino mass, and the functions \( f_6(x) \), \( \tilde{f}_6(x) \) are given by

\[
f_6(x) = \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^6} , \tag{36}
\]

\[
\tilde{f}_6(x) = \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^6} . \tag{37}
\]

The matrix elements of the operators \( Q_i \) between the \( K \)-meson states in the vacuum insertion approximation (VIA), where \( B = 1 \), can be found in Ref.[28]. The VIA generally gives only a rough estimate, so other methods, e.g. lattice QCD, are required to obtain a more realistic value. The matrix elements of the renormalized operators can be written as [29]

\[
\langle \bar{K}^0|Q_1(\mu)|K^0 \rangle = \frac{1}{3}M_Kf_K^2B_1(\mu) , \tag{38}
\]

\[
\langle \bar{K}^0|Q_2(\mu)|K^0 \rangle = -\frac{5}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_Kf_K^2B_2(\mu) , \tag{39}
\]

\[
\langle \bar{K}^0|Q_3(\mu)|K^0 \rangle = -\frac{1}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_Kf_K^2B_3(\mu) , \tag{40}
\]

\[
\langle \bar{K}^0|Q_4(\mu)|K^0 \rangle = -\frac{5}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_Kf_K^2B_4(\mu) , \tag{41}
\]

where \( Q_i(\mu) \) are the operators renormalized at the scale \( \mu \). The expressions of the matrix elements of the operators \( Q_{1-3} \) are valid for the operators \( \tilde{Q}_{1-3} \) [29], and for \( \mu = 2 \) GeV we have [29]

\[
B_1(\mu) = 0.60, \; B_2(\mu) = 0.66, \; B_3(\mu) = 1.05, \; B_4(\mu) = 1.03, \; B_5(\mu) = 0.73. \tag{42}
\]

Using these values, the gluino contribution to the \( K^0 - \bar{K}^0 \) can be calculated via Eq.(29). As mentioned above, for universal soft scalar masses the LL and RR insertions are generated only through the RG running and can be neglected. The LR and RL mass insertions appear at the tree level and may have tangible effects. It is worth mentioning that, the RL and LR mass insertions contribute with the same sign in Eq.(35) and for Hermitian A-terms they are almost equal, so no cancellation between these two contributions occurs.

In Fig. 2 we plot the values of \( |\varepsilon_K| \) versus the phase \( \varphi_{12} \) for \( \delta_i = 1 \) and the other parameters are chosen as in Fig. 1. From this figure, we conclude that the SUSY contribution with Hermitian Yukawa and universal soft scalar masses can not account for the
experimentally observed indirect CP violation in the Kaon system. In Ref.[14] values for $\varepsilon_K \sim 10^{-3}$ have been obtained but these values require light gaugino mass ($m_{1/2} \sim 100$ GeV) which is now excluded by the new experimental limits on the mass of the lightest Higgs. Also it requires that the magnitude of the off–diagonal entries of the $A$–terms should be much larger (at least five times larger) than the diagonal ones, which looks unnatural.

A possible way to enhance $\varepsilon_K$ is to have non–universal soft squark masses at GUT scale. As mentioned above, the soft scalar masses are not necessarily universal in generic SUSY models and their non–universality is not constrained by the EDMs. Thus for $\delta_i \neq 1$ the mass insertion $(\delta^{d}_{12})_{RR}$ is enhanced and we can easily saturate $\varepsilon_K$ through the gluino contribution. To see this more explicitly, let us consider the LL and RR squark mass matrices in the super–CKM basis

$$\begin{align*}
(M^2_{\tilde{d}})^{LL} \sim & \quad V^{d\dagger} M^2_Q V^d, \\
(M^2_{\tilde{d}})^{RR} \sim & \quad V^{d\dagger} (M^2_D)^T V^d.
\end{align*}$$

Due to the universality assumption of $M^2_Q$ at GUT scale, the matrix $(M^2_{\tilde{d}})^{LL}$ remains approximately universal and the mass insertions $(\delta^{d}_{12})_{LL}$ are sufficiently small ($\text{Im}(\delta^{d}_{12})_{LL} \sim 10^{-5}$). However, since the masses of the squark singlets $M^2_D$ are not universal, Eq.(27), sizeable off–diagonal elements in $(M^2_{\tilde{d}})^{RR}$ are obtained. We find that Re$(\delta^{d}_{12})_{RR}$ is enhanced from $\sim 10^{-7}$ in the universal case ($\delta_i = 1$) to $\sim 10^{-3}$ for $\delta_i \sim 0.7$ while the
imaginary part remains the same, of order $10^{-7}$. Thus, in this case, we have
\[
\sqrt{\left| \text{Im} \left( (\delta_{12})_{LL}(\delta_{12})_{RR} \right) \right|} \simeq \sqrt{\left| \text{Re}(\delta_{12})_{RR}\text{Im}(\delta_{12})_{LL} \right|} \simeq 10^{-4}
\]
which is the required value in order to saturate the observed result of $\varepsilon_K$ [28].

