Intermediate Symmetries in the Spontaneous Breaking of Supersymmetric $SO(10)$

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Abstract

We study the supersymmetric spontaneous symmetry breaking of $SO(10)$ into $SU(3) \otimes SU(2) \otimes U(1)$ for the most physically interesting cases of $SU(5)$ or flipped $SU(5) \otimes U(1)$ intermediate symmetries. The first case is more easily realized while the second one requires a fine-tuning condition on the parameters of the superpotential. This is because in the case of $SU(5)$ symmetry there is at most one singlet of the residual symmetry in each $SO(10)$ irreducible representation. We also point out on more general grounds in supersymmetric GUT’s that some intermediate symmetries can be exactly realized and others can only be approximated by fine-tuning. In the first category, there could occur some tunneling between the vacua with exact and approximate intermediate symmetry. The flipped $SU(5) \otimes U(1)$ symmetry improves the unification of gauge couplings if $(B-L)$ is broken by $||B-L|| = 1$ scalars yielding right handed neutrino masses below $10^{14}$ GeV.
The experimental data suggest at least two new high scales in particle physics. On one hand, the interpretation of the solar [1] and atmospheric [2] neutrino anomalies in terms of oscillations [3] require (mass)$^2$ differences which can be accounted for in the framework of the see-saw mechanism with very heavy right-handed neutrinos. Their Majorana masses settle a high scale, $M_R$, to be associated with the violation of the lepton numbers. On the other hand, the extrapolation of the three running gauge couplings of the Standard Model, suggest that they converge towards a common value at a very high scale, $M_U$, giving evidence for a grand-unifying symmetry. The extraction of $M_R$ from the neutrino data suffers from uncertainties [4], while $M_U$ depends on the physical states that are assumed at intermediate energies to improve the three-to-one convergence of the gauge couplings. Still, they should not differ by more than a few orders of magnitude, not so much as compared to the huge hierarchy between these scales and the electroweak symmetry breaking scale. It is tempting to associate these two scales to the spontaneous breaking of some very high energy gauge symmetries.

Despite the relative vicinity of the two scales, there is no compelling reason to embed the gauge symmetries of $SU(3) \otimes SU(2) \otimes U(1)$ into a larger one. In particular, this is not necessary to explain the gauge coupling unification in a string theoretical framework. Nevertheless, grand-unification symmetries are a very attractive hypothesis (as far as one has control of the proton lifetime) with predictive power. The natural GUT symmetry encompassing both the Standard Model gauge group $SU(3) \otimes SU(2) \otimes U(1)$ and a gauged $B - L$ symmetry is $SO(10)$ [5]. It goes without saying, this is not the only motivation for a $SO(10)$ GUT, and many other aspects are to be found in the huge literature on this subject [6].

The study of the spontaneous breaking of the non-supersymmetric $SO(10)$ models with the present values of the strong coupling $\alpha_s$, shows that the $(B - L)$ symmetry has to be broken at an intermediate scale around $10^{10} - 10^{12}$ GeV [7] to allow for the $SO(10)$ unification. The consistency of this relatively low value with neutrino mass patterns has also been discussed in this context [4].

In this paper we present a reappraisal of these matters in the framework of supersymmetric $SO(10)$ GUT’s. We point out that with the same set of fields that can produce the breaking of $SO(10)$ into $SU(3) \otimes SU(2) \otimes U(1)$, there are other vacua with intermediate gauge symmetries, e.g., the Georgi-Glashow $SU(5)$. When the $SU(3) \otimes SU(2) \otimes U(1)$ vacuum approaches the $SU(5)$ one it has an approximate Georgi-Glashow symmetry, but if they get too close, the physical $SU(3) \otimes SU(2) \otimes U(1)$ vacuum would be tunneled into the $SU(5)$ one. Instead, there are other possible intermediate symmetries, e.g., flipped $SU(5) \otimes U(1)$, which can only be approximated by tuning the parameters in the superpotential, so that they are not expected to be well realized. An approximate $SU(5)$ symmetry would correspond to the breaking of $(B - L)$ above the gauge coupling unification scale, an approximate flipped $SU(5) \otimes U(1)$ symmetry to the opposite situation. In either cases, a big difference in these scales would conflict with the seesaw interpretation of the neutrino oscillation data. Hence a control of either the tunneling or the tuning is needed. Fortunately, the gauge coupling unification points in the direction of a moderate difference in these scales, as the neutrino oscillations seem to do as well.

