Reexamining nonstandard interaction effects on supernova neutrino flavor oscillations

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Abstract

Several extensions of the standard electroweak model allow new four-fermion interactions $\mathcal{L}_{\alpha\beta} \propto G_F$ with strength $\epsilon_{\alpha\beta} G_F$, where $\alpha, \beta$ are flavor indices. We revisit their effects on flavor oscillations of massive (anti)neutrinos in supernovae, in order to achieve, in the region above the protoneutron star, an analytical treatment valid for generic values of the neutrino mixing angles $(\omega, \phi, \psi) = (\theta_{12}, \theta_{13}, \theta_{23})$. Assuming that $\epsilon_{\alpha\beta} \ll 1$, we find that the leading effects on the flavor transitions occurring at high ($H$) and low ($L$) density along the supernova matter profile can be simply embedded through the replacements $\phi \rightarrow \phi + \epsilon_H$ and $\omega \rightarrow \omega + \epsilon_L$, respectively, where $\epsilon_H$ and $\epsilon_L$ are specific linear combinations of the $\epsilon_{\alpha\beta}$’s. Similar replacements hold for eventual oscillations in the Earth matter. From a phenomenological point of view, the most relevant consequence is a possible uncontrolled bias $(\phi \rightarrow \phi + \epsilon_H)$ in the value of the mixing angle $\phi$ inferred by inversion of supernova neutrino data. Such a drawback, however, does not preclude the discrimination of the neutrino mass spectrum hierarchy (direct or inverse) through supernova neutrino oscillations.

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I. INTRODUCTION

Atmospheric and solar neutrino data provide convincing evidence for lepton flavor non-conservation in the channels $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\mu, \tau$, respectively [1]. The Maki-Nakagawa-Sakata-Pontecorvo hypothesis of $\nu$ masses and mixings [2] suffices to explain such phenomena in terms of flavor oscillations [1], with no need for additional neutrino properties or interactions.

However, new neutrino interactions are often predicted (together with neutrino masses and mixings) by several extensions of the standard electroweak theory, in the form of effective (low-energy) four-fermion operators $O_{\alpha\beta} \sim \nu_\alpha \nu_\beta \bar{f}f$ with strength $G_{f\alpha\beta}^{\alpha\beta}$, inducing either flavor-changing ($\alpha \neq \beta$) or flavor non-universal ($\alpha = \beta$) neutrino transitions. Models with left-right symmetry [3] or with supersymmetry with broken $R$-parity [4, 5] provide widely studied realizations of such operators. Their net effect in ordinary matter (with fermion density $N_f(x)$ at position $x$) is to provide extra $\nu$ interaction energies proportional to the following dimensionless couplings

$$\epsilon_{\alpha\beta}(x) \equiv \sum_{f=e,u,d} G_{f\alpha\beta}^{\alpha\beta} N_f(x) / G_F N_e(x)$$

$$= \epsilon_{\alpha\beta}^e + \epsilon_{\alpha\beta}^u N_u(x) / N_e(x) + \epsilon_{\alpha\beta}^d N_d(x) / N_e(x)$$

$$= \epsilon_{\alpha\beta}^e + \epsilon_{\alpha\beta}^u - \epsilon_{\alpha\beta}^d + (\epsilon_{\alpha\beta}^u + 2\epsilon_{\alpha\beta}^d) / Y_e(x) ,$$

where $\epsilon_{\alpha\beta}^f = G_{f\alpha\beta}^{\alpha\beta} / G_F$, and $Y_e(x)$ is the electron/nucleon number fraction.

As a consequence, in the flavor basis $\nu_\alpha = (\nu_e, \nu_\mu, \nu_\tau)$, the standard interaction hamiltonian in matter [6, 7]

$$H_{\text{int}}^{\text{std}} = \text{diag}(V, 0, 0) ,$$

can be generalized as

$$(H_{\text{int}})_{\alpha\beta} = (H_{\text{int}}^{\text{std}})_{\alpha\beta} + V \epsilon_{\alpha\beta} ,$$

where

$$V(x) = \sqrt{2} G_F N_e(x) .$$

The interaction hamiltonian for antineutrinos is obtained by replacing $G_{f\alpha\beta}^{\alpha\beta}$ with $-(G_{f\alpha\beta}^{\alpha\beta})^\ast$, namely, $V$ with $-V$ and $\epsilon_{\alpha\beta}$ with $\epsilon_{\alpha\beta}^\ast$. [2]

Nonstandard neutrino interactions in matter, and their interplay with the oscillation phenomenon, have been investigated in many different contexts. An incomplete list includes analyses related to the solar neutrino problem [1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], to the atmospheric neutrino anomaly [22], to the LSND oscillation results [23], to neutrinoless double beta decay [24], to supernova neutrinos [18, 25], to the production or detection of laboratory neutrinos [26, 27], and to future (very) long baseline projects [28, 29, 30, 31]. In a nutshell, these works demonstrate that nonstandard neutrino interactions in matter can (sometimes profoundly) modify the interpretation of current and future oscillation searches, and that their possible effects deserve particular attention, whenever inferences about standard mass-mixing parameters are made from experimental data.

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1 Up to terms proportional to the unit matrix, and discarding negligible one-loop corrections [8].
2 Notice also that $\epsilon_{\alpha\beta}^\ast = \epsilon_{\beta\alpha}$ for hermiticity of $H_{\text{int}}$. 

