Keywords: Wilson–Taylor identities, Lagrange transformation, effective action.

and gives predictions for the physical part of the effective action.

derivation of the solution. The solution results from the explicit part of the effective action

and satisfies the conditions, so that all the effective fields are described by linear functions

under the theory. The solution can be parametrized by a limited number of functions of

of the theory. We present a way to solve Shifman–Taylor identities in a general nonperturbative-

Abstract

e-mail: m.schmidt@sun.ac.za

Department of Physics, University of Stellenbosch, Stellenbosch, South Africa.

Lefr Kobesbash, Corned Crete, and Lenn Schmidt

An Approach to Solve Shifman–Taylor Identities

UIST-97-122
1 Introduction

The effective action is an important quantity of the quantum theory. Defined as the Legendre transformation of the path integral, it provides us with an instrument to find the true vacuum state of the theory under consideration and to study its behavior taking into account quantum corrections. Slavnov–Taylor (ST) identities are also an important tool to prove the renormalizability of gauge theories in four spacetime dimensions [1, 2]. They generalize Ward–Takahashi identities of quantum electrodynamics to the non-Abelian case and can be derived starting from the property of invariance of the tree-level action with respect to BRST symmetry [3, 4]. ST identities for the effective action have been derived in Ref. [5].

Slavnov–Taylor identities are equations involving variational derivatives of the effective action. The effective action contains all the information about quantum behavior of the theory, and in quantum field theory it is the one particle irreducible diagram generator. Searching for the solution to Slavnov–Taylor identities can be considered as a complementary method to the existing non-perturbative methods of quantum field theory such as the Dyson–Schwinger and Bethe–Salpeter equations. The solution to Slavnov–Taylor identities in the four-dimensional supersymmetric theory has been proposed recently [6]. In the procedure to derive that solution, the no-renormalization theorem for superpotential [7, 8] has been used extensively. In this paper we will suggest that this point is not crucial and that arguments similar to those given before [6] can be used in the nonsupersymmetric case. In the approach developed below there are no restrictions on the number of dimensions and renormalizability of the theory. We require only that the theory under consideration can be regularized in such a way that the Slavnov–Taylor identities are valid and that BRST symmetry is anomaly free, as in the case, e.g., in QCD.

We argue that the functional structure of the auxiliary ghost-ghost $L e^2$ correlator in nonsupersymmetric gauge theories is fixed by Slavnov–Taylor identities in a unique way. In this correlator $L$ is a nonpropagating background field and it is coupled at the tree level to the BRST transformation of the ghost field $c$. According to our assumption, the vertex $L e^2$ is invariant with respect to ST identities and this then gives the following quantum structure for it:

$$\int dx' dx' dy dz \, G_c(x' - x) \, G_c^{-1}(x' - y) \, G_c^{-1}(x' - z) \, \frac{i}{2} f^{bc} L^c(x) e^b(y) e^c(z).$$

(1)

As one can see, the main feature of this result is that the effective ghost field $c$ is dressed by the unknown function $G_c^{-1} (x - y)$. This dressing contains all the quantum information about this correlator. We can use the structure of this correlator as a starting point to find the solution for the total effective action.

The solution to the Slavnov–Taylor identities found in the present paper imposes restrictions on the ghost part of the effective action. For example, it means that the gluon-ghost-antighost vertex can be read off from our result for the effective action (67):

$$G_m(q, p) = i q_m \frac{\tilde G_A(q^2)}{G_A(k^2) G_c(p^2)},$$

(2)

where $\tilde G_A$ is the Fourier image of a function that dresses gauge field, while $G_m(q, p)$ is the gluon-ghost-antighost vertex, $q$ is the momentum of the antighost field $b$ and $p$ is the momentum of the ghost field $c$, and $p + k + q = 0$. Another feature of the result obtained
here is that the physical part of the effective action (67) is gauge invariant in terms of the effective fields dressed by the dressing functions $G$. In the result (67) for the effective action information about quantum behaviour of the theory is encoded in a finite number of dressing functions and in the running function of the coupling.

The paper is organized in the following way. In Section 2 we review some basic aspects of BRST symmetry and Slavnov–Taylor identities for the irreducible vertices. In Section 3 we show how to obtain the functional structure (1) of the $Lc^2$ correlator. In Section 4 we obtain the correlator linear in another nonpropagating background field $K_m$, thus fixing the terms in the effective action which contain ghost and antighost effective fields. In Section 5 we describe higher correlators in $K_m$ and $L$. In Section 6 we make a conjecture about the form of the physical (pure gluonic) part of the effective action and then in Section 7 we consider renormalization of it to remove infinities. A brief summary is given at the end. The questions of consistency of this effective action within perturbative QCD are investigated in a second paper [9]. For simplicity, in the present paper we focus on pure gauge theories in four spacetime dimensions with $SU(N)$ gauge group. No matter field is included in the consideration, although their addition does not change our results.

2 Preliminaries

We consider the traditional Yang–Mills Lagrangian of the pure gauge theory

$$S = -i \int dx \frac{1}{2g^2} \text{Tr}[F_{mn}(x)F_{mn}(x)] \quad (3)$$

The gauge field is in the adjoint representation of the gauge group. A nonlinear local (gauge) transformation of the gauge fields exists which keeps theory (3) invariant. This symmetry must be fixed, Faddeev–Popov ghost fields [10] must be introduced and finally the BRST symmetry can be established for the theory that in addition to the classical action (3) contains a Faddeev–Popov ghost action and the gauge-fixing term.

To be specific, we choose the Lorentz gauge fixing condition

$$\partial_m A_m(x) = f(x). \quad (4)$$

Here $f$ is an arbitrary function in the adjoint representation of the gauge group that is independent on the gauge field. The normalization of the gauge group generators is

$$\text{Tr} \left( T^a T^b \right) = \frac{1}{2} \delta^{ab}, \quad (T^a)^\dagger = T^a, \quad [T^b, T^c] = if^{abc} T^a,$$

and we use notation $X = X^a T^a$ for all the fields in the adjoint representation of the gauge group, like gauge fields themselves, ghost fields, and their respective sources.

The conventional averaging procedure with respect to $f$ is applied to the path integral with the weight

$$e^{-i \int dx \text{Tr} \frac{1}{\alpha} f^2(x)}$$

and as the result we obtain the path integral

$$Z[J, \eta, \rho, K, L] = \int dA \, dc \, db \, \exp i \left\{ S[A, b, c] \right\}$$

2
\[ +2 \text{Tr} \left( \int dx \, J_m(x)A_m(x) + i \int dx \, \eta(x)e(x) + i \int dx \, \rho(x)b(x) \right) \]
\[ +2 \text{Tr} \left( i \int dx \, K_m(x)\nabla_m e(x) + \int dx \, L(x)e^2(x) \right) \}
\]

in which
\[ S[A, b, c] = \int d^4x \left[ \frac{1}{2g^2} \text{Tr} [F_{mn}(x)F_{mn}(x)] - \text{Tr} \left( \frac{1}{\alpha} [\partial_m A_m(x)]^2 \right) - 2 \text{Tr} \left( i b(x)\partial_m \nabla_m e(x) \right) \right]. \]

Here the ghost field \( e \) and the antighost field \( b \) are Hermitian, \( b^\dagger = b, \ e^\dagger = e \) in the adjoint representation of the gauge group. They possess Fermi statistics.

The infinitesimal transformation of the gauge field \( A_m \) is defined by the fact that it is a gauge connection,
\[ A_m \rightarrow A_m - \nabla_m \lambda, \]
where \( \lambda(x) \) is an infinitesimal parameter of the gauge transformation. This transformation comes from the transformation of covariant derivatives,
\[ \nabla_m \rightarrow e^{i\lambda} \nabla_m e^{-i\lambda}, \quad \nabla_m = \partial_m + iA_m, \quad \phi \rightarrow e^{i\lambda} \phi, \]
where \( \phi \) is some representation of the gauge group. To obtain the BRST symmetry we have to substitute \( i \ \epsilon(x) \ \varepsilon \) instead of \( \lambda \). Here \( \varepsilon \) is Hermitian Grassmannian parameter, \( \epsilon^\dagger = \epsilon, \ \epsilon^2 = 0 \). Thus, the BRST transformation of the gauge field is
\[ A_m \rightarrow A_m - i \nabla_m \epsilon \varepsilon. \]

In order to obtain the BRST transformation of the ghost field \( e \) we have to consider two subsequent BRST transformations
\[ \nabla_m \rightarrow e^{-i\kappa} \nabla_m e^{i\kappa}, \quad \nabla_m \epsilon \varepsilon^{i\kappa} \epsilon^{i\kappa} = e^{-i\kappa \epsilon \varepsilon^{i\kappa} \epsilon^{i\kappa}} \nabla_m e^{i\kappa \epsilon \varepsilon^{i\kappa} \epsilon^{i\kappa}}, \]
where \( \kappa \) is a Grassmannian parameter too, \( \kappa^2 = 0 \). This transformation again is equivalent to an infinitesimal transformation of the gauge field in covariant derivatives,
\[ A_m \rightarrow A_m - i\nabla_m \left[ (\epsilon \varepsilon + \epsilon \kappa + (\epsilon \varepsilon)(\epsilon \kappa) \right]. \]

It means that we can consider the inner BRST transformation (with \( \varepsilon \)) as the substitution (7) in the outer BRST transformation (with \( \kappa \)). The second term after the covariant derivative is a transformation of \( A_m \) under the outer BRST transformation while the third term after the covariant derivative is the transformation of \( i\nabla_m \epsilon \kappa \) and can it be cancelled by the transformation of the second term \( \epsilon \kappa \)
\[ e \rightarrow e + e^2 \varepsilon. \]

Thus, the transformations (7) and (9) together leave the covariant derivative of the ghost field unchanged. Such a symmetry is very general and always exists if the gauge fixing procedure has been performed in the path integral for any theory with nonlinear local
symmetry. The noninvariance of the gauge fixing term is cancelled by the corresponding transformation of the antighost field \( b \).