In Fig. 3 we show the dependence of $|\varepsilon_K|$ on the parameters $\delta_i$. There, the two curves, from top to bottom, correspond to the values of $|\varepsilon_K|$ versus $\delta_1$ (for $\delta_2 = 1$) and $\delta_2$ (for $\delta_1 = 1$) respectively. As explained above, in this scenario the main contribution of $\varepsilon_K$ is due to LL and RR sectors and the LR sector has essentially no effect on $\varepsilon_K$. We also see that any non–universality between the soft scalar masses of the third and the first two generations can not lead to significant contribution to $\varepsilon_K$ and some splitting between the scalar masses of the first two generations is necessary. This stems from the fact that the effect of the third generation on the mass insertion $(\delta_{12})_{RR}$ is suppressed by $V_{13} \sim \mathcal{O}(10^{-2})$ while $V_{12} \sim \sin \theta_C$. Finally, as we can see from this figure, in order to avoid over saturation of the experimental value of $\varepsilon_K$, the parameter $\delta_1$ should be of order 0.8.

### 3.2 Direct CP violation

Next let us consider SUSY contributions to $\varepsilon'/\varepsilon$. The ratio $\varepsilon'/\varepsilon$ is a measure of direct CP violation in the $K \to \pi\pi$ decays and is given by
\[
\varepsilon'/\varepsilon = -\frac{\omega}{\sqrt{2} |\varepsilon| \text{Re}A_0} \left( \text{Im}A_0 - \frac{1}{\omega} \text{Im}A_2 \right),
\]
where $A_{0,2}$ are the amplitudes for the $\Delta I = 1/2, 3/2$ transitions, and $\omega \equiv \text{Re}A_2/\text{Re}A_0 \simeq 1/22$ reflects the strong enhancement of $\Delta I = 1/2$ transitions over those with $\Delta I = 3/2$. 

The value of $|\varepsilon_K|$ as a function of the parameters $\delta_1$ (upper curve) and $\delta_2$ (lower curve) for $\tan \beta = 5$, and $m_0 = m_{1/2} = 200$ GeV.
Experimentally it has been found to be $\text{Re}(\varepsilon'/\varepsilon) \simeq 1.9 \times 10^{-3}$ which provides firm evidence for the existence of direct CP violation. The $\text{Im}A_{0,2}$ are calculated from the general low energy effective Hamiltonian for $\Delta S = 1$ transition,

$$H^{\Delta S=1}_{\text{eff}} = \sum_i C_i(\mu)O_i + \text{h.c.},$$  \hspace{1cm} (45)

where $C_i$ are the Wilson coefficients and the list of the relevant operators $O_i$ for this transition is given in Ref.[32–34]. Let us recall here that these operators can be classified into three categories. The first category includes dimension six operators: $O_{1,2}$ which refer to the current-current operators, $O_{3–6}$ for QCD penguin operators and $O_{7–10}$ for electroweak penguin operators [32]. The second category includes dimension five operators: magnetic- and electric-dipole penguin operators $O_g$ and $O_\gamma$ which are induced by the gluino exchange [33]. The third category includes the only dimension four operator $O_Z$ generated by the $\tilde{s}dZ$ vertex which is mediated by chargino exchanges [34]. In addition, one should take into account $\tilde{O}_i$ operators which are obtained from $O_i$ by the exchange $L \leftrightarrow R$.