It is well known that coupling unification is – almost – realized by the minimal supersymetrization of the Standard Model degrees of freedom around 1 TeV, consistently with a $SU(5)$ unification. Therefore, any intermediate symmetry between $SU(3) \otimes SU(2) \otimes U(1)$ and $SO(10)$ should approximately preserve the SUSY $SU(5)$ prediction for the gauge couplings.
Actually, an accurate evaluation of the gauge coupling running at two-loops displays a strong model dependence on the supersymmetric particle thresholds. For instance, a recent analysis [8] shows that in the MSSM with universal gaugino and scalar masses at the TeV scale, the exact two-loop coupling unification would occur for $\mu \sim 10^4$ GeV, where $\mu$ is the usual MSSM higgsino mass parameter. For lower values, $\alpha_s(M_Z)$ comes out slightly higher than the experimental data. The first point we would like to make here is the possibility to improve the prediction of $\alpha_s(M_Z)$ if one assumes that $SO(10)$ is broken into the “flipped” $SU(5) \otimes U(1)$ symmetry which is then broken at a slightly smaller scale.\footnote{The flipped $SU(5) \otimes U(1)$ gauge symmetry has other well-known appealing aspects [9], including an elegant mechanism to obtain the doublet-triplet splitting.}

Although the present precision on $\alpha_s(M_Z)$ requires a two-loop calculation, a one-loop study is sufficient for the qualitative argument presented here. Let us first define the standard combinations of $b$ parameters that control the approach to coupling unification, namely, the running of $\alpha/\alpha_s$ and $\sin \theta_W$, respectively: $\Delta_s b = b_3 - 3b_2/8 - 5b_1/8$ and $\Delta_w b = b_2 - b_1$. In the Standard Model, the ratio $\Delta_s b / \Delta_w b = 1.15$, so that, for non-supersymmetric grandunification, one needs new physics at intermediate scales with a larger value of $\Delta_s b / \Delta_w b$. With the addition of the supersymmetric partners within the MSSM, this ratio increases to $1.34$. If the supersymmetry threshold is below 1 TeV, the two-loop prediction for $\alpha_s(M_Z)$ turns out to be just a bit above the experimental value. Hence, one can improve the gauge coupling unification by adding new degrees of freedom at a scale just below $M_U$, with a relatively low value of $\Delta_s b / \Delta_w b$.

The flipped $SU(5) \otimes U(1)$ model has $\Delta_s b / \Delta_w b = 5/8$, as $b_3 = b_2 = b_5$. Therefore, with the symmetry breaking pattern given by $SO(10) \to$ flipped $SU(5) \otimes U(1)$ at the scale $M_U$, and then flipped $SU(5) \otimes U(1) \to SU(3) \otimes SU(2) \otimes U(1)$, at the scale $M_R$, one can tune $\alpha_s(M_Z)$ toward its experimental value. The ratio $r = M_R / M_U$ depends on the effective supersymmetry threshold $T_{SUSY}$, that incorporates the various supersymmetric particle masses. Roughly speaking one is correcting the supersymmetry threshold dependence with the approximate flipped $SU(5) \otimes U(1)$ threshold effects and $r$ cannot be very small.

This symmetry breaking scheme can be implemented by introducing fields in either a $45$ or a $210$ representation, for the first breaking at $M_U$, and fields in either a $16 \oplus 16^* \oplus 54$, or a $126 \oplus 126^* \oplus 54$ representation, for the second one, at $rM_U$. However, the $126 \oplus 126^*$ which would yield a $\parallel \Delta(B - L) \parallel = 2$ breaking, cannot be added, since it would give $\Delta_s b < 0$. It remains the only possibility of a $16 \oplus 16^*$ breaking with $\parallel \Delta(B - L) \parallel = 1$. With $r < 1$ this is the only simple pattern that can improve the gauge coupling unification with a reasonable sparticle spectrum and that would correspond to a breaking of $(B - L)$ slightly below the gauge coupling unification scale. In particular, the option of left-right symmetric sub-groups of $SO(10)$ would lead to gauge coupling unification only if the sparticle masses would be above $10^4$ GeV, a situation requiring fine-tuning of the MSSM parameters to yield the electroweak symmetry breaking scale.