To collect all things together, the action (6) is invariant with respect to the BRST symmetry transformation with Grassmannian parameter \( \varepsilon \),

\[
A_m \rightarrow A_m - i \nabla_m c \varepsilon, \\
c \rightarrow c + c^2 \varepsilon, \\
b \rightarrow b - \frac{1}{\alpha} \partial_m A_m \varepsilon. 
\] (10)

The external source \( K \) and \( L \) of the BRST transformations of the fields are BRST invariant by definition, so the last two lines in the Eq. (5) are BRST invariant with respect to the transformations (10).

The effective action \( \Gamma \) is related to \( W = i \, \ln Z \) by the Legendre transformation\(^\dagger\)

\[
A_m \equiv - \frac{\delta W}{\delta J_m}, \quad ic \equiv - \frac{\delta W}{\delta \eta}, \quad ib \equiv - \frac{\delta W}{\delta \rho} 
\] (11)

\[
\Gamma = -W - 2 \text{Tr} \left( \int dx \ J_m(x) A_m(x) + \int dx \ i\eta(x) \ c(x) + \int dx \ i\rho(x) \ b(x) \right) \\
\equiv -W - 2 \text{Tr}(X \varphi), \\
(X \varphi) \equiv i^{G(k)}(X^k \varphi^k), \\
X \equiv (J_m, \eta, \rho), \quad \varphi \equiv (A_m, c, b),
\] (12)

where \( G(k) = 0 \) if \( \varphi^k \) is Bose field and \( G(k) = 1 \) if \( \varphi^k \) is Fermi field. We use throughout the paper notation

\[
\frac{\delta}{\delta X} \equiv T^a \frac{\delta}{\delta X^a}
\]

for any field \( X \) in the adjoint representation of the gauge group. Iteratively all equations (11) can be reversed,

\[
X = X[\varphi, K_m, L]
\]

and the effective action is defined in terms of new variables, \( \Gamma = \Gamma[\varphi, K_m, L] \). Hence, the following equalities take place

\[
\frac{\delta \Gamma}{\delta A_m} = -J_m, \quad \frac{\delta \Gamma}{\delta K_m} = -\frac{\delta W}{\delta K_m}, \quad \frac{\delta \Gamma}{\delta c} = i\eta, \quad \frac{\delta \Gamma}{\delta b} = i\rho, \quad \frac{\delta \Gamma}{\delta L} = -\frac{\delta W}{\delta L}. 
\] (13)

If the change of fields (10) in the path integral (5) is made one obtains the Slavnov–Taylor identity as the result of invariance of the integral (5) under a change of variables,

\[
\text{Tr} \left[ \int dx J_m(x) \frac{\delta}{\delta K_m(x)} - \int dx i\eta(x) \left( \frac{\delta}{\delta L(x)} \right) + \int dx i\rho(x) \left( \frac{1}{\alpha} \frac{\delta}{\delta J_m(x)} \right) \right] W = 0, 
\] (14)

\(^\dagger\)We have traditionally used in this paper the same notation for variable of the effective action and for variable of integration in the path integral coupled to the corresponding source [2].
or, taking into account the relations (13), we have [2]

\[
\text{Tr} \left[ \int d x \frac{\delta \Gamma}{\delta A_m(x)} \frac{\delta \Gamma}{\delta K_m(x)} + \int d x \frac{\delta \Gamma}{\delta \epsilon(x)} \frac{\delta \Gamma}{\delta L(x)} \right. \\
\left. - \int d x \frac{\delta \Gamma}{\delta b(x)} \left( \frac{1}{\alpha} \partial_m A_m(x) \right) \right] = 0. \quad (15)
\]

The problem is to find the most general functional \( \Gamma \) of the variables \( \varphi, K_m, L \) that satisfies the ST identity (15). Before doing it, we need in addition to ST identities also the ghost equation that can be derived by shifting the antighost field \( b \) by an arbitrary field \( \varphi(x) \) in the path integral (5). The consequence of invariance of the path integral with respect to such a change of variable is (in terms of the variables (11)) [2]

\[
\frac{\delta \Gamma}{\delta b(x)} + \partial_m \frac{\delta \Gamma}{\delta K_m(x)} = 0. \quad (16)
\]

The ghost equation (16) restricts the dependence of \( \Gamma \) on the antighost field \( b \) and on the external source \( K_m \) to an arbitrary dependence on their combination

\[
\partial_m b(x) + K_m(x). \quad (17)
\]

This equation together with the third term in the ST identities (15) is responsible for the absence of quantum corrections to the gauge-fixing term. Stated otherwise, when expressing \( \delta \Gamma / \delta b(x) \) in the third term in the ST identity (15) as \( -\partial_m (\delta \Gamma / \delta K_m(x)) \) by Eq. (16), the sum of the first and the third term in (15) can be rewritten as

\[
\text{Tr} \int d x \frac{\delta \Gamma'}{\delta A_m(x)} \frac{\delta \Gamma'}{\delta K_m(x)},
\]

where \( \Gamma' \equiv \Gamma - S^{(K)} \), and \( S^{(\varphi, \epsilon, L)} = -(1/\alpha) \text{Tr} \int d x [\partial_m A_m(x)]^2 \) is the gauge-fixing part of the classical action (6). In fact, all the other terms in the ST identity (15) can be rewritten with \( \Gamma' \) instead of \( \Gamma \), yielding

\[
\text{Tr} \left[ \int d x \frac{\delta \Gamma'}{\delta A_m(x)} \frac{\delta \Gamma'}{\delta K_m(x)} + \int d x \frac{\delta \Gamma'}{\delta \epsilon(x)} \frac{\delta \Gamma'}{\delta L(x)} \right] = 0. \quad (18)
\]

This shows explicitly that the gauge-fixing part of \( \Gamma \) remains unaffected by quantum corrections (\( \Gamma = \Gamma' + \Gamma^{(\varphi, \epsilon, L)}; \Gamma^{(\varphi, \epsilon, L)} = S^{(\varphi, \epsilon, L)} \)).

### 3 Functional structure of Lcc vertex

One can consider the part of the effective action that depends only on the fields \( L \) and \( \epsilon \). We write generally

\[
\Gamma_{L, \epsilon} = \int d x_1 dy_1 \Gamma^{(\varphi, \epsilon)}_{L, \epsilon}(x_1; y_1) \epsilon^{i_1}(y_1) e^{i_2}(y_2) + \ldots \\
+ \int d x_1 \ldots d x_n dy_1 \ldots dy_{2n} \Gamma^{(\varphi, \epsilon)}_{L, \epsilon}(x_1, \ldots, x_n; y_1, \ldots, y_{2n}) L^{a_1}(x_1) \times \\
\times \ldots L^{a_n}(x_n) e^{i_1}(y_1) \ldots e^{i_{2n}}(y_{2n}) + \ldots \quad (19)
\]
We assume that the first term is invariant with respect to the second operator in the identities (15) which is
\[
\text{Tr} \int d x \frac{\delta \Gamma}{\delta c(x)} \frac{\delta \Gamma}{\delta L(x)} = 0.
\] (20)

This assumption is based on the following. In perturbation theory the first term of (19) can be understood as the classical term plus a quantum correction to the vertex \( \text{Lee} \) (nothing forbids us to consider the auxiliary field \( L \) as a non-propagating background field). The operator (20) can be considered as an infinitesimal substitution in the effective action
\[
c(x) \rightarrow c(x) + \frac{\delta \Gamma}{\delta L(x)}.
\] (21)

In other words, one can consider the result of such a substitution as the difference
\[
\Gamma \left[ L, c(x) + \frac{\delta \Gamma}{\delta L(x)} \right] - \Gamma [L, c(x)],
\]
to linear order in \( \frac{\delta \Gamma}{\delta L(x)} \). As one can see, the application of the substitution (21) to the vertex \( \text{Lee} \) of the effective action \( \Gamma \) gives a variation of order \( \text{Lee} c \). Another contribution of the same order \( \text{Lee} c \) comes into the variation from the monomial \( \text{Lee} A \) of the effective action \( \Gamma \) due to the first term in the ST identity (15). Indeed, one can consider the first term in (15) as the substitution
\[
A_m(x) \rightarrow A_m(x) + \frac{\delta \Gamma}{\delta K_m(x)},
\]
or, in other words, such a substitution can be considered as the difference
\[
\Gamma \left[ K_m, A_m(x) + \frac{\delta \Gamma}{\delta K_m(x)} \right] - \Gamma [K_m, A_m(x)]
\]
to linear order in \( \frac{\delta \Gamma}{\delta K_m(x)} \). Application of such a substitution to the monomial \( \text{Lee} A \) of the effective action \( \Gamma \) gives a contribution of order \( \text{Lee} c \) in effective fields and this contribution comes from full ghost propagator of order in fields \( K_m \partial_m c \),
\[
\text{Lee} A \rightarrow \text{Lee} \frac{\delta \Gamma}{\delta K_m} \sim \text{Lee} \frac{\delta (K_m \partial_m c)}{\delta K_m} \sim \text{Lee} c.
\]
Thus, there are only these two possible contributions in variation \( \text{Lee} \). Schematically, total \( \text{Lee} \) variation can be presented as
\[
\langle \text{Lee} \rangle \times \langle L c \rangle + \langle \text{Lee} A \rangle \times \langle K_m \partial_m c \rangle = 0
\] (22)
where brackets mean \( \text{v.e.} \text{e.} \text{v.s.} \) of the vertices. This is a schematic form of the ST identity relating the \( L c \) and \( \text{Lee} A \) field monomials. The precise form of this relation can be obtained by differentiating the identity (15) with respect to \( L \) and three times with respect to \( c \) and then by setting all the variables of the effective action to zero. The brackets in (22) mean that we have taken functional derivatives with respect to fields in the corresponding
brackets and then have put all the effective fields to zero. Of course, this sum (22) should be zero since on the r.h.s. of the ST identity (15) we have zero. One can consider the identity (22) order by order in $g^2$. At tree level, the second contribution is absent since the $L_{ccA}$ term is absent in the classical action. For the first one we obtain the Jacobian identity. At one loop level, we have one loop $L_{cc}$ times tree level $L_{cc}$ plus one loop $L_{ccA}$ times tree level $K \partial c$. However, one loop $L_{ccA}$ is superficially convergent and does not depend on the normalization point $\mu$. In the asymptotic region one loop $L_{cc}$ depends on first degree of $\ln(p^2/\mu^2)$ where we have taken the symmetric point in momentum space, that is all the external momentums of the vertex $L_{cc}$ are $\sim p^2$. This means that the first degree of $\ln(p^2/\mu^2)$ in one loop $L_{cc}$ is invariant with respect to the operator (20). In other words, the dependence on $\ln(p^2/\mu^2)$ is cancelled within the first term of the identity (22). We can consider two loop approximation for the identity (22) in the same manner. Indeed, at two loop level of the identity (22) one has two loop $L_{cc}$ times tree level $L_{cc}$ plus one loop $L_{ccA}$ times tree level $K \partial c$ plus one loop $L_{ccA}$ times one loop $K \partial c$ and all this should be zero. However, one can see that the second degree of $\ln(p^2/\mu^2)$ is determined again by only the first term in the schematic identity (22) since two loop $L_{ccA}$ does not have superficial divergences and is divergent only in subgraphs. Thus, the second degree of $\ln(p^2/\mu^2)$ is also determined by the invariance with respect to the first term in the identity (22). We can go further in this logical chain and we will always conclude that the highest degree of $\ln(p^2/\mu^2)$ in $L_{cc}$ is invariant itself with respect to ST identity. This is the main source of the intuitive motivation to consider the $L_{cc}$ correlator separately from the other field monomial $L_{ccA}$.