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In this work, we revisit the effects of subleading nonstandard four-fermion interactions on the flavor oscillation of massive and mixed (anti)neutrinos in supernovae. We shall limit ourselves to the region above the protoneutron star, where neutrinos propagate freely, and all relevant fermion densities are monotonically decreasing (roughly as the third power of the radius).\[3\] For this case (previously studied, e.g., in \[18, 25\]) we aim to a completely analytical treatment, valid for generic values of the neutrino mixing angles and for both the direct and inverse mass spectrum hierarchies allowed by the current \( \nu \) phenomenology. Such treatment has been recently proposed in \[38\] for the case of standard supernova \( \nu \) oscillations.

In the present work, we make a few simplifying assumptions about the nonstandard couplings \( \epsilon_{\alpha\beta} \): (1) they are taken as real (either positive or negative), so that \( \epsilon_{\alpha\beta} = \epsilon_{\beta\alpha} \); (2) their possible variation with \( x \) through \( Y_e \) [see Eq. (3)] is neglected;\[4\] and (3) they are assumed to be small, 
\[ |\epsilon_{\alpha\beta}| \ll 1 , \] (7)
say, \( |\epsilon_{\alpha\beta}| < \text{few} \times 10^{-2} \), so that terms of \( O(\epsilon^2) \) can be safely neglected in the calculations.

Under the above assumptions, we find that the main effect of nonstandard interactions on supernova neutrino oscillations is to induce a shift in the relevant mixing angles, as compared to the standard case. Such a shift is discussed first in the two-neutrino case (Sec. II) and then in the three-neutrino case (Sec. III). Nonstandard effects in the Earth matter are briefly discussed in Sec. IV. Phenomenological consequences are illustrated in Sec. V. Conclusions are drawn in Sec. VI.

A final remark is in order. The assumption in Eq. (7) is consistent with phenomenological upper bounds \[18, 19, 23, 40\], derived from charged lepton processes under \( SU(2) \) symmetry assumptions. Such bounds are typically of \( O(10^{-1}) \) for the (differences of) diagonal couplings and of \( O(10^{-2}) \) for the off-diagonal ones; only the upper limit on \( \epsilon_{e\mu} \) is much more severe \( [O(10^{-5})] \). However, if allowance is made for substantial \( SU(2) \) breaking, and if the analysis is performed in a more model-independent way, the previous constraints \[18, 19, 23, 40\] can be significantly relaxed \[11, 12\]. Accordingly, scenarios with (some) large \( \epsilon_{\alpha\beta} \)’s \( [\sim \text{few} \times 10^{-1}] \) can be built (see, e.g., \[11, 13\]), but they will not be considered in this paper.

II. TWO-NEUTRINO TRANSITIONS

In this section, we study the effects of nonstandard interactions on \( 2\nu \) oscillations, in the region well above the neutrinosphere. We show that the \( 2\nu \) evolution equation can be formally cast in an effective (and solvable) standard form \( (\epsilon_{\alpha\beta} = 0) \), through a proper redefinition of the neutrino mixing angle. The traditional formulae, derived through resonance-condition arguments \[16, 17, 18, 23 \] (inapplicable for mixing angles \( \gtrsim \pi/4 \), see \[38\] and references therein), are shown to be recovered in the limit of small mixing. At the end of this section, we also comment on a different mechanism that might induce nonstandard flavor transitions near the neutrinosphere.

In the flavor basis, the two-family evolution equation for free-streaming neutrinos can be

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3 Possible flavor transitions induced by nonstandard interactions near the neutrinosphere \[31, 32\] will be commented at the end of Sec. II.

4 Detailed supernova simulations show that \( Y_e \sim 0.5 \) above the protoneutron star. Significant variations (reductions) of \( Y_e \) are confined to regions near the neutrinosphere during the cooling phase \[38\].
written as
\[ i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = [H_{\text{kin}}(\theta) + H_{\text{int}}(x)] \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}, \]  
(8)
where \( \alpha = \mu \) or \( \tau \), and \( H = H_{\text{kin}}(\theta) + H_{\text{int}}(x) \) is the total hamiltonian, split into kinetic and interaction energy terms. The kinetic term can be expressed in terms of the neutrino mixing angle \( \theta \), of the squared mass difference \( \Delta m^2 = m_2^2 - m_1^2 \), and of the energy \( E \) as
\[ H_{\text{kin}} = U(\theta) \begin{pmatrix} -k/2 & 0 \\ 0 & +k/2 \end{pmatrix} U^\dagger(\theta), \]  
(9)
where \( k = \Delta m^2 / 2E \) is the neutrino oscillation wave number, and
\[ U(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]  
(10)
is the mixing matrix. The interaction term can be written as
\[ H_{\text{int}}(x) = V(x) \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{ea} \\ \epsilon_{ea} & \epsilon_{aa} \end{pmatrix} \]  
(11)
\[ \equiv \frac{V(x)}{2} \begin{pmatrix} 1 + \epsilon_{ee} - \epsilon_{aa} & 2\epsilon_{ea} \\ 2\epsilon_{ea} & -1 + \epsilon_{ee} + \epsilon_{aa} \end{pmatrix}, \]  
(12)
where we have made \( H_{\text{int}} \) traceless in the last equation, \( V(x) \) being defined as in Eq. (6). For \( \epsilon_{\alpha\beta} = 0 \) (standard case), the interaction term reduces to
\[ H_{\text{std}} = \text{diag}(+1, -1) \cdot V(x)/2. \]  
(13)
The term \( H_{\text{int}} \) in Eq. (12) is diagonalized through
\[ H_{\text{int}} = U(\eta)^\dagger \text{diag}(+1, -1) U(\eta) \cdot f V(x)/2, \]  
(14)
where
\[ f = \sqrt{(1 + \epsilon_{ee} - \epsilon_{aa})^2 + 4\epsilon_{ea}^2}, \]  
(15)
while \( U(\eta) \) is defined as in Eq. (10), but with argument
\[ \eta = \frac{1}{2} \arctan \frac{2\epsilon_{ea}}{1 + \epsilon_{ee} - \epsilon_{aa}} \]  
(16)
\[ = \epsilon_{ea} + O(\epsilon^2), \]  
(17)
where terms of \( O(\epsilon^2) \) can be neglected under the assumption in Eq. (7).

