On the Strong Coupling Dynamics of

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We study the strong coupling dynamics of the heterotic $E_8 \times E_8$ string theory on the orbifolds $T^6/Z_3$ and $C^3/Z_3$ using the duality with the Horava-Witten M-theory picture. This leads us to a conjecture about the low energy description of the five dimensional $E_0$-theory (the CFT that describes the the singularity region of M-theory on $C^3/Z_3$ compactified on $S^1/Z_2$).

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Introduction

Recently, models of 5D space-time bounded by two end-of-the-world branes attracted attention both as a laboratory for the phenomenology of elementary particle physics as well as of novel cosmological scenarios. A class of such models is inspired by compactifications of the Horava Witten (HW) M-theory HW1,HW2. The understanding of the underlying 5D bulk physics is thus a key ingredient for the study of such models.

It turns out, however, that these 5D bulk theories associated with orbifold compactifications, are generally not well understood. This situation is demonstrated by the following puzzle. Consider the HW duals of the 6d $T^4/Z_N$ heterotic orbifolds. In this case there is a localized gauge symmetry $G_1 \in E_8$ on one of the 6D end-of-the-world branes, and $G_2 \in E_8$ on the other one. Generically the heterotic spectrum includes massless twisted particles that seem to be charged under both $G_1$ and $G_2$. The puzzle is how to account for these states in the HW picture. The resolution of the puzzle follows the realization that in fact there is also a non-perturbative 7D bulk gauge theory $G_{\text{bulk}}$ associated with the $A_{N-1}$ singularity of the corresponding ALE space. The twisted states are in fact charged under say $G_2$ and $G_{\text{bulk}}$ FLO, KSTY and not under $G_2$ and $G_1$. Proving the consistency of this scenario, namely, that there is a full anomaly cancellation of local symmetries, requires assigning particular boundary conditions to the 6d vector and hyper multiplets associated with the 7D vector multiplet. These boundary conditions, which seem to be quite ad hoc in the HW picture, turn out to be very natural when a duality with type I’ string is invoked GKSTY. In the type I’ picture the twisted states can be traced back to strings associated with brane junctions that involve $D6$ branes, $D8$ branes $O_8$ orientifold planes and $NS5$ branes.

When analyzing the HW duals of 4D heterotic orbifold models, namely, compactifications on $T^6/Z_N$ one faces a similar puzzle. Again there are massless twisted states that are charged under both $G_1$ and $G_2$. However, since the geometry at the vicinity of the fixed points is now $R^6/Z_N$, which is not associated with an $A_{N-1}$ singularity but rather with a strongly coupled $E_0$ theory Seiberg, there is no room for a non-abelian non-perturbative 5D gauge symmetry. Thus, the mechanism that resolves the puzzle has to be of a different origin.

The goal of this paper is to explore the duality between the heterotic theory on the $T^6/Z_3$ orbifold compactification and M-theory on $(S_1/Z_2) \times (T^6/Z_3)$. In particular we would like to account for the twisted mixed states.

The compactification of M-theory on $C^3/Z_3$ was intensively explored Witten:1996qb,Seiberg,MS,KMV,Douglas:1996xp,GKSTY. The corresponding low energy field theory is the “mysterious” $E_0$ MS theory which is a strongly coupled 5D CFT with 8 supersymmetries and a one dimensional Coulomb branch. The $E_0$ theory has been explored using various different techniques including non-trivial fixed points of the renormalization flow of 5D supersymmetric theories Seiberg,MS,GMS,IMS, collapse of de Pezzo surfaces in Calabi-Yau compactifications KMV,Douglas:1996xp,MS,CV and type I’ string theoriesCV. In spite of these study efforts and due to its strongly coupled nature, the $E_0$ theory is still not well understood.