In such a case, it will be shown below that the only solution for this $L_{cc}$ term of the effective action is

$$\int dx dx_1 dy_1 dy_2 \ G(x - x_1) \ G^{-1}(x - y_1) \ G^{-1}(x - y_2) \ 2 \text{Tr}(L(x_1)c(y_1)c(y_2)). \quad (23)$$

To prove (23), we consider the proper correlator

$$\Gamma = \int dx \ dy \ dz \ \Gamma(x, y, z) T^{abc} \ L^a(x) \ c^b(y) \ c^c(z). \quad (24)$$

As we have already noted, in perturbation theory it can be understood as a correction to the vertex $L_{cc}^2$ and we consider the auxiliary field $L$ as a non-propagating background field. $T^{abc}$ is some group structure. The equation (24) is just a general parametrization of the proper correlator $L_{cc}^2$ and nothing more. Equation (24) says that $\Gamma^{(abc)}(x, y, z) = \Gamma(x, y, z) T^{abc}$, where $T^{abc}$ is a 3-tensor in the adjoint representation of the gauge group. This reflects the fact that the global symmetry of the gauge group must be conserved in the effective action. With respect to that symmetry the auxiliary fields $K^a$ and $L^a$ are vectors in the adjoint representation of the gauge group. Also,

$$\Gamma(x, y, z) T^{abc} = -\Gamma(x, z, y) T^{acb}. \quad (25)$$

This is a direct consequence of the Grassmannian nature of the ghost fields. It follows from the parametrization (24). Further, from Eq. (24) follows

$$\frac{\delta \Gamma}{\delta L^a(x)} = \int dy \ dz \ \Gamma(x, y, z) T^{abc} \ c^b(y) c^c(z).$$
By substituting this expression in the Slavnov–Taylor identity (15) we have

\[
\int dx \frac{\delta \Gamma}{\delta e^{\alpha}(x)} \frac{\delta \Gamma}{\delta L^{a}(x)} = \int dx \, dy \, dz' \, \Gamma(y, x, z') T^{dab} L^{d}(y') \frac{\delta \Gamma}{\delta L^{a}(x)} e^{b}(z')
\]

\[
- \int dx \, dy' \, dz' \, \Gamma(y', x, z') T^{dab} L^{d}(y') e^{b}(z') \frac{\delta \Gamma}{\delta L^{a}(x)}
\]

\[
= \int dx \, dy \, dz \, dy' \, dz' \, \Gamma(y', x, z') T^{dab} L^{d}(y') \Gamma(x, y, z) T^{ann} e^{m}(y) e^{n}(z) e^{b}(z')
\]

\[
- \int dx \, dy \, dz \, dy' \, dz' \, \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z')
\]

\[
- \int dx \, dy \, dz \, dy' \, dz' \, \Gamma(y', y, x) \Gamma(x, z, z') T^{dab} T^{ann} L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z')
\]

\[
= \int dx \, dy \, dz \, dy' \, dz' \left[ \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} \right] L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z') = 0.
\]

Taking into account (25) the last two lines can be re-written as

\[
\int dx \, dy \, dz \, dy' \, dz' \left[ \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} \right] L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z')
\]

\[
- \Gamma(y', y, x) \Gamma(x, z, z') T^{dab} T^{ann} L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z')
\]

\[
= \int dx \, dy \, dz \, dy' \, dz' \left[ \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} \right] L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z') = 0.
\]

Now one can make total symmetrisation with respect to pairs \((m, y), (n, z), \) and \((b, z').\) It results in

\[
\int dx \, dy \, dz \, dy' \, dz' \left[ \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} + \Gamma(y', x, y) \Gamma(x, z, z') T^{dab} T^{ann} + \Gamma(y', x, z) \Gamma(x, y, z') T^{dab} T^{ann}\right] L^{d}(y') e^{m}(y) e^{n}(z) e^{b}(z') = 0.
\]

Thus, one comes to the equation

\[
\int dx \, dy \, dz \, dy' \, dz' \Gamma(y', x, z') \Gamma(x, y, z) T^{dab} T^{ann} + \int dx \, dy \, dz \, dy' \, dz' \Gamma(y', x, y) \Gamma(x, z, z') T^{dab} T^{ann} + \int dx \, dy \, dz \, dy' \, dz' \Gamma(y', x, z) \Gamma(x, y, z') T^{dab} T^{ann} = 0.
\]

(26)

As one can see, at tree level \(T^{dab} \sim f^{a\bar{b}d}\) and

\[
\Gamma_{tree}(x', y, z) = \int dx' \delta(x' - x) \delta(x' - y) \delta(x' - z)
\]

and, hence, the identity (26) is Jacobi identity. We consider in this paper gauge theories with \(SU(N)\) gauge group and we have noted this in Introduction. The structure constants
$f^{abc}$ are completely antisymmetric in such a case. With help of identities

$$f^{ABC} f^{CDE} f^{EBF} = -\frac{1}{2} N f^{ABF}$$

which are consequences of Jacobi identity, one can reduce the group structure of one loop diagram $L_{bc}$ to $f^{ABC}$ and that is true for all loops. Thus, it is natural to assume that $T^{abc} \sim f^{bca}$ and the identity (26) is

$$\int dx \Gamma(y', x, z') \Gamma(x, y, z) f^{abd} f^{mna} + \int dx \Gamma(y', x, y) \Gamma(x, z, z') f^{amd} f^{nba} + \int dx \Gamma(y', x, z) \Gamma(x, z', y) f^{amd} f^{bma} = 0.$$  

Because of Jacobi identity only two group structures are independent here:

$$\left[ \int dx \Gamma(y', x, z') \Gamma(x, y, z) - \int dx \Gamma(y', x, y) \Gamma(x, z, z') \right] f^{abd} f^{mna} + \left[ \int dx \Gamma(y', x, z) \Gamma(x, z', y) - \int dx \Gamma(y', x, y) \Gamma(x, z, z') \right] f^{amd} f^{bma} = 0.$$  

Since these two group structures are independent, we come to the equations

$$\int dx \Gamma(y', x, z') \Gamma(x, y, z) = \int dx \Gamma(y', y, x) \Gamma(x, z, z')$$

$$= \int dx \Gamma(y', x, z) \Gamma(x, z', y).$$  

(28)

We can solve at the beginning the first one:

$$\int dx \Gamma(y', x, z') \Gamma(x, y, z) = \int dx \Gamma(y', y, x) \Gamma(x, z, z')$$  

(29)

and then to check that the second equality is also satisfied. In writing this equation we have used the symmetry properties (25). We introduce Fourier transformations$^2$

$$\Gamma(x, y, z) = \int dp_1 \, dq_1 \, dk_1 \delta(p_1 + q_1 + k_1) \bar{\Gamma}(p_1, q_1, k_1) \exp(ip_1 x + iq_1 y + ik_1 z)$$

$$\Gamma(y', x, z') = \int dp_2 \, dq_2 \, dk_2 \delta(p_2 + q_2 + k_2) \bar{\Gamma}(p_2, q_2, k_2) \exp(ip_2 y' + iq_2 x + ik_2 z')$$

$$\Gamma(y', y, x) = \int dp_3 \, dq_3 \, dk_3 \delta(p_3 + q_3 + k_3) \bar{\Gamma}(p_3, q_3, k_3) \exp(ip_3 y' + iq_3 y + ik_3 x)$$

$$\Gamma(x, z, z') = \int dp_4 \, dq_4 \, dk_4 \delta(p_4 + q_4 + k_4) \bar{\Gamma}(p_4, q_4, k_4) \exp(ip_4 x + iq_4 z + ik_4 z')$$