Equation (14) differs from Eq. (13) in two respects: (a) the potential \( V \) is multiplied by a factor \( f \), and (b) there is an additional rotation matrix \( U(\eta) = U(\epsilon_{ea}) + O(\epsilon^2) \). The point (a) is equivalent to rescale the supernova rotation matrix by less than a few percent \( |f = 1 + O(\epsilon)| \), and does not lead to realistically measurable consequences; in fact, the difference \( f - 1 \) is conceivably smaller than the uncertainties associated to the normalization and shape of inferred or simulated supernova density profiles; moreover, the oscillation physics depends on \( V \) mainly through its logarithmic derivative \( [38] \). The point (b) can instead be important and, as we shall see, it leads to a bias in the “effective mixing angle” governing flavor transitions along the supernova matter profile.
Summarizing the above discussion, the leading nonstandard effect on $2\nu$ propagation in supernovae is governed by the off-diagonal coupling $\epsilon_{ea}$, and amounts to write $H_{\text{int}}$ in the form

$$H_{\text{int}}(x) \simeq \frac{V(x)}{2} U^\dagger(\epsilon_{ea}) \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} U(\epsilon_{ea}) .$$

It is then easy to check that, in the new basis

$$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_a \end{pmatrix} = U(\epsilon_{ea}) \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} ,$$

the neutrino evolution equation in matter [Eq. (8)] takes an effective “standard” form,

$$i \frac{d}{dx} \begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_a \end{pmatrix} = \left[ H_{\text{kin}}(\tilde{\theta}) + H^\text{std}_{\text{int}}(x) \right] \begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_a \end{pmatrix} ,$$

where $\tilde{\theta} = \theta + \epsilon_{ea}$. Roughly speaking, nonstandard effects can be “rotated away” from $H_{\text{int}}$ (which becomes $H^\text{std}_{\text{int}}$) and “transferred” to $H_{\text{kin}}$ through the replacement $\theta \rightarrow \tilde{\theta} = \theta + \epsilon_{ea}$. Since accurate analytical solutions of Eq. (20) exist [expressed, e.g., in terms of the survival probability $P(\tilde{\nu}_e \rightarrow \tilde{\nu}_e)$], the task is reduced to a proper back-rotation of known results (applied to $\tilde{\nu}_a$) to the physical flavor basis $\nu_\alpha$.

To do so, one defines the usual mixing angle in matter $\tilde{\theta}_m$ diagonalizing the total hamiltonian in Eq. (20),

$$\cos 2\tilde{\theta}_m = \frac{\cos 2\tilde{\theta} - V/k}{\sqrt{(\cos 2\tilde{\theta} - V/k)^2 + \sin^2 2\tilde{\theta}}}$$

and the associated basis of effective mass eigenstates in matter $\tilde{\nu}_{m,i}$,

$$\tilde{\nu}_\alpha = U(\tilde{\theta}_m) \tilde{\nu}_{m,i} .$$

The formal solution of Eq. (20), evolving the state $(\nu_\alpha)$ from the starting point $(x = x_o)$ to the detection point $(x = x_d)$, can then be factorized as

$$\begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}_d = U(\tilde{\theta}) \cdot T_V \cdot T_S(\tilde{\theta}) \cdot U^\dagger(\tilde{\theta}_m^o) U(\epsilon_{ea}) \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}_o ,$$

where, from right to left, $U(\epsilon_{ea})$ rotates the initial state $\nu_\alpha$ to the $\tilde{\nu}_\alpha$ basis of Eq. (20), $U^\dagger(\tilde{\theta}_m^o)$ rotates $\tilde{\nu}_\alpha$ to the initial basis of mass eigenstates $\tilde{\nu}_{m,j}$ in matter, $T_S(\tilde{\theta})$ embeds the transitions $\tilde{\nu}_{m,j} \rightarrow \tilde{\nu}_{m,i}$ up to the supernova surface (where $V = 0$ and $\tilde{\nu}_{m,i} = \nu_i$), $T_V$ further propagates the $\nu_i$’s in vacuum until they arrive at the detector (up to Earth matter effects), and a final rotation $U(\theta)$ brings the $\nu_i$ states back to the physical flavor basis $\nu_\alpha$. Equation (24), obtained by grouping the first two rotations, represents the formal solution of the $\nu$ evolution equation, in a format analogous to the standard case (i.e., initial rotation, propagation in matter, propagation in vacuum, final rotation).