In spite of the presence of this large number of operators that in principle can contribute to $\varepsilon'/\varepsilon$, it is remarkable that few of them can give significant contributions. As we will discuss below, this is due to the suppression of the matrix elements and/or the associated Wilson coefficients of most of the operators. The SM contribution to $\varepsilon'/\varepsilon$ is dominated by the operators $Q_6$ and $Q_8$, and can be expressed as

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{SM}} = \frac{\text{Im} (\lambda_t \lambda_u^*)}{|\lambda_u|} F_{\varepsilon'},$$  \hspace{1cm} (46)

where $\lambda_i = V_{is}^*V_{id}$ and the function $F_{\varepsilon'}$ is given in Ref.[30]. By using our Hermitian Yukawa in Eq.(11) we get

$$\varepsilon'/\varepsilon \simeq 7.5 \times 10^{-4}.$$  \hspace{1cm} (47)

Again, the SM prediction is below the observed value. Nevertheless, this estimation cannot be considered as a firm conclusion for a new physics beyond the SM since there are significant hadronic uncertainties involved.

The supersymmetric contribution to $\varepsilon'/\varepsilon$ depends on the flavor structure of the SUSY model. It is known that, in a minimal flavor SUSY model, it is not possible to generate a sizeable contribution to $\varepsilon'/\varepsilon$ even if the SUSY phases are assumed to be large. Recently, it has been pointed out that with non–degenerate $A$–terms the gluino contribution to $\varepsilon'/\varepsilon$ can naturally be enhanced to saturate the observed value [4, 5]. Indeed, in this scenario, the LR mass insertions can have large imaginary parts and the chromomagnetic operator
\( O_g \) gives the dominant contribution to \( \varepsilon'/\varepsilon \)

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)^g \approx \frac{11\sqrt{3}}{64\pi^2\varepsilon} |\text{Re} A_0| \frac{m_s + m_d}{m_s} \frac{F_k^2}{F_\pi^3} m_K^2 \frac{m_d^2}{m_s} \text{Im} \left[ C_g - \tilde{C}_g \right], \tag{48}
\]

where \( C_g \) is the Wilson coefficient associated with the operator \( O_g \), given by

\[
C_g = \frac{\alpha_s \pi}{m_q^2} \left[ (\delta_{12}^d)_{LL} \left( -\frac{1}{3} M_5(x) - 3M_4(x) \right) + (\delta_{12}^d)_{LR} \frac{m_d^2}{m_s} \left( -\frac{1}{3} M_1(x) - 3M_2(x) \right) \right], \tag{49}
\]

where the functions \( M_i(x) \) can be found in Ref. [28] and \( x = \frac{m_g^2}{m_q^2} \).

Using these relations, one finds that in order to saturate \( \varepsilon'/\varepsilon \) from the gluino contribution the imaginary parts of the LR mass insertions for \( x \approx 1 \) should satisfy \( \text{Im}(\delta_{12}^d)_{LR} \sim 10^{-5} \). Such values can easily be obtained in this class of models. However, as mentioned above, in the case of Hermitian \( A \)-terms and Yukawa couplings we have \( (\delta_{12}^d)_{LR} \approx (\delta_{12}^d)_{RL} \), hence we get \( \text{Im} \left[ C_g - \tilde{C}_g \right] \sim \text{Im} \left[ (\delta_{12}^d)_{LR} - (\delta_{12}^d)_{RL} \right] \sim 10^{-6} \) which leads to a negligible gluino contribution to \( \varepsilon'/\varepsilon \) [14]. It is worth noticing that, due to the universality assumption of \( M_{Q_L}^2 \), the imaginary part of the mass insertion \( (\delta_{12}^d)_{LL} \) is of order \( 10^{-5} \). So its contribution to \( C_g \) is negligible with respect to the LR one which is enhanced by the ratio \( m_g/m_s \). To achieve the required contribution to \( \varepsilon'/\varepsilon \) from the LL sector, one has to relax this universality assumption to get \( \text{Im}(\delta_{12}^u)_{LL} \sim 10^{-2} \). However, as we will discuss below, in this case the chargino contribution is also enhanced and becomes dominant.

Now we turn to the chargino contributions. The dominant contribution is found to be due to the terms proportional to a single mass insertion [31].

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)^{\chi^\pm} = \text{Im} \left( \sum_{a,b} K_{a2}^* \delta_{ab}^u (\delta_{ab}^u)_{LL} K_{b1} \right) F_{\varepsilon'}(x_{q\chi}). \tag{50}
\]

The function \( F_{\varepsilon'}(x_{q\chi}) \), where \( x_{q\chi} = m_\chi^2/m_q^2 \), is given in [31]. We find that the contributions involving a double mass insertion, like those arising from the supersymmetric effective \( sdZ \), can not give any significant contribution, however we take them into account. The above contribution is dominated by \( (\delta_{12}^u)_{LL} \) and in order to account for \( \varepsilon'/\varepsilon \) entirely from the chargino exchange the up sector has to employ a large LL mixing. Again, with universal \( M_{Q_L}^2, \text{Im}(\delta_{12}^u)_{LL} \sim 10^{-6} \) and the chargino contributions (as the gluino one) to \( \varepsilon'/\varepsilon \) is negligible.

