***T*-odd correlations in charged K_{l4} decays**

Alessandra Retico

*Theory Division, CERN, CH-1211 Geneva 23, Switzerland**and INFN, Sezione di Roma and Dipartimento di Fisica, Università di Roma "La Sapienza," P. le A. Moro 5, I-00185 Rome, Italy*

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We analyze the sensitivity to physics beyond the standard model of *T*-odd correlations in K_{l4} decays, which do not involve lepton polarization. We show that a combined analysis of $K_{\mu 4}^+$ and $K_{\mu 4}^-$ decays can lead to new constraints about *CP* violation in $\Delta S=1$ charged-current interactions, complementary to those obtained from the transverse muon polarization in $K_{\mu 3}$ and of comparable accuracy.

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Charged kaon decays provide an ideal framework to study *direct CP* violation, or *CP* violation of pure $\Delta S=1$ origin [1]. Within the standard model (SM), this is expected to be very suppressed (at the 10^{-5} – 10^{-6} level) in nonleptonic decays, such as $K^\pm \rightarrow (3\pi)^\pm$, and absolutely negligible in charged-current semileptonic decays, such as $K_{l3}^\pm$ and $K_{l4}^\pm$. These processes therefore offer a clean testing ground for nonstandard scenarios, which could soon be studied with high precision at both the KLOE [2] and NA48 experiments [3], where large fluxes of both K^+ and K^- are expected.

The simplest *CP*-violating observable in charged kaon decays is the charge asymmetry of the decay widths $\Gamma(K^+ \rightarrow f) - \Gamma(K^- \rightarrow \bar{f})$, or, equivalently, the charge asymmetry of any *T*-even differential decay distribution. However, these observables can be different from zero only in the presence of at least two weak amplitudes with different final-state interaction (FSI) phases. In order to avoid this condition, which provides a severe model-independent suppression of charge asymmetries in nonleptonic modes [1] (and would prevent the study of *CP* violation in K_{l3}), *CP* violation can be tested by means of *T*-odd correlations. The most well-known example of such correlations is the transverse muon polarization in $K_{\mu 3}^+$, $P_\mu^T = \langle \vec{\sigma}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi) / |\vec{p}_\mu \times \vec{p}_\pi| \rangle$, which has been extensively discussed in the literature [4] (see also [5] and references therein) and has been found to be consistent with zero below the 10^{-2} level [6].

Contrary to the K_{l3} case, K_{l4} modes offer the possibility of studying *T*-odd correlations even without measuring the lepton polarization, which would not be accessible in experiments such as KLOE and NA48. In particular, the expectation value of the triple product $\vec{p}_l \cdot (\vec{p}_{\pi_1} \times \vec{p}_{\pi_2})$ provides a *T*-odd correlation accessible in K_{l4} decays only. Indeed, similar *T*-odd correlations have been extensively studied in the case of the rare decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ [7,8]. Here a nonvanishing result has also been recently observed [9]. However, in neutral modes the contribution from indirect *CP* violation is largely dominant and, experimentally, it is very hard to isolate a possible direct *CP*-violating effect [7,8].

In general a nonvanishing *T*-odd correlation does not necessarily imply evidence of *CP* violation: this is true only in the absence of FSI phases. The latter are known to be ex-

tremely suppressed in $K_{\mu 3}^+$ [10], but $\mathcal{O}(10^{-1})$ strong phases are certainly present in K_{l4} . However, this problem can be overcome by the analysis of both K^+ and K^- modes, whose *T*-odd correlations can be combined to construct a pure *CP*-violating observable. Therefore this study appears particularly appealing for high statistics experiments with both K^+ and K^- fluxes, such as KLOE and NA48.

In this Brief Report we present a general analysis of the above-mentioned *T*-odd correlation in K_{l4} decays. We shall compare the sensitivity of this observable with respect to P_μ^T in testing the most general structure of $\Delta S=1$ charged currents. As we shall show, K_{l4} decays can be used to extract new information about nonstandard sources of *CP* violation.