The condition (29) in momentum space is

$$\int dx \, dp_1 \, dq_1 \, dp_2 \, dq_2 \, dk_2 \delta(p_1 + q_1 + k_1) \delta(p_2 + q_2 + k_2) \bar{\Gamma}(p_1, q_1, k_1) \times \bar{\Gamma}(p_2, q_2, k_2) \exp(ip_1 x + iq_1 y + ik_1 z + ip_2 y' + iq_2 x + ik_2 z')$$

$$= \int dx \, dp_3 \, dq_3 \, dp_4 \, dq_4 \, dk_4 \delta(p_3 + q_3 + k_3) \delta(p_4 + q_4 + k_4) \bar{\Gamma}(p_3, q_3, k_3) \times \bar{\Gamma}(p_4, q_4, k_4) \exp(ip_3 y' + iq_3 y + ik_3 x + ip_4 x + iq_4 z + ik_4 z').$$

$^2$We do not write factors $2\pi$ in these Fourier transformations since at the end of the calculations we will go back to coordinate space in which all the factors $2\pi$ will disappear.
It can be transformed to
\[
\int dp_1dq_1dk_1dp_2dk_2 \delta(p_1 + q_1 + k_1)\delta(p_2 - p_1 + k_2)\hat{\Gamma}(p_1, q_1, k_1) \times
\hat{\Gamma}(p_2, -p_1, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z')
\]
\[
= \int dp_3dq_3dk_3dq_4dk_4 \delta(p_3 + q_3 + k_3)\delta(-k_3 + q_4 + k_4)\hat{\Gamma}(p_3, q_3, k_3) \times
\hat{\Gamma}(-k_3, q_4, k_4) \exp(iq_3y' + iq_4y + ik_3z + ik_4z'),
\]
and then by momentum redefinitions in the second integral one obtains
\[
\int dp_1dq_1dk_1dp_2dk_2 \delta(p_1 + q_1 + k_1)\delta(p_2 - p_1 + k_2)\hat{\Gamma}(p_1, q_1, k_1) \times
\hat{\Gamma}(p_2, -p_1, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z')
\]
\[
= \int dp_4dq_4dk_4dp_2dk_2 \delta(p_2 + q_1 + k_2)\delta(-k_3 + k_1 + k_2)\hat{\Gamma}(p_2, q_1, k_3) \times
\hat{\Gamma}(-k_3, k_1, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z').
\]
By removing one of delta functions in each part one obtains
\[
\int dq_1dp_2dk_2 \delta(p_2 + q_1 + k_1)\hat{\Gamma}(p_2 + q_1, k_1)\hat{\Gamma}(p_2, -p_2 - k_2, k_2) \times
\hat{\Gamma}(p_2, -p_2 - k_2, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z')
\]
\[
= \int dp_4dq_1dk_1dp_2dk_2 \delta(p_2 + q_1 + k_1 + k_2)\hat{\Gamma}(p_2, q_1, k_1 + k_2) \times
\hat{\Gamma}(-k_1 - k_2, k_1, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z').
\]
By making the last simplification one obtains
\[
\int dk_1dp_2dk_2 \hat{\Gamma}(p_2 + k_2, -p_2 - k_2 - k_1, k_1)\hat{\Gamma}(p_2, -p_2 - k_2, k_2) \times
\hat{\Gamma}(p_2, -p_2 - k_2 - k_1, k_1) \exp(iq_1y + ik_1z + ip_2y' + ik_2z')
\]
\[
= \int dp_4dk_1dk_2 \hat{\Gamma}(p_2, -p_2 - k_2 - k_1, k_1 + k_2)\hat{\Gamma}(-k_1 - k_2, k_1, k_2) \times
\hat{\Gamma}(-k_1 - k_2, k_1, k_2) \exp(iq_1y + ik_1z + ip_2y' + ik_2z').
\]
Thus, finally the condition (29) takes the form
\[
\hat{\Gamma}(p_2 + k_2, -p_2 - k_2 - k_1, k_1)\hat{\Gamma}(p_2, -p_2 - k_2, k_2)
= \hat{\Gamma}(p_2, -p_2 - k_2 - k_1, k_1 + k_2)\hat{\Gamma}(-k_1 - k_2, k_1, k_2). \tag{30}
\]
This is an equation for a function of three variables, which will be solved below. First we show that there is simple ansatz which satisfies Eq. (30). Indeed, by choosing ansatz
\[
\hat{\Gamma}(p, q, k) = \frac{\tilde{G}(q^2)\tilde{G}(k^2)}{G(p^2)}, \tag{31}
\]
where \(\tilde{G}\) is the Fourier image of some function \(G_z^{-1}\), we can substitute this expression in Eq. (30):
\[
\frac{\tilde{G}(p_2 + k_2^2)}{G(p_2^2)} \times \frac{\tilde{G}(k_2^2)}{G(k_2^2)} = \frac{\tilde{G}(p_2 + k_2 + k_1^2)}{G(p_2^2)} \times \frac{\tilde{G}(k_2^2)}{G(k_2^2)} = \frac{\tilde{G}(p_2 + k_2 + k_1^2)}{G(p_2^2)} \times \frac{\tilde{G}(k_2^2)}{G(k_2^2)} = \tag{32}
\]
\[
\frac{\tilde{G}(p_2 + k_2 + k_1^2)}{G(p_2^2)} \times \frac{\tilde{G}(k_2 + k_1^2)}{G((k_2 + k_1)^2)} = \frac{\tilde{G}(p_2 + k_2 + k_1^2)}{G(p_2^2)} \times \frac{\tilde{G}(k_2 + k_1^2)}{G((k_2 + k_1)^2)}.
\]
This is an identity. That is, for the ansatz (31), Eq. (30) is valid. Now we will demonstrate that this ansatz is unique solution.

In general, the function \( \tilde{\Gamma}(p, q, k) \) is a function of three independent Lorentz invariants, since the moments \( p, q \) and \( k \) are not independent but related by conservation of the moments, \( p + q + k = 0 \). We can choose \( p^2, q^2 \) and \( k^2 \) as those independent invariants,

\[
\tilde{\Gamma}(p, q, k) \equiv f(p^2, q^2, k^2).
\]

Therefore, we can rewrite the basic equation (30) as

\[
f \left( (p_2 + k_2)^2, (p_2 + k_2 + k_1)^2, k_1^2 \right) \times f \left( p_2^2, (p_2 + k_2)^2, k_2^2 \right)
= f \left( p_2^2, (p_2 + k_2 + k_1)^2, k_1^2 \right) \times f \left( (k_1 + k_2)^2, k_1^2, k_2^2 \right).
\]

Let us introduce into the equation (33) new independent variables,

\[
(p_2 + k_2)^2 = x, \quad (p_2 + k_2 + k_1)^2 = y,
\]

\[
k_1^2 = z, \quad p_2^2 = u,
\]

\[
k_2^2 = v, \quad (k_1 + k_2)^2 = \lambda^2.
\]

The number of the independent variables is six, since in (33) we have only three independent Lorentz vectors \( p_2, k_2, k_1 \). Using these vectors we can construct six Lorentz-invariant values above. In terms of these new independent variables the basic equation (33) looks like

\[
f(x, y) \times f(u, x, v) = f(u, y, w) \times f(w, z, v).
\]

We consider equation (35) as an equation for an analytical function of three variables in \( R^3 \) space. We observe that the r.h.s. of (35) does not depend on \( x \) for any values of \( y, z, u, v \). There is the unique solution to this - the dependence on \( x \) must be factorized in the following way:

\[
f(x, y, z) = \frac{1}{\varphi(x)} F_1(y, z), \quad f(u, x, v) = \varphi(x) F_2(u, v),
\]

where \( \varphi(x) \) is some function, and \( F_1(y, z) \) and \( F_2(u, v) \) are other functions. The rigorous prove of this statement is given below. The two equations in (36) imply

\[
f(x, y, z) = \frac{\varphi(y)}{\varphi(x)} \times F(z),
\]

where \( F(z) \) is some function. By substituting this in Eq. (35) we immediately infer that \( F(z) = \text{constant} \times \varphi(z) \). Rescaling \( \varphi(z) \) by an appropriate constant, we obtain:

\[
f(x, y, z) = \frac{\varphi(y) \varphi(z)}{\varphi(x)}.
\]

Let us give a rigorous proof that the factorization (36) of the \( x \) dependence is the unique solution to equation (35). Denote \( \hbar \equiv \ln f \). Applying logarithm to (35), we have

\[
\hbar(x, y, z) = -\hbar(u, x, v) + \text{terms independent on } x.
\]
Applying \( \frac{\partial^m}{\partial x_1^m} \), \( m = 1, 2, \ldots \) to (38), we obtain

\[
\frac{\partial^m h(x_1, y, z)}{\partial x_1^m} \bigg|_{x_1=x} = -\frac{\partial^m h(u, x, v)}{\partial x_1^m} \bigg|_{x_2=x}
\]

This means that the Taylor expansions in \( x \) around the point \( x = 0 \) for functions \( h(x, y, z) \) and \( h(u, x, v) \) are

\[
\begin{align*}
  h(x, y, z) &= h(0, y, z) - \varphi(x, y, z), \\
  h(u, x, v) &= h(u, 0, v) + \varphi(x, y, z),
\end{align*}
\]

where

\[
\varphi(x, y, z) = -\sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n h(x_1, y, z)}{\partial x_1^n} \bigg|_{x_1=0}.
\]

Applying exponent to both the sides of (39) and (40), we obtain

\[
\begin{align*}
  f(x, y, z) &= f(0, y, z) \varphi(x, y, z), \\
  f(u, x, v) &= f(u, 0, v) \varphi(x, y, z),
\end{align*}
\]

where \( \varphi(x, y, z) = \exp \varphi(x, y, z) \). In (42) the l.h.s. is \( y \)- and \( z \)-independent. Hence, \( \varphi(x, y, z) \) is also \( y \)- and \( z \)-independent: \( \varphi(x, y, z) \equiv \varphi(x) \). Thus, we can rewrite Eqs. (41) and (42) as Eq. (36), where

\[
\begin{align*}
  F_1(y, z) &= f(0, y, z), \\
  F_2(u, v) &= f(u, 0, v).
\end{align*}
\]