Finally, we consider another possibility proposed by Kagan and Neubert to obtain a large contribution to \( \varepsilon'/\varepsilon \) [35]. It is important to note that in the previous mechanisms to generate \( \varepsilon'/\varepsilon \) one is tacitly assuming that \( \Delta I = 1/2 \) transitions are dominant and that the \( \Delta I = 3/2 \) ones are suppressed as in the SM. However, in Ref. [35] it was shown that it is possible to generate a large \( \varepsilon'/\varepsilon \) from the \( \Delta I = 3/2 \) penguin operators. This mechanism
relies on the LL mass insertion \((\delta_{21}^d)_{LL}\) and requires isospin violation in the squark masses \((m_u \neq m_d)\). In this case, the relevant \(\Delta S = 1\) gluino box diagrams lead to the effective Hamiltonian [35]

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=3}^6 \left( C_i(\mu)Q_i + \tilde{C}_i\tilde{Q}_i \right) + \text{h.c.} \tag{51}
\]

where

\[
Q_1 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A}, \quad Q_2 = (\bar{d}_\alpha s_\beta)_{V-A} (\bar{q}_\beta s_\alpha)_{V+A}, \tag{52}
\]

\[
Q_3 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A}, \quad Q_4 = (\bar{d}_\alpha s_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A}, \tag{53}
\]

and the operators \(\tilde{Q}_i\) are obtained from \(Q_i\) by exchanging \(L \leftrightarrow R\). It turns out that the SUSY \(\Delta I = 3/2\) contribution to \(\varepsilon'/\varepsilon\) is given by [35]

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{\Delta I=3/2} \simeq 19.2 \left( \frac{500 \text{GeV}}{m_{\tilde{g}}} \right)^2 B_8^{(2)}(x_u^L, x_u^R, x_d^R) \text{Im}(\delta_{12}^d)_{LL}. \tag{54}
\]

Here, \(x_u^{L,R} = \left( \frac{m_{\tilde{u}_{L,R}}}{m_{\tilde{g}}} \right)^2\), \(x_d^{L,R} = \left( \frac{m_{\tilde{d}_{L,R}}}{m_{\tilde{g}}} \right)^2\), \(B_8^{(2)}(m_c) \simeq 1\), and the function \(K(x,y,z)\) is given by

\[
K(x,y,z) = \frac{32}{27} [f(x,y) - f(x,z)] + \frac{2}{27} [g(x,y) - g(x,z)], \tag{55}
\]

where \(f(x,y)\) and \(g(x,y)\) are given in Ref.[35]. It is clear from the above equation that for \(m_{\tilde{d}_R} = m_{\tilde{u}_R}\), the function \(K(x_u^L, x_u^R, x_d^R)\) vanishes identically. Thus, a mass splitting between the right–handed squark mass is necessary to get large contributions to \(\varepsilon'/\varepsilon\) through this mechanism. Furthermore, the \(\text{Im}(\delta_{12}^d)_{LL}\) has to be of order \(O(10^{-3} - 10^{-2})\) to saturate the observed value of \(\varepsilon'/\varepsilon\). It is clear that, with universal \(M_Q^2\), this contribution can not give any significant value for \(\varepsilon'/\varepsilon\).

Now we relax the universality assumption of \(M_Q^2\) at GUT scale to enhance the mass insertion \((\delta_{12}^d)_{LL}\) and saturate the experimental value of \(\varepsilon'/\varepsilon\). As mentioned above, the non–universality between the first two generation of \(M_Q^2\) is very constrained by \(\Delta M_K\) and \(\varepsilon_K\). Therefore we just assume that the mass of third generation is given by \(\delta_3 m_0\) while the masses of the first two generations remain universal and equal to \(m_0\). This deviation from universality provides enhancement to both \(\varepsilon_K\) and \(\varepsilon'/\varepsilon\). We have chosen the parameters \(\delta_i\) so that the total contributions of \(\varepsilon_K\) from chargino and gluino are consistent with the experimental limits.