With these motivations for our study of the supersymmetric spontaneous breaking of $SO(10)$ into $SU(3) \otimes SU(2) \otimes U(1)$ with $SU(5)$ or flipped $SU(5) \otimes U(1)$ intermediate symmetries, let us first stress some general properties of the gauge symmetry breaking in supersymmetric theories. The minimum conditions:

$$ W_i(z^0) = \frac{\partial W(z)}{\partial z^i}(z^0) = 0 , \quad (1) $$
where \( W(z) \) is the superpotential and the \( z^i \)'s stand for the components of the complex scalar fields, are non-trivial only for the components \( W_i(z^0) \) along the directions invariant under the little group \( H_{z^0} \) of \( z^0 \). This follows from the invariance of \( W(z) \) under \( H_{z^0} \). The gradient directions along all \( H_{z^0} \) singlet fields must be considered. Therefore, the number \( n \) of non-trivial equations is equal to the number of \( H_{z^0} \) singlets in the representation of the chiral multiplets. Generically, the solutions of the resulting system of \( n \) equations and \( n \) variables are linear combinations of all the singlet fields, in proportions fixed by the parameters in the superpotential. (We concentrate here on gauge symmetries, but this remark applies to the – complexified of the – global symmetries of the superpotential as well.)

If the initial gauge group is \( G_U \) and \( H_{z^0} \) is \( G_0 \) (we shall consider later on the case where they are \( SO(10) \) and \( SU(3) \otimes SU(2) \otimes U(1) \), respectively) we may look for solutions with symmetry \( G_I \), with \( G_0 \subset G_I \subset G_U \), if there are \( p < n \) \( G_I \) singlets among the fields, since in this case the number of non-trivial equations also reduces to \( p \). We may also look for solutions whose exact symmetry is \( G_0 \) which possess an approximate symmetry \( G_I \) because the predominant \( vev's \) are the \( G_I \) singlets. When these solutions approach the corresponding one with exact \( G_I \) symmetry, a tunneling between the two vacua may become possible. Instead, a vacuum with an approximate symmetry \( G_I \) has not necessarily a counterpart with exact symmetry \( G_I \).

Let us illustrate these situations in a model with \( G_U = SO(10) \) and \( G_0 = SU(3) \otimes SU(2) \otimes U(1) \), with the Higgs chiral multiplets in a \( 16 \otimes 16^* \otimes 45 \otimes 54 \) representation of \( SO(10) \), corresponding to the spinors \( \psi \) and \( \bar{\psi} \), the antisymmetric matrix \( A \) and the symmetric matrix \( S \), respectively. Their \( SU(3) \otimes SU(2) \otimes U(1) \) singlets are the following five directions:

\( a) \) \( \psi_1 \in 16 \) and \( \bar{\psi}_1 \in 16^* \), with little group \( SU(5) \), which by definition is the Georgi-Glashow one,

\( b) \) \( A_1 \) and \( A_{24} \) in the \( 45 \) transforming as a singlet and a \( 24 \) under this \( SU(5) \), respectively,

\( c) \) \( S_0 \in 54 \), with Pati-Salam little group \( SO(6) \otimes SO(4) = SU(4) \otimes SU(2) \otimes SU(2) \).

The \( 45 \) components \( A_1 \) and \( A_{24} \) can be rearranged into a singlet and a \( 24 \) component with respect to the flipped \( SU(5) \otimes U(1) \) as follows:

\[
A'_1 = \frac{1}{5} A_1 + \frac{2\sqrt{6}}{5} A_{24} \\
A'_{24} = \frac{2\sqrt{6}}{5} A_1 - \frac{1}{5} A_{24}
\]  

(2)

As a linear combination of \( A_1 \) and \( A_{24} \), \( A'_1 \) belongs to the same critical orbit as \( A_1 \), \text{i.e.}, they are related by a \( SO(10) \) rotation which does not leave \( \psi_1 \) invariant.