This proves (36) and thus (37). Thus, we can conclude from (37) that (31) is the unique solution for \( \Gamma(p, q, k) \). To go back to the coordinate representation, we have to perform a Fourier transformation of (31),

\[
\Gamma(x, y, z) = \int \frac{dp dq dk}{2\pi} \delta(p + q + k) \tilde{\Gamma}(p, q, k) \exp(ipx + iky + ikz)
\]

\[
= \int \frac{dp dq dk}{2\pi} \delta(p + q + k) \frac{\hat{G}(q^2)\hat{G}(k^2)}{G(p^2)} \exp(ipx + iky + ikz)
\]

\[
= \int dx' dp dq dk \exp(-i(p + q + k)x') \frac{\hat{G}(q^2)\hat{G}(k^2)}{G(p^2)} \exp(ipx + iky + ikz)
\]

\[
= \int dx' G_c(x' - x) G_c^{-1}(x' - y) F_1(x' - z).
\]

By substituting this result in the second of equalities (28), we can see that it is also satisfied by this solution. One can take the correct tree level normalization of \( T^{abc} \)

\[
T^{abc} = \frac{i}{2} f^{bca}
\]

and present the final result for the functional structure of \( L e^2 \) proper correlator in the following form:

\[
\begin{align*}
  \int dx \ dy \ dz \ \Gamma(x, y, z) T^{abc} L^a(x) e^b(y) e^c(z) &= \int dx' dx' dy dz G_c(x' - x) G_c^{-1}(x' - y) G_c^{-1}(x' - z) \frac{i}{2} f^{bca} L^a(x) e^b(y) e^c(z).
\end{align*}
\]
As we have mentioned above, the natural assumption (44) about the group structure of the proper correlator $Lc^2$ has been done. However, we could avoid this assumption. Indeed, if all the group structures in (26) are independent, we obtain from there, instead of (28), three equations

$$\int dx \Gamma(y', x, z') \Gamma(x, y, z) = \int dx \Gamma(y', y, x) \Gamma(x, z, z')$$

$$= \int dx \Gamma(y', x, z) \Gamma(x, z', y) = 0$$

which are not true even at tree level as can be seen from Eq. (27). This means that at most two of the group structures must be independent to have a consistent solution. In such a case we come again to the necessity to solve Eq. (29) that has unique solution (43) as we have demonstrated above. Substituting this solution in Eq. (26) we obtain Jacobi identities for $T^{abc}$ which means that they are structure constants. In detail, this procedure can be done as follows. We can substitute the result (43) in (24)

$$\int dx \Gamma(y', x, z) T^{abc} L^a(x)^c(y) c'(z)$$

$$= \int dx' dy dz G_c(x' - x) G^{-1}_c(y' - y) G^{-1}_c(z' - z) T^{abc} L^a(x)^c(y) c'(z)$$

and then redefine fields $L$ and $c$

$$c^a(x) = \int dx' G_c(x - x') \tilde{c}^a(x')$$

$$L^a(x) = \int dx' G^{-1}_c(x - x') \tilde{L}^a(x')$$

$$\int dx' G^{-1}_c(x' - x) G_c(x' - x') = \delta(x - x').$$

The second term in Slavnov-Taylor identity (15) is covariant with respect to this change of variables,

$$\int dx \frac{\delta \Gamma[L, c]}{\delta c^a(x)} \frac{\delta \Gamma[L, c]}{\delta L^a(x)} = \int dx \frac{\delta \Gamma[L, c]}{\delta c^a(x)} \frac{\delta \Gamma[L, c]}{\delta L^a(x)} \frac{\delta \Gamma[L, c]}{\delta c^a(x)}$$

as can be explicitly checked, but the expression (45) takes the local form,

$$\Gamma = \int dx T^{abc} L^a(x)^c(x) \tilde{c}(x).$$

By substituting this in the ST operator (46) one concludes that

$$T^{abc} = \frac{i}{2} f^{abc}$$

solves it. The reason for this is that this $f^{abc}$ structure appears also at the level of the classical action

$$2Tr \int dx \ L(x)^c(x) = \frac{i}{2} f^{hca} L^a(x)^c(x) \tilde{c}(x),$$
and we already know that this structure satisfies the ST operator (46). Furthermore, there can be no other solution for $T^{abc}$, because (47) is the only one 3-tensor in the adjoint representation of the gauge group that is antisymmetric in the last two indices and satisfies Jacobi identities. Thus, the final result for the functional structure of $Lc^3$ proper correlator is

$$\int dx \ dy \ dz \ \Gamma(x, y, z) T^{abc} L^a(x) e^b(y) e^c(z) = \int dx' dy' dz' G_c(x' - x) G^{-1}_c(x' - y) G^{-1}_c(x' - z) \frac{i}{2} \int f^{bca} L^a(x) e^b(y) e^c(z).$$

(48)

In conclusion of this section we present arguments that the form (48) of the $Lc^3$ correlator remains unchanged even if corrections from $Lc^4$ correlator are allowed to contribute to the $\sim Lc^3$ term in the ST equation, i.e., the first term in the ST identity (15) contributes as well. This results in corrections to Eq. (26). In such a case we can demonstrate that the basic equation (35) will be modified to the following form:

$$f(x, y, z) \times f(u, v, w) - f(u, v, w) \times f(x, y, z) \times f(x, y, z) = \int d^{2}x' d^{2}y' dz' G_c(x' - x) G^{-1}_c(x' - y) G^{-1}_c(x' - z) \frac{i}{2} \int f^{bca} L^a(x) e^b(y) e^c(z).$$

(49)

The new function $f_2$ of the variables (34) parameterizes the contribution from the $Lc^4$ correlator. As one can see, there is a four-dimensional subspace of the six-dimensional space (34) with coordinates $x, y, z, u, v, w$ which is intersection of two hyperplanes $x = u + z + y + v - x - w$ and $v = z$ where the contribution of $Lc^4$ in Eq. (49) disappears. In this four-dimensional subspace Eq. (49) takes the same form as the basic equation (35) takes in the six-dimensional space

$$f \left( \frac{u + 2z + y - w}{2}, y, z \right) \times f \left( \frac{u + 2z + y - w}{2}, z \right) - f(u, v, w) \times f(w, z, z) = 0.$$

(50)

Unfortunately, at present we do not have a clear proof that the factorization (37) is the only solution to this equation. However, there are several strong indications in favor of uniqueness of the factorization. Indeed, one of them is that if we reduce the subspace in consideration further to $u = y = z$ and $v = 4\alpha z$, where $\alpha$ is an arbitrary real parameter, we obtain

$$f \left( 2(1 - \alpha)z, z, z \right) \times f \left( z, 2(1 - \alpha)z, z \right) \times f \left( z, 4\alpha z, z \right) = 0.$$

This suggests

$$\frac{d}{d\alpha} f(z, z, \alpha z) \times f(\alpha z, z, z) = 0.$$

As we have shown above, the factorization (37) is the only solution for such type of equations.

Another indication in favor of the factorization (37) is that for the region of the four-dimensional subspace in consideration where $z$ is much larger than each of $u, y, \alpha$ and $v$ we have in the leading order of $u/z$ and $y/z$ the equation

$$f(z, z, z) \times f(u, z, z) - f(u, y, w) \times f(w, z, z) = 0.$$

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that also requires the factorization (37) as the only solution since the information about \( w \) disappears on the l.h.s.

As the third indication, we can decompose logarithm of Eq. (50) in Taylor expansion in vicinity of any point in the four-dimensional subspace with coordinates \( u, y, z, w \). We then obtain, for the function \( h = \ln f \) at the quadratic order of Taylor expansion, separability of the variables as the only solution. But the separability for \( h \) means the factorization for \( f \). Further, we have indications that the separability must occur at any order of Taylor expansion.

Thus, we have shown that there are at least three arguments in favor of the factorization (37) being the only solution also for the Eq. (50), where this latter equation takes into account possible corrections from the \( LecA \) correlator to the basic equation (35).

4 Solution to the correlator of \( K_m A_m c \) type

Starting from this point we can repeat the method that has been used in Ref. [6] for deriving the solution to ST identities for supersymmetric theories. As it has been noted at the end of Introduction, the antighost equation (16) restricts the dependence of \( \Gamma \) on the antighost field \( b \) and on the external source \( K_m \) to an arbitrary dependence on their combination

\[
\partial_m b(x) + K_m(x).
\]

We can present this dependence of the effective action on the external source \( K_m \) in terms of a series

\[
\Gamma = \mathcal{F}_0 + \sum_{n=1} \int d x_1 d x_2 \ldots d x_n \mathcal{F}^{m_1 m_2 \ldots m_n}_n(x_1, x_2, \ldots, x_n) (\partial_{m_1} b(x_1) + K_{m_1}(x_1)) \times \\
\times (\partial_{m_2} b(x_2) + K_{m_2}(x_2)) \ldots (\partial_{m_n} b(x_n) + K_{m_n}(x_n)),
\]

where we assume contractions in spacetime indices \( m_j \). The coefficient functions of this expansion are in their turn functionals of other effective fields (11),

\[
\mathcal{F}^{m_1 m_2 \ldots m_n}_n = \mathcal{F}^{m_1 m_2 \ldots m_n}_n[A_m, c, L],
\]

whose coefficient functions for example in case \( L = 0 \) are ghost-antighost-vector correlators. \( \mathcal{F}_0 \) is a \( K_m \)-independent part of the effective action. The spacetime indices \( m_j \) of \( \mathcal{F}_n \) will be omitted everywhere below since they are not important in the present analysis.