In Fig. 4 we present the different gluino and chargino contributions to the \(\varepsilon'/\varepsilon\) and also the total contribution versus the imaginary part of the mass insertion \((\delta_{12}^d)_{LL}\). As explained above, there are two sources for the gluino contributions to \(\varepsilon'/\varepsilon\): the usual \(\Delta I = 1/2\) chromomagnetic dipole operator and the new \(\Delta I = 3/2\) penguin operators. Additionally, there are two sources for the chargino contribution to \(\varepsilon'/\varepsilon\): the usual gluon
Figure 4: The $\varepsilon'/\varepsilon$ contributions versus the Imaginary part of the mass insertion ($\delta_{12}^{d}_{LL}$).

and electroweak penguin diagrams with single mass insertion and the contribution from the SUSY effective $sdZ$ vertex. As can be seen from this figure, the dominant contribution to $\varepsilon'/\varepsilon$ is due to the chargino exchange with one mass insertion. It turns out that the chargino contribution with two mass insertions is negligible. As expected the gluino contribution via the chromomagnetic operator is also negligible due to the severe cancellation between the LR and RL contributions. The contribution from the $\Delta I = 3/2$ operators does not lead to significant results for $\varepsilon'/\varepsilon$. It even becomes negative (opposite to the other contributions) for $\text{Im}(\delta_{12}^{d})_{LL} > 2.5 \times 10^{-3}$.

From this figure we conclude that a non–universality among the soft scalar masses is necessary to get large values of $\varepsilon'/\varepsilon$ and $\varepsilon_{K}$.

4 CP violation in the $B$–system

Recent results from the $B$–factories have confirmed, for the first time, the existence of CP violation in the $B$–meson decays. In particular, the measurements of the CP asymmetry in the $B_d \rightarrow \psi K_s$ decay [12, 13] have verified that CP is significantly violated in the $B$–sector. The time dependent CP–asymmetry $a_{\psi K_s}(t)$ is given by

$$a_{\psi K_s}(t) = \frac{\Gamma(B_d^0(t) \rightarrow \psi K_s) - \Gamma(\bar{B}_d^0(t) \rightarrow \psi K_s)}{\Gamma(B_d^0(t) \rightarrow \psi K_s) + \Gamma(\bar{B}_d^0(t) \rightarrow \psi K_s)} = -a_{\psi K_s} \sin(\Delta m_{B_d} t),$$

where $\Delta m_{B_d}$ is the mass difference between the two mass eigenstates of the $B_d^0 - \bar{B}_d^0$ system, given by $\Delta m_{B_d} = 0.484 \pm 0.010 \text{ (ps)}^{-1}$ [30]. The measurements of this asymmetry
is given by

\[ a_{\psi K_s} = 0.59 \pm 0.14 \pm 0.05 \quad \text{(BaBar)}, \]
\[ a_{\psi K_s} = 0.99 \pm 0.14 \pm 0.06 \quad \text{(Belle)}. \]  

This large CP asymmetry implies that CP is not an approximate symmetry in nature and that the CKM mechanism is the dominant source of CP violation.

In the framework of the SM, the unitarity of the CKM matrix implies the following relation

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \]  

which can be represented as a unitarity triangle in the complex plane \((\bar{\rho}, \bar{\eta})\) [11], where

\[ \bar{\rho} = \frac{1}{2}(1 + R_b^2 - R_t^2), \quad \bar{\eta} = \sqrt{R_b^2 - \bar{\rho}}, \]

and

\[ R_b = \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|}, \quad R_t = \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|}. \]

The angle \(\beta\) of this unitarity triangle is related to the complex phase of element \(V_{td}\) and is defined as [11]

\[ \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}. \]

The asymmetry \(a_{\psi K_s}\) in the SM is given by \(a_{\psi K_s}^{SM} = \sin 2\beta\). Using the Hermitian Yukawa ansatz of Eq. (11), we find that the SM contribution to \(\sin 2\beta\) is given by

\[ \sin 2\beta^{SM} \simeq 0.59. \]