Even though the full description of the $E_0$ theory is lacking, partial results, based on educated guesses, about the low energy description of the compactified theory can be obtained. This is similar to the situation with the 6D $(2,0)$ theory where after compactification on $T^2$, the low-energy description of the theory is
given by the $N = 4$ Super-Yang-Mills field theory. This field theory is interacting and is believed to correctly describe all excitations as long as their energy is much lower than the compactification scale.

In this paper we study the compactification of the $E_6$ theory on the segment, $S^1/Z_2$, with certain boundary conditions that preserve $N = 1$ supersymmetry in 4D. We will propose a Lagrangian that (presumably) describes the low energy excitations at a scale below the compactification scale.

The motivation for this Lagrangian comes from the study of the strong coupling dynamics of the heterotic string theory on the orbifolds $T^6/Z_3$ and $C^3/Z_3$. The notation $C^3/Z_3$ and $T^6/Z_3$ is somewhat ambivalent because there are several ways to specify the action of $Z_3$ on the $E_8$ gauge degrees of freedom. In this paper we concentrate mainly on the orbifolds that break the $E_8 \times E_8$ gauge group down to $SU(3) \times E_6 \times SU(3) \times E_6$.

We assume that the volume of $T^6/Z_3$ is large so that worldsheet instantons can be neglected. We analyze the moduli space of these orbifolds from the heterotic string and the low energy supergravity pictures. We discuss the local anomaly cancellation in the various scenarios.

The paper is organized as follows. The moduli space of the $T^6/Z_3$ orbifold is discussed in section 2 from the heterotic theory point of view. We start with a brief description of the model, its spectrum, superpotential and D- term. We then analyze the F-term flatness condition for the $R^6/Z_3$ case, and this condition combined with the D-term flatness for the compact case. We show that the moduli space for the non-compact case is a blow-down at the zero section of a certain line bundle over $P^2 \times P^2$. Section 3 is devoted to a brief reminder of the geometry of the blow-up of the fixed point of the $C^3/Z_3$ orbifold. In particular the metric, complex structure and the Euler number are written down. The moduli space is then reproduced from the supergravity description in the large blow-up limit. For completeness, we discuss gauge instantons for our model as well as the other $T^6/Z_3$ orbifolds. The strong coupling limit as inferred from the Horava Witten dual theory is the topic of section 6. We identify the two scales in the systems, namely, the compactification scale and the scale of the expectation value of the scalar field. We write down the $N = 1$ supersymmetric 5d $E_6$ theory in terms of a 4D $N = 1$ chiral and vector superfields. We then compactify this theory on $S^1/Z_2$ first in the limit of an expectation value which is much larger than the inverse of the compactification scale. In this regime we reduce the 11D HW supergravity to that of a 5D theory in the form of a non-linear sigma model. We then rewrite it in terms of a linear sigma model and determine the relations between the linear an non-linear descriptions. We then conjecture about the theory in the opposite regime where the compactification scale is larger than the inverse of the expectation value of the scalar field. Section 6 is devoted to a discussion of the anomaly cancellation in both the compact and non compact cases. In the former case the cancellation is between the contribution of the twisted states and that of the untwisted states after division by the number of fixed points. In the latter case the integration over the zero mode of the orbifold operation results in an identical division. In section 7 we summarize our results and state several open questions. The moduli space from heterotic string theory The model is the heterotic string on a $T^6/Z_3$ orbifold. The $T^6$ is of the form $T^2 \times T^2 \times T^2$ where each $T^2$ is a quotient of the complex plane with the root lattice of $SU(3)$, namely, $T^2 = \frac{C}{\Lambda_{SU(3)}}$. The tori are characterized by the identifications $z_i \sim z_i + 1$ and $z_i \sim z_i + e^{\pi i / 3}$ where $i = 1, 2, 3$ and admit a $Z_3$ generated by the transformations equation $\Omega_1: z_i \rightarrow \alpha(z_i) e^{2\pi i / 3} z_i$; \quad r_i = (1, 1, -2)$