Our purpose is to restrict the expansion (51) by using the ST identities (15). Let us consider for the moment the terms of (51) without the field \( L \). The noninvariance of these terms with respect to the ST identities (15) must be compensated by the first term (23) of the series (19) or possible interactions of this term with physical effective fields because \( \delta \Gamma / \delta L(x) \) of such terms only has no \( L \). The total degree of the ghost fields \( c \) in \( \mathcal{F}_n \) must be equal to \( n \) since each proper graph contains equal number of ghost and antighost fields among its external legs.

Let us consider terms in the effective action whose variations are cancelled by variations of the ghost field caused by the first term (23) of the series (19). To start we consider the
\( \mathcal{F}_1(x_1) \) coefficient function in the expansion (51). The corresponding term of the lowest order in fields in (51) is

\[
\int d\,x \,d\,x' \,2i \,\text{Tr} \left[ (\partial_m \,b(x) + K_m(x)) \,\partial_m \,G(x - x') \,e(x') \right],
\]

where \(-i\Box\ G(x - x')\) is a 2-point ghost-antighost proper correlator. It is an Hermitian kernel of the above integral,

\[
G^\dagger = G.
\]

We can make any change of variables in the effective action \( \Gamma \). Let us make the following change of variables:

\[
A_m(x) = \int d\,x' \,G_A(x - x') \,\tilde{A}_m(x'), \quad K_m(x) = \int d\,x' G_A^{-1}(x - x') \,K_m(x'),
\]

\[
e(x) = \int d\,x' \,G_e(x - x') \,\tilde{e}(x'), \quad L(x) = \int d\,x' \,G_e^{-1}(x - x') \,\tilde{L}(x'),
\]

\[
b(x) = \int d\,x' \,G_A^{-1}(x - x') \,\tilde{b}(x').
\]

Here \( G_X(x - x') \) are some dressing functions,

\[
\int d\,x' \,G_X^{-1}(x - x') \,G_X(x' - x'') = \delta(x - x''),
\]

In terms of new variables the effective action

\[
\bar{\Gamma}[\varphi, \bar{K}_m, \bar{L}] = \Gamma[\varphi(\tilde{\varphi}), K_m(\tilde{K}_m), L(\tilde{L})]
\]

must satisfy the identity

\[
\text{Tr} \left[ \int d\,x \,\frac{\delta \bar{\Gamma}}{\delta A_m(x)} \,\delta \Gamma + \int d\,x \,\frac{\delta \bar{\Gamma}}{\delta c(x)} \,\delta \Gamma \right] + \int d\,x \,\frac{\partial}{\partial x'} \,\delta \bar{\Gamma}^{(x - x')} G_A(x - x') \left( \frac{1}{\alpha} \,\partial_m \,\tilde{A}_m(x') \,G_A(x - x') \right) = 0,
\]

which is the identity (15) re-expressed in terms of the new variables according to (53). As one can see the ST operator is covariant with respect to this change of variables except for the gauge fixing term, which remains unaffected by quantum corrections anyway as mentioned earlier.

One can make the change of variables (53) in the integral (52).

\[
\int d\,x \,d\,x' \,d\,x'' \,d\,x''' \,2i \,\text{Tr} \left[ (\partial_m \,\tilde{b}(x''') + \tilde{K}_m(x''')) \,G_A^{-1}(x''' - x) \,G(x - x') \times \right.
\]

\[
\times \,G_e(x' - x''') \,\partial_m \,\tilde{e}(x''') \right].
\]

While the dressing function \( G_e(x - x') \) has been defined through the solution (23) to the operator (20), the dressing function \( G_A(x - x') \) has not been defined yet. We define it from the requirement

\[
\int d\,x \,d\,x' \,d\,x'' \,d\,x''' \,2i \,\text{Tr} \left[ (\partial_m \,\tilde{b}(x''') + \tilde{K}_m(x''')) \,G_A^{-1}(x''' - x) \,G(x - x') \times \right.
\]

\[
\times \,G_e(x' - x''') \,\partial_m \,\tilde{e}(x''') \right] = \delta(x'' - x''').
\]

\(^3\)The formula (54) does not mean that both the functions \( G_X^{-1}(x - x') \) and \( G_X(x' - x'') \) are \( \delta \)-functions. It means only that the product of their Fourier transforms is equal to 1.
In such case the term (56) after the change of variables (53) simplifies to
\[
\int d^4 x \, 2i \, \text{Tr} \left[ \left( \partial_m \tilde{b}(x) + \tilde{K}_m(x) \right) \, \partial_m \tilde{c}(x) \right].
\]  
(57)

The first term in ST identities (55) can also be expanded in terms of \( \partial_m \tilde{b}(x) + \tilde{K}_m(x) \),
\[
\int d^4 x \, \frac{\delta \tilde{\Gamma}}{\delta A_m(x)} \frac{\delta \tilde{\Gamma}}{\delta K_m(x)} = \mathcal{M}_0 + \sum_{n=1} \int d^4 x_1 d^4 x_2 \ldots d^4 x_n \mathcal{M}_{m_1 m_2 \ldots m_n}^n \left( x_1, x_2, \ldots, x_n \right)
\times \left( \partial_{m_1} \tilde{b}(x_1) + \tilde{K}_{m_1}(x_1) \right) \left( \partial_{m_2} \tilde{b}(x_2) + \tilde{K}_{m_2}(x_2) \right) \times \ldots \left( \partial_{m_n} \tilde{b}(x_n) + \tilde{K}_{m_n}(x_n) \right),
\]
(58)

where we assume contractions in spacetime indices \( m_j \). Again, the spacetime indices \( m_j \) of \( \mathcal{M}_0 \) will be omitted everywhere below since they are not important in the present analysis. \( \mathcal{M}_0 \) is the \( \tilde{K}_m \)-independent part of (58). We can consider that the l.h.s. of (58) is the result of an infinitesimal transformation in \( \tilde{\Gamma} \), in which instead of \( \tilde{A}_m(x) \) we have substituted
\[
\tilde{\tilde{A}}_m(x) \rightarrow \tilde{A}_m(x) + \frac{\delta \tilde{\Gamma}}{\delta K_m(x)}.
\]
(59)

In other words, one can consider the result of such a substitution as the difference
\[
\tilde{\Gamma} \left[ \tilde{K}_m, \tilde{A}_m(x) + \frac{\delta \tilde{\Gamma}}{\delta K_m(x)} \right] - \tilde{\Gamma} \left[ \tilde{K}_m, \tilde{\tilde{A}}_m(x) \right]
\]
to linear order in \( \frac{\delta \tilde{\Gamma}}{\delta K_m(x)} \). Eq. (59) implies that the “gauge” transformation (59) can be rewritten as
\[
\delta \tilde{\tilde{A}}_m(x) \sim i \partial_m \tilde{c}(x) + \text{higher terms},
\]

The sum of the part quadratic in \( \tilde{A} \) of \( \mathcal{F}_0 \) and the \( \mathcal{F}_1 \)-type term (57) contributes to \( \mathcal{M}_0 \) by yielding a term \( \sim \tilde{A} \tilde{\tilde{c}} \). However, \( \mathcal{M}_0 \) must be equal to zero\(^4\). Hence, the part quadratic in \( \tilde{A} \) of \( \mathcal{F}_0 \) must be invariant, at quadratic order, under the aforementioned “gauge” transformation, implying the form
\[
- \int d^4 x \ Z_{g^2}^2 \, \frac{1}{2g^2} \, \text{Tr} \left( \partial_m \tilde{\tilde{A}}_n(x) - \partial_n \tilde{\tilde{A}}_m(x) \right) \, \mathcal{O} \left( \partial_m \tilde{\tilde{A}}_n(x) - \partial_n \tilde{\tilde{A}}_m(x) \right),
\]
(60)

where \( Z_{g^2} \) is a number that depends on couplings and a regularization parameter of the theory, and \( \mathcal{O} \) is some differential operator. \( g \) will see how the ST identities put restrictions on such an operator.

Having fixed the form of the quadratic term (57) in \( \mathcal{F}_1 \), we consider the vertex of next order in fields in \( \mathcal{F}_1 \), which looks like \( \sim \left( \partial_m \tilde{b} + \tilde{K}_m \right) \tilde{\tilde{A}}_m \tilde{\tilde{c}} \). We will show now that the

\(^4\) In principle, another term \( \sim \tilde{A} \tilde{\tilde{c}} \) can appear in the third term there, coming from the \( \sim b \partial_m \nabla_m \tilde{c} \) part of \( \tilde{\Gamma} \). However, the third term in (55) [and in (15)] is only responsible for the absence of corrections to the gauge fixing term in \( \tilde{\Gamma} \), as we have already noted at the end of Section 2.
structure of the vertex \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \tilde{A}_m \tilde{c} \) is fixed completely by the quadratic term (57) and by the term (23). According to the Slavnov–Taylor identity (55), the contribution of \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \tilde{A}_m \tilde{c} \) of the \( \mathcal{F}_1 \) part of the effective action into \( \mathcal{M}_1 \) caused by the quadratic term (57) due to the substitution (59) must be cancelled by the variation of the ghost field caused in (57) by the first term (23) of the series (19) due to substitution (21). According to our conjecture, the term (23) has the form

\[
2 \text{Tr} \int dx \, \tilde{L}(x) \tilde{c}^2(x).
\]