This result is in good agreement with the most recent world average \(a_{\psi K_s} = 0.79 \pm 0.10\). Hence, any new contribution to the CP asymmetry \(a_{\psi K_s}\) should be very limited. In supersymmetric theories the \(\Delta B = 2\) transition, the off–diagonal element of the \(B_d\) mass matrix, can be written as

\[ M_{12}(B_d) = \frac{\langle B_d^0|H_{\text{eff}}|B_d^0\rangle}{2m_{B_d}} = M_{12}^{SM}(B_d) + M_{12}^{\text{SUSY}}(B_d). \]

The effect of supersymmetry can be described by a dimensionless parameter \(r_d^2\) and a phase \(2\theta_d\):

\[ r_d^2 e^{2i\theta_d} = \frac{M_{12}(B_d)}{M_{12}^{SM}(B_d)}, \]

where \(\Delta m_{B_d} = 2|M_{12}^{SM}(B_d)|r_d^2\). Thus, in the presence of SUSY contributions, the CP asymmetry \(B_d \to \psi K_s\) is modified, and now we have

\[ a_{\psi K_s} = \sin(2\beta + 2\theta_d). \]
Therefore, the measurement of $a_{\psi K_S}$ would not determine $\sin 2\beta$ but rather $\sin(2\beta + 2\theta_d)$, where

$$2\theta_d = \arg \left(1 + \frac{M_{12}^{\text{SUSY}}(B_d)}{M_{12}^{\text{SM}}(B_d)}\right).$$  

(66)

From Eqs. (57) and (62) we find that the allowed range for $\sin 2\theta_d$ is as follow

$$-0.22 \lesssim \sin 2\theta_d \lesssim 0.8.$$  

(67)

In the following, we will analyse the impact of the SUSY flavor off-diagonal phases on the allowed values of $a_{\psi K_S}$ and the implication of the above $\sin 2\theta_d$ constraints for the SUSY model we are considering. The effective Hamiltonian for $\Delta B = 2$ processes can be expressed as

$$H_{\text{eff}}^{\Delta B=2} = \sum_i C_i(\mu)Q_i + h.c.,$$  

(68)

where the Wilson coefficients and the relevant operators $Q_i$ can be found in Ref.[9, 10].

The gluino contribution $M_{12}^{\text{SUSY}}(B_d)$ receives significant contributions from the gluino and chargino exchange box diagrams (we also included the charged Higgs contribution in our numerical analysis). The gluino contribution is given by [36]

$$M_{12}^{\chi}(B_d) = \frac{\alpha_w^2 f_{B_d}^2}{24} f_{B_d}^2 \eta^{6/23} M_{B_d}^3 \sum_{i,j=1}^6 \sum_{h,k=1}^2 \frac{1}{m_{\tilde{\chi}_j}^2} A_{ijk} A_{ihj} G' (x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{\chi}_i \tilde{\chi}_j}),$$  

(69)

where the function $A_{ijk}$ and $G'(x, y, z)$ can be found in Ref.[36]. In most of the parameter space, we found that the chargino contribution gives the dominant effect to the CP asymmetry $\sin 2\theta_d$, while the gluino is sub-dominant. This result can be understood by using the mass insertion method. The gluino amplitude receives a leading contribution from $(\delta_{13})_{LL}$, since the mass insertions $(\delta_{13})_{LR}$ and $(\delta_{13})_{RR}$ are much smaller. However, for $m_0 = m_{1/2} \sim 200$ GeV, the $|((\delta^d_{13})_{LL}| \sim 10^{-3}$ which is two orders of magnitude below the required value to saturate the experimental value of $a_{\psi K_S}$ [10], so that one has negligible gluino contribution to $\sin 2\theta_d$. The chargino amplitude is proportional to $(\delta^u_{13})_{LL}$ which can be enhanced by a light stop mass.

Our results for the total SUSY contribution to the CP asymmetry $\sin 2\theta_d$ as a function of the $|((\delta^d_{13})_{LL}|$ are presented in Fig. 5. Here we have also assumed $m_0 = m_{1/2} = 200$ GeV and $\tan \beta = 5$. The parameters $A_{ij}, \varphi_{ij}$ and $\delta_{1,2}$ are varied in their allowed regions fixed by the experimental limits on the EDMs, $\varepsilon_K$ and $\varepsilon/\varepsilon$. As expected, in order to have significant SUSY contributions to the CP asymmetry $a_{\psi K_S}$ a large mixing in the LL sector is required (in order to enhance the dominant chargino contribution). However, such mixing is not allowed in our model with Hermitian Yukawas, as Fig. 5 confirmed.
This, in fact, is due to the $\varepsilon_K$ constraint that severely cuts off any enhancement though the non-universality of the soft scalar masses.