One can then average out unobservable oscillating terms (mainly associated to $T_V$), so as to propagate classical (real) probabilities $P_{\alpha\beta}$ rather than complex amplitudes. The averaging can be accomplished [44] by replacing in Eq. (24) each entry in the matrices
$U(\theta)$, $T_S(\tilde{\theta})$, and $U^\dagger(\tilde{\theta}_m - \epsilon_{ea})$, with its squared modulus. In our case, the initial (high density) condition $V(x_o)/k \gg 1$ leads to $\sin^2(\tilde{\theta}_m - \epsilon_{ea}) \simeq 1 + O(\epsilon_{ea}^2)$, effectively cancelling any “memory” of the initial point $x_o$ above the protoneutron star. Therefore, the electron neutrino survival probability finally reads

$$P_{ee} = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \left( \begin{array}{cc} \cos^2 \theta & \sin^2 \theta \\ \sin^2 \theta & \cos^2 \theta \end{array} \right) \left( \begin{array}{cc} 1 - P_c(\tilde{\theta}) \\ P_c(\tilde{\theta}) \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right),$$

namely,

$$P_{ee} = \cos^2 \theta P_c(\tilde{\theta}) + \sin^2 \theta [1 - P_c(\tilde{\theta})],$$

where we have adopted the usual notation for the square moduli in $T_S$, in terms of the so-called crossing probability $P_c$ (see, e.g., [44]).

Equation (26) is formally analogous to the one in the standard case ($\epsilon_{ea} = 0$), modulo the replacement $\theta \rightarrow \tilde{\theta} = \theta + \epsilon_{ea}$ in $P_c$. We recall that, in the standard case, $P_c(\theta)$ is accurately described by a simple analytical expression,

$$P_c^{std}(\theta) = \frac{e^{2\pi r k \cos^2 \theta} - 1}{e^{2\pi r k} - 1},$$

where

$$r = \left(- \left(\frac{d \ln V}{dx}\right)_{x=x_p}\right)^{-1}$$

is the density scale factor, to be evaluated at the point $x_p$ where the potential equals the wave number $k$.

The above prescription for $P_c^{std}(\theta)$, inspired by the condition of maximum violation of adiabaticity [38, 40, 47] (more general than the traditional condition of resonance [44]) and by the double-exponential representation for the crossing probability [48], holds accurately for generic supernova density profiles, in the whole 2ν mass-mixing parameter space [38].

In the presence of nonstandard interactions, the previously discussed replacement $\theta \rightarrow \theta + \epsilon_{ea}$ leads then to the following analytical form for $P_c$:

$$P_c(\tilde{\theta}) = \frac{e^{2\pi r k \cos^2(\theta + \epsilon_{ea})} - 1}{e^{2\pi r k} - 1},$$

where $r$ takes the same value as in Eq. (28), the position of $x_p$ [Eq. (29)] being unaffected by the shift $\theta \rightarrow \theta + \epsilon_{ea}$.\(^6\)

Equations (26) and (30) represent our main result for the two-flavor neutrino case. They provide a compact description of nonstandard interaction effects on $P_{ee}$, valid for generic values of $\theta$ and $\Delta m^2$ (provided that $\epsilon_{ea} \ll 1$). The calculation of $P_{ee}$ for antineutrinos

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\(^5\) We remind the reader that $P_{ee}$ is the most relevant quantity in supernova $\nu$ oscillations, due to the practical indistinguishability of $\nu_\mu$ and $\nu_\tau$ in the initial and final states [44]. See, however, Ref. [38] for a recent discussion of possible differences between $\nu_\mu$ and $\nu_\tau$ fluxes at the origin.

\(^6\) The definition of $r(x_p)$ through Eqs. (28) and (29) is $\theta$-independent.
is strictly analogous: given the analytical expression reported in Sec. II.C of [38], one has simply to replace $\theta$ with $\theta + \epsilon_{ea}$ within $P_c(\bar{\nu})$.\footnote{Notice that, outside $P_c$, the angle $\theta$ is unshifted, as in the coefficients $\sin^2 \theta$ and $\cos^2 \theta$ of Eq. (20).}

It can easily be shown that Eq. (30) generalizes previous results about nonstandard interaction effects, valid in a more limited range of applicability. The traditional analytical approach to $P_c$ localizes the nonadiabatic transitions at the so-called “resonance” point $x_r$, where mixing in matter is maximal ($\sin 2\theta_m(x_r) = 1$) \cite{16, 17, 18, 25}. The resonance approach is certainly valid at small $\theta$, where it should coincide with ours. Indeed, in the resonance approach, the standard ($\epsilon_{ea} = 0$) Landau-Zener form for $P_c$ at small mixing ($\ln P_c \propto \theta^2$) \cite{44} is modified as $\ln P_c \propto \theta^2(1 + 2\epsilon_{ea} \cot 2\theta)^2 \simeq (\theta + \epsilon_{ea})^2$ \cite{16, 17, 18, 25}, in agreement with our prescription $\theta \rightarrow \theta + \epsilon_{ea}$. For $\theta$ approaching $\pi/4$, however, the resonance condition generally fails to localize the right point $x$ for the evaluation of $r(x)$, and becomes eventually not applicable for $\theta > \pi/4$ (both in the standard \cite{38} and in the nonstandard case). Conversely, Eq. (30) has no such applicability restrictions. Finally, in the subcase $\theta \equiv 0$, we also recover from Eq. (30) the double-exponential form for $P_c$ found analytically in \cite{11} in the scenario of massless (solar) neutrinos with nonstandard interactions. Therefore, Eq. (30) correctly generalizes previous limiting cases.