The most general superpotential with quadratic and cubic invariants has the form:

\[
W = m\bar{\psi}\psi + Mtr A^2 + \mu tr S^2 + h\bar{\psi}A\psi + \lambda tr A^2 S + \kappa tr S^3
\]  

(3)

In Table 1 only the contributions of the \( SU(3) \otimes SU(2) \otimes U(1) \) invariant fields to these invariants are written.

The non-trivial equations obtained from the conditions (1) for the superpotential (3) are in correspondance with the five singlets, \( \psi_1, \bar{\psi}_1, A_1, A_{24}, \) and \( S_0 \). The vanishing of the \( D \)–terms requires \( |\psi_1| = |\bar{\psi}_1| \), and since \( W \) is symmetric under \( \psi_1 \leftrightarrow \bar{\psi}_1 \), the number of relevant equations and singlets is reduced to four. According to the previous general discussion, one finds the following solutions:

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(i) a generic vacuum with $SU(3) \otimes SU(2) \otimes U(1)$ symmetry and components along the five directions in proportions that are fixed by the couplings in $W$;

(ii) a $SU(5)$ symmetric vacuum with vanishing components along $A_{24}$ and $S_0$—there are two non-trivial equations for the components $|\psi_1| = |\bar{\psi}_1|$ and $A_1$;

(iii) a $U(3) \otimes SO(4)$ solution with $\psi_1 = \bar{\psi}_1 = 0$, $A_1/A_{24} = \sqrt{2}/3$, as $S_0$ is invariant under $SO(6) \otimes SO(4) \supset U(3) \otimes SO(4)$ (two non-trivial equations);

(iv) a $SO(6) \otimes U(2)$ solution with $\psi_1 = \bar{\psi}_1 = 0$, $A_1/A_{24} = -\sqrt{3}/2$, as $S_0$ is invariant under $SO(6) \otimes SO(4) \supset SO(6) \otimes U(2)$ (two non-trivial equations).

Even if our choice of chiral multiplets is motivated by the physical requirement of the breaking of $SO(10)$ into $SU(3) \otimes SU(2) \otimes U(1)$, there are also extrema of $W$ where the residual invariance does not contain $SU(3) \otimes SU(2) \otimes U(1)$: e.g., a $SO(7)$ invariance that corresponds to the little group of a critical orbit of $16 \oplus 16^*$ and is associated to the $45$ and the $54$ along their critical orbits with a $SO(8) \otimes SO(2)$ symmetry.

In the cases (ii)–(iv), there is at most one singlet of the residual symmetry in each $SO(10)$ irreducible representation which correspond to a critical orbit. The total number of singlets and equations is reduced to two as a result of the increased symmetry.

An approximate intermediate symmetry $G_I =$ flipped $SU(5) \otimes U(1)$ can be obtained by deforming the vacuum by a fine-tuning on the superpotential couplings, $(\sqrt{5} \kappa M + 2 \lambda \mu) \to 0$, so that $A_{24} \to 2\sqrt{3}A_1$, which gives the flipped $SU(5) \otimes U(1)$ singlet in the $45$. The parameters in $W$ are chosen such that $A'_{24}$ takes the largest vev, yielding an intermediate flipped $SU(5) \otimes U(1)$ symmetry. The usual Georgi-Glashow $SU(5)$ is defined by the $\psi_1$ vev which breaks $(B - L)$. The exact symmetry, once all vev’s are taken into account, is $SU(3) \otimes SU(2) \otimes U(1)$.

The implementation of an intermediate flipped $SU(5) \otimes U(1)$ symmetry requires, besides the obvious hierarchy in the parameters to define an intermediate scale, a tuning of the couplings in the model. This approximate symmetry solution does not have an exact symmetry solution counterpart, because in the $S_0 \to 0$, $\bar{\psi}_1 \psi_1 \to 0$ limit even the $45$ component vanishes: This is related to the absence of a cubic invariant for $A$.