Also as can be seen from Fig. 5, the predicted values of $\sin 2\theta_d$ reside within the allowed range defined in Eq.(67). Hence, in this class of models, the SM gives the leading contribution to the CP asymmetry $a_{\psi K_S}$, while the supersymmetric contribution is very small. Therefore, there is no constraint can be set on the non-universality of this class of models by the recent BaBar and Belle measurements.

We conclude this section with some remarks on the CP asymmetry in $B \to X_s\gamma$. It is known that the SM prediction for the CP asymmetry is very small, less than 1%. Thus, the observation of sizeable asymmetry in this decay would be a clean signal of new physics. The most recent result reported by the CLEO collaboration for the CP asymmetry in these decays is $-27\% < A_{CP}^{b\to s\gamma} < 10\%$ [37]. The SUSY predictions for $A_{CP}^{b\to s\gamma}$ are strongly dependent on the flavor structure of the soft breaking terms. In the universal case, as in the minimal supergravity models, the asymmetry is found to be less than 2% [6]. However, it was shown that the non-universal $A$–terms can result in a large CP–asymmetry in the $B \to X_s\gamma$ [8] and these effects are correlated with the EDMs.

The enhancement of $A_{CP}^{b\to s\gamma}$ is due to important contributions from gluino–mediated diagrams, in this scenario, in addition to the usual chargino and charged Higgs contributions. The expression for the asymmetry $A_{CP}^{b\to s\gamma}$, corrected to next–to–leading order is given by [7]

\[
A_{CP}^{b\to s\gamma} = \frac{4\alpha_s(m_b)}{9|C_7|^2} \left( \frac{10}{9} - 2z \left(v(z) + b(z, \delta)\right) \right) \text{Im} \left[ C_2 C_7^* + \tilde{C}_2 \tilde{C}_7^* \right] + \text{Im} \left[ C_7 C_8^* + \tilde{C}_7 \tilde{C}_8^* \right]
\]
+ \frac{2}{3} z b(z, \delta) \text{Im} \left[ C_2 C_8^8 + \tilde{C}_2 \tilde{C}_8^8 \right],

(70)

where \( z = m_c^2/m_b^2 \). The functions \( v(z) \) and \( b(z, \delta) \) and the Wilson coefficients \( C_i \) can be found in Ref.[7, 8]. In the EDM-free models we are considering, we found that the flavor dependent phase \( \varphi_{23} \) gives a large contribution to the CP asymmetry (since the Wilson coefficients are proportional to \( (\delta_{23}^d)_{LR} \) which receives a dominant contribution from \( A_{23} \) entry [8]). The effect of the other flavor dependent phases on \( A_{CP}^{b\rightarrow s\gamma} \) is found to be very small.

In our SUSY model with Hermitian \( A \)-terms, we found that the gluino contribution to CP asymmetry \( A_{CP}^{b\rightarrow s\gamma} \) can be as large as 10\%, which can be accessible at the \( B \) facrories.

5 Conclusions

In this paper, we have analysed the CP violation in supersymmetric models with Hermitian Yukawa and trilinear couplings. We emphasized that in most of the parameter space of this class of models the EDM of the neutron and mercury atom are two orders of magnitude smaller than the experimental limits. Furthermore, we have shown that Hermitian Yukawas can naturally be implemented in SUSY models with \( SU(3) \) flavor symmetry.

We have studied the CP violation in the Kaon system. We found that in order to saturate \( \varepsilon_K \) a small non–universality between the squark soft masses is required. We investigated the effects of the EDM–free, flavor off–diagonal, phases on the direct CP violation observable \( \varepsilon'/\varepsilon \). A large SUSY contribution to this observable is possible via the chargino contribution.

Finally, we considered the \( a_{\psi K_S} \) and \( A_{CP}^{b\rightarrow s\gamma} \) CP asymmetries in \( B \)-meason decays. We verified that the SM contribution to \( a_{\psi K_S} \) is in agreement with the recent measurements by BaBar and Belle, while the SUSY contributions are very small and hence no further constraint is imposed on the non–universality of these models. In contrast, with \( A_{CP}^{b\rightarrow s\gamma} \) the SM contribution is negligible and the SUSY contribution can be as large as \( \pm 10\% \) which can be accessible at \( B \)-factories.

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References