A final comment is in order. Additional nonstandard flavor transitions (different from those described in this section) might independently occur in inner layers \cite{36, 37}, as a result of a decrease of $Y_e(x)$ to $O(10^{-2})$, localized just above the neutrinosphere during the cooling phase \cite{37, 39}. The $\epsilon_{\alpha\beta}(x)$'s can then locally increase to $O(1)$ [see Eq. (1)], making the diagonal and off-diagonal terms in the interaction hamiltonian [Eq. (11)] comparable in magnitude, and dominant (due to the very high matter density) with respect to the kinetic terms [Eq. (3)]. As a consequence, near the neutrinosphere (and thus prior to the flavor oscillations discussed in this paper), nonstandard interactions might induce additional flavor transitions of (effectively massless) $\nu$ and $\bar{\nu}$ during the cooling phase \cite{36, 37}. Our results, applicable to the region of decreasing fermion density above the protoneutron star (where $Y_e \approx 0.5$) should be thus considered as complementary to the possible independent effects \cite{36, 37} possibly induced by a local (preceding) drop of $Y_e$. A combination of the two nonstandard interaction effects (still lacking in the literature, to our knowledge) is, however, beyond the scope of this paper.\footnote{We note in passing that, being confined to a region just above the neutrinosphere (where collective and collision effects can still be non negligible), the additional transitions at low $Y_e$ might require a more advanced description than the free-particle formalism used in \cite{37} (e.g., in terms of kinetic and density matrix equations \cite{49}).}

**III. THREE-NEUTRINO TRANSITIONS**

In the 3$\nu$ case, the interaction term in the Hamiltonian is given by Eq. (3), while the kinetic term $H_{\text{kin}}$ is

$$H_{\text{kin}} = U(\omega, \phi, \psi) \begin{pmatrix} -k_L/2 & 0 & 0 \\ 0 & +k_L/2 & 0 \\ 0 & 0 & k_H \end{pmatrix} U^\dagger(\omega, \phi, \psi),$$

(31)
where \( k_L = \delta m^2 / 2E \) and \( k_H = m^2 / 2E \) (\( \delta m^2 \) and \( m^2 \) being the “solar” and “atmospheric” neutrino squared mass differences, see [1, 35]), while the 3 \( \times \) 3 mixing matrix \( U(\omega, \phi, \psi) \) is factorized (discarding a possible CP violating phase) into three real rotations [44, 50]

\[
U(\omega, \phi, \psi) = U_{23}(\psi)U_{13}(\phi)U_{12}(\omega) ,
\]

(32)

where \((\omega, \phi, \psi) = (\theta_{12}, \theta_{13}, \theta_{23})\). The subscripts \( H \) and \( L \) remind that two transitions can occur along the supernova profile when \( V \sim O(k_H) \) and \( V \sim O(k_L) \), namely, at high \((H)\) and low \((L)\) density, respectively.

The phenomenological assumption of hierarchical squared mass differences,

\[
\delta m^2 \ll m^2 \leftrightarrow k_L \ll k_H ,
\]

together with Eq. (11), makes the 3\( \nu \) dynamics factorizable into two (almost decoupled) 2\( \nu \) subsystems for the \( H \) and \( L \) transitions, in the same way as for the standard case [14].

To isolate the dynamics of the \( H \) transition, one rotates the starting neutrino (flavor) basis by \( U_{23}(\psi) \), and extracts the submatrix with indices \((1, 3)\) [25, 44]. The effective 2\( \nu \) sub-dynamics is then governed by \((k, \theta) \equiv (k_L, \omega)\), and the analogous of \( \epsilon_{ea} \) is given by the off-diagonal (nonstandard) term, which reads

\[
\epsilon_H = \epsilon_{e\mu} \sin \psi + \epsilon_{e\tau} \cos \psi .
\]

(34)

Using the results of the previous section and those of Ref. [38], the crossing probability for the \( H \) transition is thus described in terms of an effective \((1, 3)\) mixing angle \( \phi = \phi + \epsilon_H \), so that

\[
P_H = \frac{e^{2\pi r_H k_H \cos^2(\phi + \epsilon_H)} - 1}{e^{2\pi r_H k_H} - 1} ,
\]

(35)

\( r_H \) being the density scale factor [Eq. (28)] evaluated at the point \( x_H \) where \( V(x_H) = k_H \).

Analogously, to isolate the dynamics of the \( L \) transition, one rotates the starting neutrino (flavor) basis by \( U_{13}(\phi)U_{23}(\psi) \), and then extracts the submatrix with indices \((1, 2)\). The effective 2\( \nu \) sub-dynamics is then governed by \((k, \theta) \equiv (k_L, \omega)\) and by the off-diagonal (nonstandard) term, which now reads\(^9\)

\[
\epsilon_L = \cos \phi (\epsilon_{e\mu} \cos \psi - \epsilon_{e\tau} \sin \psi) \\
+ \sin \phi [ (\epsilon_{e\tau} - \epsilon_{\mu\mu}) \sin \psi \cos \psi - \epsilon_{\mu\tau} \cos 2\psi] .
\]

(36)

The crossing probability for the \( L \) transition is thus described in terms of an effective \((1, 2)\) mixing angle \( \tilde{\omega} = \omega + \epsilon_L \), so that

\[
P_L = \frac{e^{2\pi r_L k_L \cos^2(\omega + \epsilon_L)} - 1}{e^{2\pi r_L k_L} - 1} ,
\]

(37)

\( r_L \) being evaluated at the point \( x_L \) where \( V(x_L) = k_L \). The same shift \((\omega \rightarrow \omega + \epsilon_L)\) applies to the expression of \( P_L \) for antineutrinos (as given in [38]).