Assuming purely left-handed neutrinos, the most general dimension-6 effective Hamiltonian relevant to K_{l3} and K_{l4} decays can be written as

$$\mathcal{H}^{\text{eff}} = 2\sqrt{2}G_F V_{us}^* \sum_i C_i O_i + \text{H.c.} \quad (1)$$

where the operator basis is given by

$$\begin{aligned} O_L^V &= \bar{s}_L \gamma^\mu u_L \bar{\nu}_L \gamma_\mu l_L, \\ O_R^V &= \bar{s}_R \gamma^\mu u_R \bar{\nu}_L \gamma_\mu l_L, \\ O_L^S &= \bar{s}_R u_L \bar{\nu}_L l_R, \\ O_R^S &= \bar{s}_L u_R \bar{\nu}_L l_R, \\ O_L^T &= \bar{s}_R \sigma^{\mu\nu} u_L \bar{\nu}_L \sigma_{\mu\nu} l_R, \\ O_R^T &= \bar{s}_L \sigma^{\mu\nu} u_R \bar{\nu}_L \sigma_{\mu\nu} l_R. \end{aligned} \quad (2)$$

Within the SM, all the C_i are negligible except for $C_L^V=1$ and, in general, all the C_i can be assumed to be real in the absence of *CP* violation.

The hadronic matrix elements relevant to the $K_{\mu 3}^+$ decay can be written as

$$\langle \pi^0(p_\pi) | \bar{s} \gamma^\mu u | K^+(p_K) \rangle = \frac{1}{\sqrt{2}} [(p_k + p_\pi)^\mu f_+(q^2) + (p_k - p_\pi)^\mu f_-(q^2)],$$

$$\langle \pi^0(p_\pi) | \bar{s} u | K^+(p_K) \rangle = \frac{1}{\sqrt{2}} m_K B_S(q^2),$$

$$\langle \pi^0(p_\pi) | \bar{s} \sigma^{\mu\nu} u | K^+(p_K) \rangle = i \frac{B_T(q^2)}{\sqrt{2} m_K} (p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu), \quad (3)$$

where $q^2 = (p_k - p_\pi)^2 = (p_l + p_\nu)^2$. The form factors are all real (neglecting tiny electromagnetic effects [10]) and their dependence on q^2 starts only beyond lowest order in the chiral expansion (see, e.g., Ref. [11]). Concerning scalar and vector form factors, the lowest-order expressions, which already provide a reasonable approximation for our purposes, are given by¹

$$f_+ = 1, \quad f_- = 0, \quad B_S = \frac{m_K}{m_s + m_u} \approx 4. \quad (4)$$

The tensor form factor, which cannot be directly related to observable quantities or fundamental parameters (such as m_s), has recently been measured on the lattice obtaining $B_T = 1.23 \pm 0.09$ (with a negligible q^2 dependence) [13], in good agreement with the estimate $B_T \approx 1$ made in [14]. As the scalar and the tensor operators have non-null anomalous dimensions, B_S and B_T scale according to the renormalization group equations. The values given above refer to a renormalization scale around 2 GeV. With these notations the transverse muon polarization in $K_{\mu 3}^+$ is given by

$$P_\mu^T = \frac{1}{2} \frac{m_\mu}{m_K} \left\langle \frac{\vec{p}_\nu \times \vec{p}_\mu}{E_\nu E_\mu} \text{Im } \xi \right\rangle, \quad (5)$$

where

$$\xi = \left[\left(1 - \frac{f_-}{f_+} \right) + \frac{m_K}{m_\mu} \frac{C_+^S B_S}{C_+^V f_+} + \frac{2 p_K (p_\nu - p_\mu)}{m_\mu m_K} \frac{C_+^T B_T}{C_+^V f_+} \right] \times \left[1 - \frac{m_\mu}{m_K} \frac{C_+^T B_T}{C_+^V f_+} \right]^{-1} \quad (6)$$

and $C_\pm^i = C_R^i \pm C_L^i$. Neglecting the kinematical dependence of $\text{Im } \xi$, the experimental information on P_μ^T implies $\text{Im } \xi$

$= -0.013 \pm 0.016$ or $|\text{Im } \xi| < 0.033$ (90% C.L.) [6]. Because of the chiral enhancement of the scalar matrix element, the most stringent constraints are obtained on the CP -violating phases of $C_{L,R}^S$: $|\text{Im } C_{L,R}^S| \lesssim 10^{-3}$ (assuming C_+^V to be real). This holds in the absence of cancellations between left-handed and right-handed terms. However, since the parity-violating combination $\text{Im } C_-^S$ is not accessible in $K_{\mu 3}$, in principle larger values of $\text{Im } C_L^S$ and $\text{Im } C_R^S$, such that $|\text{Im } C_-^S| \gg |\text{Im } C_+^S|$, cannot be excluded yet.