Instead, the Georgi-Glashow $SU(5)$ supersymmetric vacuum is generically present, without any tuning of the parameters. If one introduces some hierarchy in the couplings in $W$, namely $M \gg \mu$ and $h \gg \lambda$, another vacuum possesses an approximate $SU(5)$ symmetry as $A_1 > S_0 > A_{24}$. There could be some tunneling between these two vacua when they get close. We may conclude that the flipped $SU(5) \otimes U(1)$ is a less natural intermediate symmetry unless the required fine tuning is provided by some mechanism, e.g., the existence of a fixed point.

Although the $45$ has a flipped $SU(5) \otimes U(1)$ invariant solution (in the same orbit as the Georgi-Glashow solution), there is no solution with that symmetry since the only non-trivial invariants with $A$ in $W$ are linear in $\psi_1$ and $\bar{\psi}_1$ or $S$ which have no flipped $SU(5) \otimes U(1)$ invariant direction. Indeed the direction of $A$ in the $(A_1, A_{24})$ space is settled by a compromise between the alignment to $\psi_1$ and $\bar{\psi}_1$ to give the Georgi-Glashow $SU(5)$ and the alignment to $S_0$ along either the $U(3) \otimes SO(4)$ or the $SO(6) \otimes U(2)$ directions. Therefore, only a particular tuning brings the $45$ along the flipped $SU(5) \otimes U(1)$ direction. The inclusion of higher degree polynomial invariants does not prevent the need for a tuning in the parameters of the superpotential, which gets more involved. However, the presence of a quartic non trivial invariant,
tr $A^4$, allows for a solution with exact flipped $SU(5) \otimes U(1)$. Nevertheless, for $|\psi_1| = |\bar{\psi}_1| \neq 0$, the 45 chooses the $SU(5)$ invariant direction, $A_1$.

The physical interest of an approximate flipped symmetry seems a motivation to include chiral multiplets transforming in the 210 (namely, an antisymmetric tensor of rank = 4), $\Phi$, since a cubic $SO(10)$ invariant $\Phi^3$ exists which does not vanish along the flipped $SU(5) \otimes U(1)$ invariant direction of $\Phi$. There are three $SU(3) \otimes SU(2) \otimes U(1)$ invariant directions in the 210, $\Phi_1$, $\Phi_{24}$ and $\Phi_{75}$, transforming as a singlet, a 24 and a 75 under $SU(5)$, respectively.

The following linear combinations:

$$\Phi'_1 = -\frac{1}{5} \Phi_1 + \frac{2}{5} \Phi_{24} + \frac{2}{\sqrt{5}} \Phi_{75}$$
$$\Phi'_{24} = \Phi_1 + \frac{13}{15} \Phi_{24} - \frac{2}{3\sqrt{5}} \Phi_{75}$$
$$\Phi'_{75} = \frac{2}{\sqrt{5}} \Phi_1 - \frac{2}{3\sqrt{5}} \Phi_{24} + \frac{1}{3} \Phi_{75}$$

(4)

transform as 1, 24, 75 under flipped $SU(5) \otimes U(1)$, respectively.

Let us first concentrate on the $16 + 16^* + 210$ chiral multiplets and the generic superpotential

$$\tilde{W} = m\bar{\psi}\psi + \tilde{M}\Phi^2 + g\bar{\psi}\Phi\psi + \kappa\Phi^3$$

(5)

In Table 1, the cubic invariants are explicitly written in terms of only the $SU(3) \otimes SU(2) \otimes U(1)$ invariant components of the various $SO(10)$ multiplets discussed here, and the corresponding expressions in terms of the flipped $SU(5) \otimes U(1)$ relevant directions are displayed in brackets for the first two invariants and are given by the same expressions with $A, \Phi \rightarrow A', \Phi'$ for the others. The fact that the invariants containing $\psi_1$ and $\bar{\psi}_1$ couple only to $A_1, \Phi_1$, but to all $A', \Phi'$ components disfavours the flipped $SU(5) \otimes U(1)$ invariant solution $A'_1$ as we now turn to discuss.