The above discussion implicitly assumes direct hierarchy of neutrino masses \((m_{1,2} < m_3)\). The case of inverse hierarchy \((m_3 < m_{1,2})\) is, however, strictly analogous; indeed, the

\(^9\) An analogous result for \( \epsilon_L \) has been recently reported in the context of solar neutrinos [21]. The expression of \( \epsilon_L \) for supernova neutrinos, in the subcase \( \epsilon_{\mu\tau} = \epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 0 \), is reported in [25].
symmetry arguments used in [38] to derive $P_{ee}$ analytically in the case of inverse hierarchy (in terms of $P_{ee}$ for direct hierarchy) are unaffected by the presence of nonstandard interactions. Therefore, the prescription remains the same, irrespective of the spectrum type: take the standard expressions of $P_{ee}$ from [38], and replace $\omega \to \omega + \epsilon_L$ and $\phi \to \phi + \epsilon_H$ in the crossing probabilities for both neutrinos and antineutrinos. Embedding nonstandard interaction effects through appropriate shifts of $\omega$ and $\phi$ within $P_L$ and $P_H$, respectively, represents our main results for the case of three-flavor mixing of supernova neutrinos (up to Earth matter effects).

IV. EARTH MATTER EFFECTS

In the standard case ($\epsilon_{\alpha\beta} = 0$), it is well known that possible Earth matter effects before supernova $\nu$ detection can be embedded through a specific transition probability $P_E = P(\nu_2 \to \nu_e)$ (see [38, 51, 52] and references therein). In the hierarchical approximation [Eq. (33)], $P_E$ is basically a function of the $L$-transition parameters ($\omega, \delta m^2$) only.\(^{10}\)

The dependence of $P_E$ on $\omega$ is both explicit (through an $\omega$-rotation in vacuum), and implicit [through the mixing angle in the Earth matter, $\omega_m = \omega_m(\omega)$], so that

$$P_E = P_E(\omega, \omega_m(\omega), \delta m^2), \text{ standard case .}$$

(38)

In particular, Sec. IV of [38] reports known representations of the function $P_E(\omega, \omega_m(\omega), \delta m^2)$ in the case of Earth mantle (+ core) crossing, in the same notation as in the present paper.

In the nonstandard case, since it is $\epsilon_{\alpha\beta} \neq 0$ only in matter, the previously found shift $\omega \to \omega + \epsilon_L$ applies only to $\omega_m$, namely,

$$P_E = P_E(\omega, \omega_m(\omega + \epsilon_L), \delta m^2), \text{ nonstandard case ,}$$

(39)

and similarly for antineutrinos.\(^{11}\) We have also checked the above prescription by explicitly repeating the derivation of $P_E$ in the cases of one-layer or two-layer model for the Earth crossing (omitted).

V. DISCUSSION

Let us discuss in more detail the effects of nonzero $\epsilon_L$ and $\epsilon_H$ on supernova neutrino oscillations, by considering the phenomenologically interesting case with $m^2 = 3 \times 10^{-3} \text{ eV}^2$, $\delta m^2 \ll m^2$, and $\sin^2 \phi \lesssim \text{few\%}$ [1]. We assume the supernova power-law density profile reported in Fig. 1 of [38], and a representative neutrino energy $E = 15 \text{ MeV}$. The electron neutrino survival probability, in the case of direct hierarchy, is then given by

$$P_{ee}(\nu) \simeq [(1 - P_E) P_L(k_L, \omega + \epsilon_L) + P_E (1 - P_L(k_L, \omega + \epsilon_L))] e^{-2\pi r_H k_H \sin^2(\phi + \epsilon_H)} ,$$

(40)

\(^{10}\) There is a subleading dependence on $\phi$ through the renormalization $V \to V \cos^2 \phi$, which can be safely neglected for supernova neutrinos, given the smallness of $\phi$ [1] and unavoidable normalization uncertainties of $V$.

\(^{11}\) In principle, the value of $\epsilon_L$ to be used in Eq. (39) might be slightly different from the one in Eq. (37), due to different (average) values of $Y_e$ in the Earth and in the supernova [see Eq. (3)].
where we have neglected a small (additive) $O(\phi^2)$ term, the main effect of $\phi$ being embedded in the exponential suppression factor. In the above equation, $P_L$ is given by Eq. (37), while $P_E$ is calculated as in Eqs. (38) or (39). In the case of no Earth crossing, $P_E$ is simply given by

$$P_E = \sin^2 \omega \quad \text{(no Earth effects).} \quad (41)$$

Figure 1 shows curves of iso-$P_\text{ee}(\nu)$, calculated according to Eq. (40), with and without Earth matter effects (dotted and solid lines, respectively),\textsuperscript{12} in the plane of the $L$-transition variables ($\delta m^2, \tan^2 \omega$). The three left panels refer to the case $\tan^2(\phi + \epsilon_H) = 0$ (corresponding either to $\phi = \epsilon_H = 0$ or to $\epsilon_H = -\phi$), which implies $P_H = 1$. The three right panels correspond instead to a nonzero value for $\phi + \epsilon_H$, namely, $\tan^2(\phi + \epsilon_H) = 2 \times 10^{-5}$, implying $P_H = 0.46$. The comparison of left and right panels clearly shows the suppression of $P_\text{ee}(\nu)$ due to $\phi + \epsilon_H \neq 0$.

In Fig. 1, from top to bottom, the three (left and right) panels correspond to $\epsilon_L = 0, \epsilon_L = +3 \times 10^{-2}$, and $\epsilon_L = -3 \times 10^{-2}$. The effect of nonzero $\epsilon_L$ is dramatic at small mixing angles, and leads to a distinctive and well-known pattern for the isolines of $P_\text{ee}$ through the so-called large mixing angle (LMA) solution, which appears to be elongated towards very small values of $\omega$ for positive $\epsilon_L$, and split into two branches for negative $\epsilon_L$. In our formalism, this behavior is simply captured in terms of the $\cos^2(\omega + \epsilon_L)$ argument in $P_L$: for $\epsilon_L > 0$ the argument increases monotonically for decreasing (small) $\omega$, otherwise it first reaches a maximum at $\omega = -\epsilon_L$ and then decreases. The Earth effect is also similarly “stretched” in Fig. 1.