Indeed, the only contribution to \( \mathcal{M}_1 \) of the order of \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \partial_m \tilde{c}^2 \) in \( \mathcal{M}_1 \) comes from this \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \tilde{A}_m \tilde{c} \) term in \( \mathcal{F}_1 \):

\[
\int dx \, \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_1}}{\delta A_m(x)} \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_2}}{\delta \tilde{K}_m(x)} \sim \left[ (\partial_m \tilde{\beta} + \tilde{K}_m) \tilde{c} \right] \partial_m \tilde{c} \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \partial_m \tilde{c}^2,
\]

where \( \tilde{\Gamma}|_{\mathcal{F}_1} \) is the \( \mathcal{F}_1 \) part of the effective action. One could think at first that the \( \mathcal{F}_0 \)- and \( \mathcal{F}_2 \)- type terms \( \tilde{\Gamma}|_{\mathcal{F}_0} \), \( \tilde{\Gamma}|_{\mathcal{F}_2} \) of (19) might also contribute to the term of order \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \partial_m \tilde{c}^2 \) in \( \mathcal{M}_1 \) via

\[
\int dx \, \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_0}}{\delta A_m(x)} \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_2}}{\delta \tilde{K}_m(x)}
\]

because

\[
\frac{\delta \tilde{\Gamma}|_{\mathcal{F}_2}}{\delta \tilde{K}_m(x)} \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \mathcal{F}_2[A_m, c].
\]

However, \( \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_0}}{\delta A_m(x)} \) starts with terms linear in \( \tilde{A}_m(x) \). Thus, the \( \mathcal{F}_2 \) part of the effective action does not contribute to the term of the order of \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \partial_m \tilde{c}^2 \) in \( \mathcal{M}_1 \), only the \( \mathcal{F}_1 \) part of the effective action does. Hence, the term of order \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \tilde{A}_m \tilde{c} \) in \( \mathcal{F}_1 \) is the term of the same order that is contained in \( \tilde{K}_m(x) \nabla_m \tilde{c}(x) \) because only in this case the terms \( \sim (\partial_m \tilde{\beta} + \tilde{K}_m) \partial_m \tilde{c}^2 \) in \( \mathcal{M}_1 \) will be cancelled by the second term in ST identities (55), which will result in

\[
\int dx \, 2i \, \text{Tr} \left[ (\partial_m \tilde{\beta}(x) + \tilde{K}_m(x)) \partial_m \tilde{c}^2(x) \right]
\]

due to substitution (21). Thus, the term of lowest order in fields in \( \mathcal{F}_1 \) is

\[
2i \, \text{Tr} \left[ (\partial_m \tilde{\beta}(x) + \tilde{K}_m(x)) \nabla_m \tilde{c}(x) \right], \quad \nabla_m = \partial_m + i \tilde{A}_m.
\quad (61)
\]

All the terms in \( \mathcal{F}_0 \) of higher orders in \( \tilde{A}_m(x) \) are fixed by themselves in an iterative way due to the requirement that \( \mathcal{F}_0 \) must be invariant with respect to the substitution (59). Taking into account (61), we see that the first invariant term is

\[
- \int dx \, Z g^2 \frac{1}{2 g^2} \text{Tr} \tilde{F}_{mn}(x) \tilde{F}_{mn}(x),
\]
where \( \tilde{F}_{mn}(x) \) is Yang–Mills tensor of \( A_m(x) \). That is, the physical part of the effective action can be restored from the requirement of its invariance with respect to the gauge invariance in terms of the gauge field dressed by the dressing function. Here we see that the differential operators \( O \) in Eq. (60) between two Yang–Mills tensors must be covariant derivatives. For example, the following term is allowed,

\[
\int d^4x \; f_2 \frac{1}{\Lambda^2} \text{Tr} \tilde{F}_{mn}(x) \nabla^2 \tilde{F}_{mn}(x),
\]

(62)

where \( f_2 \) is another number that depends on couplings, and \( \Lambda \) is a regularization parameter of the theory. Starting from the fourth degree of \( A_m(x) \) higher order gauge invariant contributions like

\[
\int d^4x \; f_3 \frac{1}{\Lambda^4} \text{Tr} \tilde{F}_{mn}(x) \tilde{F}_{mn}(x) \tilde{F}_{kl}(x) \tilde{F}_{kl}(x)
\]

(63)

into \( \mathcal{F}_0 \) are allowed. Here \( f_3 \) is another number that depends on couplings.

5 Furth e r steps for higher correlators in \( K_m \) and \( L \)

We consider now the coefficient functions \( \mathcal{F}_n \) with \( n \geq 1 \) in (51) for \( L = 0 \). There are two possibilities here. The first possibility is that these terms of higher degrees in \( \tilde{K} \) do not respect the gauge invariance of the physical part of (51) created by the \( \mathcal{F}_1 \) term. In such a case \( \mathcal{F}_2 \) contributes to \( \mathcal{M}_1 \) but we do not have anything that can compensate this contribution by ghost transformations induced by the second term in the ST identities (55). Hence, \( \mathcal{F}_2 = 0 \). If we consider \( \mathcal{F}_3 \), it contributes to \( \mathcal{M}_2 \) and, in general, could be compensated by ghost transformations in \( \mathcal{F}_1 \). But \( \mathcal{F}_2 \) is zero, hence, \( \mathcal{F}_3 \) is also zero. We can repeat the former argument for all higher numbers \( n \) of \( \mathcal{F}_n \). All coefficient functions \( \mathcal{F}_n \) with \( n > 1 \) are equal to zero in the first possibility. The second possibility is that the terms of higher degrees in \( \tilde{K} \) respect the gauge invariance of the physical part of (51). In such a case \( \mathcal{F}_n \) with \( n > 1 \) does not contribute to \( \mathcal{M}_n \) for any \( n \). In supersymmetric theories this possibility does not exist [6] due to chiral nature of the ghost superfields. However, in the nonsupersymmetric case one can invent, for example, \( \mathcal{F}_2 \) constructions such as the following one

\[
\int d^4x \; \text{Tr} \left[ \left( \partial_m \tilde{b}(x) + \tilde{K}_m(x) \right) \left( \partial_m \tilde{b}(x) + \tilde{K}_m(x) \right) \right].
\]

(64)

Such a term gives zero contribution to \( \mathcal{M}_1 \), since its variation with respect to \( \tilde{K} \) is proportional to \( \nabla_m \) (scalar function) and its contribution to \( \mathcal{M}_2 \) can be cancelled by the transformation of the ghost field in \( \mathcal{F}_2 \) if the coefficient before (64) has been fixed in an appropriate way. This can be proved in the same way (8) which has been used to derive the BRST transformation in Section 2.

We have considered the terms in the effective action whose variations are cancelled by variations of the ghost field caused by the first term (23) of the series (19). In general, some sophisticated interactions of the term (23) with physical fields can be introduced. However, again we can state that the higher order terms must respect the already established invariance with respect to the Slavnov–Taylor operator for the terms of lowest
degrees in fields. In our case for example we can write for interactions of the term (23) with physical fields by using the following substitution

$$\tilde{L}e^2 \rightarrow \tilde{L}e^2 \left( 1 + f_4 \frac{1}{\bar{A}^4} \text{Tr} \tilde{F}_{mn}(x) \tilde{F}_{mn}(x) \right),$$

and then making a substitution in (61):

$$\tilde{c} \rightarrow \hat{c} \left( 1 + f_4 \frac{1}{\bar{A}^4} \text{Tr} \tilde{F}_{mn}(x) \tilde{F}_{mn}(x) \right).$$

(65)

However, these terms cannot change the structure of the physical part of the effective action since it is already determined by the terms of the first order in the auxiliary field $\bar{K}_m$.

One can consider possible terms with higher degrees of $L$. For example, the sum of (23) and

$$\int dx \sum_{k=1}^{4k} \left( \tilde{L}^a(x) \tilde{L}^a(x) \right)^k \hat{c}^{b_1} \left( x \right) \ldots \hat{c}^{b_{4k}} \left( x \right) \delta_{b_1 b_2 \ldots b_{4k}}$$

(66)

satisfies the identity (20) if $4k$ is the rank of the gauge group. If these terms exist it is also necessary to consider the dependence of $\mathcal{F}_n$ on the auxiliary field $L$, since the substitution due to the second term in the ST identities would produce these terms. However, at the end we put all the auxiliary fields equal to zero, and therefore all the terms with higher degrees of $\tilde{L}$ do not have any importance. In comparison, the situation with the $K_m$ field is different. Indeed, terms with zero $K_m$ are still important since they are responsible for higher degrees of ghost-antighost correlators which may have applications in some models.

6 Conjecture for the physical part of the action

Taking into account the structure (61) of the term linear in $K_m$, one can come to a natural conjecture about the form of the part of the effective action that depends only on the gauge effective field $A_m$. Namely, due to the ST identity (55) in terms of the dressed fields, the structure of the effective action is

$$\Gamma[A_m, b, c] = \int d x \left[ -\frac{1}{2g^2} Z_g^2 \text{Tr} \left( \tilde{F}_{mn}(x) \mathcal{G} \left( \frac{\bar{V}^2}{\bar{A}^2} \right) \tilde{F}_{mn}(x) \right) \right]$$

(67)

$$- \text{Tr} \left( \frac{1}{\alpha} \left[ \partial_m A_m(x) \right]^2 \right) - 2i \text{Tr} \left( b(x) \partial_m \bar{\nabla}_m \bar{c}(x) \right) \right] + \text{irrelevant part},$$

where all auxiliary fields $K$ and $L$ are set equal to zero. It is necessary to make three comments here:

- The function $\mathcal{G}$ is a series in terms of covariant derivative with dressed gauge connection. The part of this series without gauge connection $\mathcal{G} \left( \frac{\bar{V}^2}{\bar{A}^2} \right)$ has logarithmic asymptotic in the momentum space at high momentum, $\mathcal{G} \left( \frac{\bar{V}^2}{\bar{A}^2} \right) \sim \ln \left( -\frac{\bar{V}^2}{\bar{A}^2} \right)$, while at low momentum it may be represented, e.g., by powers of $p^2 / A_{QCD}^2$ with $A_{QCD} \sim 0.1$ GeV [9].