The presence in Eq.(5) of both the quadratic and cubic invariants for the 210 representation implies the existence of a vacuum such that $\psi = \bar{\psi} = 0$ and $\Phi$ belongs to any critical orbit (excepting those like the one with $SO(6) \otimes SO(4)$ symmetry for which the cubic invariant vanishes) including the one that contains both the $SU(5) \times U(1)$ and the flipped $SU(5) \otimes U(1)$ symmetric vacua. However, for $|\psi_1| = |\bar{\psi}_1| \neq 0$, the 210 $SU(5) \otimes U(1)$ invariant vev aligns with the $SU(5)$ invariance of the $\psi_1$. Including a 54 chiral multiplet, the vanishing of the gradient of $\tilde{W}$ along $\Phi_{75}$ still implies $g\psi_1\bar{\psi}_1 = 0$. Indeed, we cannot put $g = 0$ because this coupling is the only one that links the 210 and the 16 + 16* directions.

Finally, with a 16 + 16* + 45 + 210 chiral multiplet, if $\psi_1\bar{\psi}_1 \neq 0$ both the 45 and the 210 must align to the $SU(5)$ invariant direction. To enforce the 45 or 210 vev to be along the flipped $SU(5) \otimes U(1)$ invariant direction one needs one (two, resp.) tuning conditions to reduce the number of independent equations.

The necessity of tuning conditions to get supersymmetric vacua symmetric under $SU(3) \otimes SU(2) \otimes U(1)$ with a dominant vev along the flipped $SU(5) \otimes U(1)$ invariant direction of an irreducible representation is related to the presence of other singlets of $SU(3) \otimes SU(2) \otimes U(1)$ in the same representation. This gives rise to more minimum equations than variables.
<table>
<thead>
<tr>
<th>Invariant</th>
<th>Expression limited to $SU(3) \otimes SU(2) \otimes U(1)$ singlet fields</th>
</tr>
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<tbody>
<tr>
<td>$16 \otimes 16^* \otimes 45$</td>
<td>$\psi_1 \tilde{\psi}_1 \ A_1 = \psi_1 \tilde{\psi}_1 \left( \frac{1}{5} A_1' + \frac{2\sqrt{6}}{5} A_24' \right)$</td>
</tr>
<tr>
<td>$16 \otimes 16^* \otimes 210$</td>
<td>$\psi_1 \tilde{\psi}_1 \Phi_1 = \psi_1 \tilde{\psi}_1 \left( -\frac{1}{5} \Phi_1' + \frac{2}{5} \Phi_24' + \frac{2}{\sqrt{5}} \Phi_75' \right)$</td>
</tr>
<tr>
<td>$54 \otimes 45 \otimes 45$</td>
<td>$S_0 \left( A^2_{24} - 2\sqrt{6} A_{24} A_1 \right)$</td>
</tr>
<tr>
<td>$210 \otimes 210 \otimes 210$</td>
<td>$\Phi_1^3 + \frac{7}{27} \Phi_{24}^3 + \frac{8\sqrt{6}}{27} \Phi_{75}^3 + \frac{1}{2} \Phi_{24}^2 \Phi_1 + \frac{5\sqrt{6}}{18} \Phi_{24}^2 \Phi_{75} - \Phi_{75}^2 \Phi_1 + \frac{8}{9} \Phi_{75}^2 \Phi_{24}$</td>
</tr>
<tr>
<td>$210 \otimes 45 \otimes 45$</td>
<td>$\Phi_1(2 A_{11}^2 - \frac{1}{2} A_{24}^2) + \Phi_{24}(\frac{1}{3} A_{24}^2 + \sqrt{6} A_{24} A_1) + \frac{5\sqrt{6}}{6} \Phi_{75} A_{24}^2$</td>
</tr>
<tr>
<td>$45 \otimes 210 \otimes 210$</td>
<td>$A_1(3 \Phi_1^2 - 2 \Phi_{24}^2 + \Phi_{75}^2) - \sqrt{6} A_{24}(\Phi_{24}^2 \Phi_1 + \frac{2}{9} \Phi_{24}^2 - \frac{5\sqrt{6}}{9} \Phi_{75} \Phi_{24} - \frac{8}{9} \Phi_{75}^2)$</td>
</tr>
<tr>
<td>$54 \otimes 210 \otimes 210$</td>
<td>$S_0(-\Phi_{24}^2 - 8 \Phi_{75}^2 + 18 \Phi_{24} \Phi_1 + \frac{10\sqrt{6}}{9} \Phi_{75} \Phi_{24})$</td>
</tr>
</tbody>
</table>
problem does not arise for the Georgi-Glashow $SU(5)$ since there are no other $SU(3) \otimes SU(2) \otimes U(1)$ singlets in the $16$ (or in the $\mathbf{126}$) representation. This is not a reason not to consider supersymmetric $SO(10)$ models with intermediate flipped symmetry, as far as the necessary tuning of parameters can be justified.