Let us consider in more detail the case of relatively large $\omega [\tan^2 \omega \sim O(10^{\pm 1})]$, which appears to be generally favored by recent solar neutrino data.\textsuperscript{53} In this case, independently of the value of $\delta m^2$, effects induced by $\epsilon_L \neq 0$ are hardly recognizable in Fig. 1, since the difference between $\omega$ and $\omega + \epsilon_L$ is fractionally small, while the leading effect of nonstandard interactions is embedded in the exponential suppression factor ($P_H$) of Eq. (40). In particular, for relatively large values of $\delta m^2/E$ [providing the best fit to solar neutrino data through the so-called large mixing angle (LMA) solution], it is $P_L \approx 0$, and the $\nu_e$ survival probability simply reads

$$P_{ee} \approx P_E e^{-2\pi k_H k_H \sin^2 (\phi + \epsilon_H)} \quad \text{(LMA solution).} \quad (42)$$

Figure 2 shows the function $P_E(\delta m^2)$ for three representative values of $\tan^2 \omega$ in the LMA region. Other parameters ($\epsilon_L$, neutrino energy and pathlength, mantle density and electron number fraction) are as in Fig. 1. The case of no Earth crossing [Eq. (41)] is also shown as a horizontal line in each panel. In general, Earth matter effects appear to be relevant in all cases, producing a distinctive oscillatory pattern in $P_E$. Different values of $\epsilon_L$ induce slightly different oscillation amplitudes in $P_E$, and could thus (in principle) be distinguished, especially for large values of $\delta m^2$, where the relative differences among the curves in each panel increase. However, such discrimination would require very high statistics observations. We conclude that, for mass-mixing parameters in the LMA solution, and for typical supernova density profiles and energy spectra, the main nonstandard effects are associated with $\epsilon_H$ (through $P_H$), while smaller effects are associated with $\epsilon_L$ (through $P_E$).

\textsuperscript{12} In Fig. 1, Earth matter effects refer to a representative case of mantle crossing, with pathlength $L = 8500$ km, mass density $\rho = 4.5 \text{ g/cm}^3$, and electron number fraction $Y_e = 1/2$. 

Concerning the main nonstandard effects, the survival probability in Eq. (40) shows that, in general, supernova neutrinos can be sensitive to values of \( \tan^2(\phi + \epsilon_H) \) as low as \( \sim 10^{-5} \), corresponding to a (sub)percent sensitivity in \( \phi + \epsilon_H \). Indeed, in the standard case (\( \epsilon_H = 0 \)), it has been shown that inversion of future galactic supernova data might provide upper and lower bounds on \( \phi \) in the percent range.\(^{14} \) Our results show that, in the presence of additional nonstandard interactions (\( \epsilon_H \neq 0 \)), any constraint about \( \phi \) must actually be interpreted as a constraint on \( \phi + \epsilon_H \), namely, there is a strict degeneracy between the vacuum mixing angle \( \phi = \theta_{13} \) and the nonstandard coupling \( \epsilon_H \) given in Eq. (34).

The \((\phi, \epsilon_H)\) degeneracy in supernova neutrino oscillations can be rather dangerous for the interpretation of experimental data. Supernova \( \nu \) oscillations may offer one of the few opportunities to probe experimentally very small values of \( \phi \), and any uncontrolled bias of the kind \( \phi \to \phi + \epsilon_H \) may lead to dramatic differences in the theoretical inferences from the experimental data. In extreme cases, nonstandard interactions might either completely mimic (\( \phi = 0, \epsilon_H \neq 0 \)) or completely cancel (\( \phi = -\epsilon_H \)) “standard” \( \phi \)-related effects in the \( \bar{H} \) transition. An analogous “confusion scenario,” with degeneracy between effects of nonstandard interactions and of \( \phi \neq 0 \), has recently been discussed in \( \text{[35]} \) in the context of oscillation searches at neutrino factories. The main message emerging from such results is that constraining \( \phi \) will generally be a very challenging task, if allowance is made for nonstandard (flavor changing) neutrino interactions, with strength as weak as a few percent of the standard (flavor diagonal) electroweak one.\(^{14} \)

We conclude this section with a positive remark, by showing that not everything is necessarily biased by nonstandard interactions. In particular, the spectrum hierarchy discrimination (direct vs inverse) may still be viable. Let us consider the phenomenologically interesting case of the so-called large mixing angle solution (LMA) to the solar neutrino problem,\(^{53} \) requiring \( \delta m^2 \sim O(10^{-5}) \text{ eV}^2 \) and \( \tan^2 \omega \lesssim 1 \), so that \( P_L \simeq 0 \) (adiabatic \( L \) transition). For small \( \phi \), the supernova electron (anti)neutrino survival probabilities are approximately given (up to Earth matter effects) by

\[
\begin{align*}
P_{ee}^{\text{dir}}(\nu) &\simeq \sin^2\omega e^{-2\pi \epsilon_H k_H \sin^2(\phi + \epsilon_H)} , \\
P_{ee}^{\text{dir}}(\bar{\nu}) &\simeq \cos^2\omega , \\
P_{ee}^{\text{inv}}(\nu) &\simeq \sin^2\omega , \\
P_{ee}^{\text{inv}}(\bar{\nu}) &\simeq \cos^2\omega e^{-2\pi \epsilon_H k_H \sin^2(\phi + \epsilon_H)} ,
\end{align*}
\]

where the superscripts denote the cases of direct or inverse hierarchy. From such equations one can derive the following \( \epsilon_H \)-independent inequalities,

\[
\begin{align*}
P_{ee}^{\text{dir}}(\nu) &< P_{ee}^{\text{inv}}(\nu) , \\
P_{ee}^{\text{dir}}(\bar{\nu}) &> P_{ee}^{\text{inv}}(\bar{\nu}) ,
\end{align*}
\]

and

\[
\frac{P_{ee}^{\text{dir}}(\nu)}{P_{ee}^{\text{dir}}(\bar{\nu})} < \frac{P_{ee}^{\text{inv}}(\nu)}{P_{ee}^{\text{inv}}(\bar{\nu})} ,
\]