- The physical part of the action is gauge invariant in terms of the dressed field $\bar{A}_m(x)$. 20
7 Regularization and renormalization

In a general nonsupersymmetric four-dimensional gauge theory which is regularized in a way that preserves gauge (and BRST) symmetry, the dressing functions are of the following form:

\[ G_X^{-1}(x-x') = z_X \delta(x-x') + \frac{C_1(\Lambda^2, \mu^2)}{\mu^2} \left( \Box - \mu^2 \right) \delta(x-x') \]
\[ + \frac{C_2(\Lambda^2, \mu^2)}{(\mu^2)^2} \left( \Box - \mu^2 \right)^2 \delta(x-x') + \ldots \]  
(68)

This representation means that we have expanded the Fourier transformed dressing function \( G_X^{-1}(p^2) = 1/G_X(p^2) \), \( X = A, \phi \), in the vicinity of the point \( p^2 = -\mu^2 \). Here \( z_X \) is a constant that goes to infinity if the regularization is removed, and \( C_1, C_2 \) are finite constants.\(^5\) For instance, \( z_A \) is a renormalization constant of the gauge field. To renormalize the theory we have to introduce counterterms into the classical action (6) [11]. This is equivalent to the change of the field in the classical action (6). For example, in the case of the pure gauge theory, to remove divergences from \( G_A^{-1}(x-x') \) we have to make the following redefinition of the gauge field in the classical action

\[ A_m^{\text{bare}} \rightarrow A_m^{\text{phys}} = \frac{A_m^{\text{bare}}}{z_A}. \]  
(69)

The motivation for terminology “bare” and “physical” for fields in the path integral is that introducing counterterms into the classical action (6) by the rescaling (69) of fields and couplings will result in the effective action without divergences (renormalized effective action). We can show that by such a redefinition we can make the dressing function \( G_A^{-1} \) finite. Indeed, if we represent the term with the source of the gauge field in the path integral (5) as

\[ J_m A_m = \left( J_m \ z_A \right) \frac{A_m}{z_A}, \]

then the path integral for the theory with counterterms (69) can be transformed to the form (5) by the substitution of variables of the integral \( A_m = A_m^{\text{phys}} z_A \). It means that all the previous construction can be reproduced without any change but taking into account the redefinition \( J_m \rightarrow J_m \ z_A \). In turn such a redefinition, according to definitions (11), means nothing else but that the effective fields are also redefined as in (69), which is equivalent to the redefinition of the dressing function

\[ G_A^{-1}(x-x') \rightarrow \frac{1}{z_A} G_A^{-1}(x-x'). \]  
(70)

\(^{5}\)\( G_X^{-1}(x-x') = (2\pi)^{-d} \int d^4p \exp\left[-ip(x-x')\right]/\left[1/G_X(p^2)\right] \), i.e. \( G_X^{-1}(x-x') \neq 0 \) for \( x-x' \neq 0 \) in general, although the expansion (68) might suggest otherwise.
One can consider Eq. (70) in momentum space,

\[
\frac{1}{z_A} \frac{1}{G_A(p^2)} = \frac{\tilde{G}_A(-\mu^2, A^2)}{G_A(p^2, A^2)} = \left(1 + \alpha g'^2 + \beta \left(g'^2\right)^2 + \gamma \left(g'^2\right)^3 + \ldots\right) \times \\
\times \left(1 + \tilde{G}_1(p^2) g^2 + \tilde{G}_2(p^2) \left(g^2\right)^2 + \ldots\right) \\
= 1 + \left(\alpha + \tilde{G}_1(p^2)\right) g^2 + \left(\beta + \alpha \tilde{G}_1(p^2) + \tilde{G}_2(p^2)\right) \left(g^2\right)^2 + \ldots
\]

(71)

where we have presented both factors of the l.h.s. as a series in terms of the coupling constant. In this expansion \(g^2\) is the physical coupling that stays in the classical action according to the counterterm approach [11]. All these dressing functions parametrize our result (67) for the effective action, that is, they parametrize the irreducible vertices that contain divergences. Divergences from the dressing functions must be removed. We can remove the divergences in each order in coupling constant by choosing the divergent coefficients \(\alpha, \beta, \gamma\) in \(1/z_A\) in an appropriate way, because each coefficient \(\tilde{G}_n(p^2)\) of the decomposition \(G_A^{-1}(p^2)\) in terms of the coupling constant is in turn a series in terms of \(p^2\) with only the zero order in \(p^2\) terms being divergent. This is due to the fact that

\[
\lim_{\Lambda \to \infty} \frac{\tilde{G}_A(p^2, A^2)}{G_A(-\mu^2, A^2)}
\]

is finite. As to divergent coefficients before the relevant operators, they will be compensated by counterterms from the bare couplings.

Till this moment we did not specify which regularization is used. The regularization by higher derivatives (HDR) is the most convenient from the point of view of the theoretical analysis [2]. It provides strong suppression of ultraviolet divergences by introducing additional terms with higher degrees of covariant derivatives acting on Yang–Mills fields into the classical action (6), which are suppressed by appropriate degrees of the regularization scale \(\Lambda\). In addition to this it is necessary to introduce a modification of the Pauli–Villars regularization to guarantee the convergence of the one loop diagrams [2]. To regularize the fermion cycles, the usual Pauli–Villars regularization can be used.

Thus in case of four-dimensional QCD without quarks the classical action (6) is

\[
S_{QCD}[A, b, e] = \int d x \left[ -\frac{1}{2 g^2} \text{Tr} \left[ F_{mn}(A(x)) F_{mn}(A(x)) \right] - \text{Tr} \left( \frac{1}{\alpha} \left[ \partial_m \frac{A_{mn}}{z^2 A(x)}(x) \right]^2 \right) - 2 \text{Tr} \left( i b(x) \partial_m \nabla_m(A(x)) e(x) \right) \right].
\]

In the counterterm technique [11] the coupling constant here is the physical coupling constant. The classical action with the counterterms is

\[
S_{QCD}[A, b, e] = \int d x \left[ -\frac{1}{Z g^2} \frac{1}{2 g^2} \text{Tr} \left[ F_{mn} \left( \frac{A}{z_A}(x) \right) F_{mn} \left( \frac{A}{z_A}(x) \right) \right] \right] - \text{Tr} \left( \frac{(z_A)^2}{\alpha} \left[ \partial_m \frac{A_{mn}}{z_A^2}(x) \right]^2 \right) - 2 \text{Tr} \left( i z_A b(x) \partial_m \nabla_m \left( \frac{A}{z_A} \right) \left( \frac{A}{z_A} \right) \frac{c(x)}{z^2} \right),
\]

\[\text{Even if the renormalization (71) has been done and the dressing functions are finite, the theory still has divergences in the coefficients of the relevant operators. These divergences are absorbed by the bare couplings.}\]

\[\text{A somewhat different regularization approach is applied in Ref. [3] where explicit QCD one loop dressing functions are obtained.}\]
where fields are “physical” in the sense that this classical action together with counterterms results in the effective action in which divergences are removed. Thus, we come to the conclusion that the renormalized effective action takes the form:

\[
\Gamma_{QCD}[A_m, \phi, \bar{c}] = \int d^4x \left[ -\frac{1}{2g} \text{Tr} \left( \tilde{F}_{mn}(x) G_2 \left( \frac{\nabla^2}{\mu^2} \right) \tilde{F}_{mn}(x) \right) - \text{Tr} \left( \frac{1}{\alpha} [\delta_m A_m(x)]^2 \right) - 2i \text{Tr} \left( \hat{b}(x) \partial_m \nabla \hat{a}(x) \right) \right],
\]

where all auxiliary fields \( K \) and \( L \) are set equal to zero. Here the function \( G_2 \) is defined as

\[
G_2 \left( \frac{\nabla^2}{\mu^2} \right) \equiv \lim_{\Lambda \to \infty} \tilde{G} \left( \frac{\nabla^2}{\Lambda^2} \right) / \tilde{G} \left( \frac{\mu^2}{\Lambda^2} \right).
\]

8 Summary

In this work we proposed a solution to the Slavnov–Taylor identities for the effective action of non-supersymmetric non-Abelian gauge theory without matter. The solution is expressed in terms of gauge \( A_m \) and (anti)ghost effective fields \( (\phi, b) \) convoluted with unspecified dressing functions:

\[
\tilde{A}_m(x) = \int d^4 x' G_A^{-1}(x - x') A_m(x')
\]

\[
\tilde{\phi}(x) = \int d^4 x' G_\phi^{-1}(x - x') \phi(x')
\]

\[
\tilde{b}(x) = \int d^4 x' G_b(x - x') b(x').
\]

Further, the solution is invariant under the gauge (BRST) transformation of the convoluted fields. We gave arguments which show that, under a specific plausible assumption, the terms of the effective action containing (anti)ghost fields must have the same form as those in the classical action, but under the substitution \( X \to \tilde{X} \) \( (X = c, b, A_m) \). Further, we conjectured a rather general form of the terms of the effective action which contain only the effective gauge fields and involve an additional function \( \tilde{G} \). We briefly described how regularization and renormalization are reflected in the dressing functions. The obtained effective action is assumed to contain the quantum contributions of the gauge theory, perturbative and non-perturbative, but not including the soliton-like vacuum effects. Stated otherwise, all these effects are assumed to be contained in a limited number of dressing functions \( (G_A, G_c, \tilde{G}) \). Application and consistency checks of this effective action for the case of high-momentum QCD are presented elsewhere [9].

Acknowledgements

The work of I.K. was supported by the Programa Mecesup FSM9901 of the Ministry of Education (Chile) and also by Conicyt (Chile) under grant 8000017. The work of G.C. and I.S. was supported by Fondecyt (Chile) grant No. 1010094, and 8000017, respectively. I.K. is grateful to Tim Jones for suggesting the terminology “dressing” for the dressing functions.
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