More in general, one can easily build supersymmetric patterns of symmetry breaking to $SU(3) \otimes SU(2) \otimes U(1)$ with particular intermediate symmetries, when the vev possessing that invariance belongs to a representation without other $SU(3) \otimes SU(2) \otimes U(1)$ singlets. Otherwise, in the most favourable case, a tuning condition is required.

Finally, we have not considered the $\mathbf{126} \oplus \mathbf{126}^*$ representation since it gives $\Delta_s b < 0$ in the intermediate flipped $SU(5) \otimes U(1)$ regime, opposite to what is needed to reach $SO(10)$ unification. The $\mathbf{16} \oplus \mathbf{16}^*$ vev which gives the scale $M_R$, has $||\Delta (B - L)|| = 1$ so that the $||\Delta L|| = 2$ right-handed neutrino Majorana mass matrix $M_N$, must be proportional to $M_R^2$. If appropriate dimension 5 operators could be generated when the fields with masses $O(M_U)$ are integrated out, then one would expect $M_N \sim O(M_R^2/M_U) = O(r^2) \times M_U$. However, at least in the minimal scheme discussed here, when one adds to the $SO(10)$ breaking set of fields the three generations of matter fields in three $16$'s, these dimension 5 operators are not yielded. Indeed, the $45$ or $210$ that couple to the $\mathbf{16} \oplus \mathbf{16}^*$ cannot be coupled to the matter $16$'s without spoiling the $SU(3) \otimes SU(2) \otimes U(1)$ invariant solution. This is naturally enforced by requiring the R-parity symmetry which is anyway needed at the MSSM scale. This forbids contributions $O(M_R^2/M_U)$ at the tree-level. Supersymmetry non-renormalization theorems forbid quantum-loop diagrams to generate them à la Witten [10] up to corrections proportional to supersymmetry breaking soft masses and the R-parity symmetry prevents any mixing through wave-function renormalization.

We are assuming that the cut-off scale of the $SO(10)$ theory is $M_{\text{Planck}}$, and $M_N$ must originate from non-renormalizable $SO(10)$ invariant operators, hence $M_N \sim O(M_R^2/M_{\text{Planck}}) = O(r^2) \times 10^{14}\text{GeV}$. This is consistent with the seesaw mechanism if the heavier neutrino masses are $O(\sqrt{m^2_{\text{atm}}})$. Interestingly enough, this scale is quite close to the one obtained in (non-supersymmetric) $SO(10)$ with intermediate Pati-Salam symmetry [11], even if the scale of $(B - L)$ breaking are very different. This difference might be relevant for baryogenesis through leptogenesis. Instead, with intermediate $SU(5)$ such that $M_R$ is above the unification scale, the right-handed neutrinos are expected to be heavier, especially with a $\mathbf{126} \oplus \mathbf{126}^*$ breaking of $(B - L)$. In supersymmetric theories these different patterns for right-handed neutrino masses give rise to different predictions for charged lepton flavour violating decays.

In contrast with the relatively minimal field content and the use of low dimension couplings adopted here, many papers have suggested to achieve the symmetry breaking by selecting only a few suitable couplings [6,13] through ad hoc discrete symmetries. The absence of couplings that are gauge invariant in the superpotential usually leads to new solutions and even flat directions [12] that should be carefully examined (just as the use of R-parity constraints brings about the colour breaking vacua in the supersymmetric extensions of the Standard Model).
References