\(^{13} \) We remind the reader that, even assuming restrictive bounds on the \( \epsilon_{\alpha\beta} \),\(^{18, 19, 23, 40} \) one cannot currently exclude values of \( \epsilon_H \) as large as a few percent.

\(^{14} \) At a more sophisticated level, for complex \( \epsilon_{\alpha\beta} \) there can also be confusion between standard and nonstandard CP-violation phase effects in (very) long baseline experiments; see, e.g.,\(^{31, 32} \).
which become all stronger for decreasing neutrino energy. Such relations can then be used to infer information about the mass spectrum hierarchy from general features of supernova (anti)neutrino event spectra,\footnote{15} analogously to the standard case \cite{2, 54, 55}. The hierarchy discrimination becomes impossible, however, for $\sin^2(\phi + \epsilon_H) \ll 10^{-5}$ (corresponding to $\exp[-2\pi r_H k_H \sin^2(\phi + \epsilon_H)] \simeq 1$), in which case the above three relations become equalities.

We finally remind that, under certain conditions (see the end of Sec. II), additional nonstandard transitions might take place just above the neutrinosphere \cite{30, 11}. In this case (not considered here), the global $P_{ee}$ function should be obtained by convolving such transitions with those $(H$ and $L)$ considered in this and in the previous section, leading to a more complicated phenomenology.

VI. SUMMARY

We have revisited the effects of nonstandard four-fermion interactions (with strength $\epsilon_{\alpha\beta} G_F$) on supernova neutrino oscillations occurring above the protoneutron star. Under reasonable approximations, we have found that the oscillation probability can be written in an analytical form, valid in the whole $2\nu$ parameter space, and applicable also to the $3\nu$ parameter space in hierarchical (direct or inverse) cases. Our approach generalizes previous results, which are recovered as specific limits. We find that, as far as the transitions at high $(H)$ and low $(L)$ density are concerned, the main effects of the new interactions can be embedded through (positive or negative) shifts of the relevant mixing angles $\phi = \theta_{13}$ and $\omega = \theta_{12}$, namely, $\phi \to \phi + \epsilon_H$ and $\omega \to \omega + \epsilon_L$, respectively [see Eqs. (34) and (36)]. Barring the case of small $\omega$ (disfavored by solar neutrino data), and apart from small $\epsilon_L$-induced Earth matter effects, the main phenomenological implication of such results is a strict degeneracy between standard $(\phi)$ and nonstandard $(\epsilon_H)$ effects on the $H$ supernova $\nu$ transition. However, such a degeneracy does not necessarily spoil the discrimination between direct and inverse neutrino mass spectrum hierarchy. Our work is complementary to studies \cite{37} of nonstandard transition phenomena possibly occurring just above the neutrinosphere.

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\footnote{15} The phenomenological implementation of the above inequalities requires, of course, that $P_{ee}$ is folded with $\nu$ (or $\bar{\nu}$) energy spectra and cross sections. This further step is beyond the scope of the present work.


[53] G.L. Fogli, E. Lisi, D. Montanino and A. Palazzo, Phys. Rev. D 64, 093007 (2001). The LMA solution is also strongly favored by the recent neutral current measurements at the Sudbury Neutrino Observatory; see Q.R. Ahmad et al., nucl-ex/0204008 and nucl-ex/0204004.
$3\nu$ survival probability, $E = 15$ MeV, $m^2 = 3 \times 10^{-3}$ eV$^2$

![Isolines of $P_{ee}$ for supernova neutrinos with direct mass hierarchy, in the plane of the $L$-transition variables ($\delta m^2, \tan^2 \omega$), with (dotted lines) and without (solid lines) Earth effects (calculated for $L = 8500$ km, $\rho = 4.5$ g/cm$^3$, and $Y_e = 1/2$). Nonstandard interactions are parameterized through the dimensionless couplings $\epsilon_{L,H}$. The values of $m^2$ and $E$ are fixed at $3 \times 10^{-3}$ eV$^2$ and 15 MeV, respectively. Left (right) panels refer to $\tan^2(\phi + \epsilon_H) = 0$ ($= 2 \times 10^{-5}$). The parameter $\epsilon_L$ takes the representative values 0 (upper panels), $+3 \times 10^{-2}$ (middle panels), and $-3 \times 10^{-2}$ (lower panels). See the text for details.](image-url)
FIG. 2: Earth matter effects: Curves of the probability $P_E = P(\nu_2 \rightarrow \nu_e)$ as a function of $\delta m^2$ for mantle crossing of supernova neutrinos (with $E, L, \rho$ and $Y_e$ as in Fig. 1). The chosen range of $\delta m^2$ and the three representative values of $\tan^2 \omega$ refer to the so-called LMA solution to the solar neutrino problem [53]. Nonstandard interactions affect the amplitude and the phase of the oscillatory pattern through $\epsilon_L$. The horizontal solid line represents the case of no Earth effect.