The hadronic matrix elements appearing in K_{l4} decays, within the SM, can be parametrized as [11]

$$\langle \pi^+(p_+) \pi^-(p_-) | \bar{s} \gamma^\mu u | K^+(p_K) \rangle = -\frac{H}{m_K^3} \epsilon_{\mu\nu\rho\sigma} L^\nu P^\rho Q^\sigma, \quad (7)$$

$$\langle \pi^+(p_+) \pi^-(p_-) | \bar{s} \gamma^\mu \gamma_5 u | K^+(p_K) \rangle = \frac{-i}{m_K} [P_\mu F + Q_\mu G + L_\mu R],$$

where $P_\mu = (p_+ + p_-)_\mu$, $Q_\mu = (p_+ - p_-)_\mu$, $L_\mu = (p_l + p_\nu)_\mu$, and the $\mathcal{O}(1)$ form factors H , F , G , R are complex functions of three independent kinematical variables. A precise experimental determination of these form factors has recently been obtained by the BNL-E865 Collaboration [15], updating the old result by Rosselet *et al.* [16].

Concerning the matrix element of nonstandard contributions, not determined by experiments, we shall restrict our attention to those not vanishing at lowest order in the chiral expansion (expected to be dominant):

$$\langle \pi^+(p_+) \pi^-(p_-) | \bar{s} \gamma_5 u | K^+(p_K) \rangle = iS,$$

$$\langle \pi^+(p_+) \pi^-(p_-) | \bar{s} \sigma^{\mu\nu} \gamma_5 u | K^+(p_K) \rangle = \frac{T}{m_K^2} [p_-^\mu (p_+^\nu + p_K^\nu) - p_-^\nu (p_+^\mu + p_K^\mu)]. \quad (8)$$

The lowest-order results for S (see, e.g., Ref. [11]) and T (see Ref. [14]) are given by

$$S = \frac{\sqrt{2} m_K}{3 F_\pi} B_S, \quad T = \frac{m_K}{\sqrt{2} F_\pi} B_T, \quad (9)$$

but of course these form factors also acquire a nontrivial phase beyond lowest order.

The matrix element is then given by

¹For the scalar form factor, see, e.g., Ref. [12], and for a complete next-to-leading [$\mathcal{O}(p^4)$] evaluation of the vector and axial form factors, see, e.g., Ref. [11].

$$\begin{aligned}
\mathcal{M} = \sqrt{2} G_F V_{us}^* \left\{ \left[-C_+^V H \epsilon^{\mu\nu\rho\sigma} \frac{L_\nu P_\rho Q_\sigma}{m_K^3} - i C_-^V \frac{P^\mu}{m_K} \left(F + 3 \frac{m_l}{m_K} \frac{C_-^T}{C_-^V} T \right) - i C_-^V \frac{Q^\mu}{m_K} \left(G + \frac{m_l}{m_K} \frac{C_-^T}{C_-^V} T \right) \right. \right. \\
\left. \left. - i C_-^V \frac{L^\mu}{m_K} \left(R + 2 \frac{m_l}{m_K} \frac{C_-^T}{C_-^V} T \right) \right] \bar{u}_L(p_\nu) \gamma_\mu v_L(p_l) + i \left[C_-^S S - 2 C_-^T T \frac{(p_l - p_\nu)(p_+ + p_K)}{m_K^2} \right] \bar{u}_L(p_\nu) v_R(p_l) \right. \\
\left. - 4 C_-^T T \frac{P_+^\mu P_{K-}^\nu}{m_K^2} \bar{u}_L(p_\nu) \sigma_{\mu\nu} v_R(p_l) \right\}. \quad (10)
\end{aligned}$$

The interference between vector and scalar terms leads to the following T -odd term in the matrix element squared, summed over lepton polarizations:

$$\begin{aligned}
\sum_{\text{pol}} \mathcal{M}^2(K^+) |_{T\text{-odd}} = -16 G_F^2 |V_{us}|^2 \frac{m_l}{m_K^2} \\
\times \text{Im} \left\{ C_+^V H (C_-^S S)^* \right. \\
\left. \times \left[1 - 2 \frac{(C_-^T T)^*}{(C_-^S S)^*} \frac{(p_l - p_\nu)(p_+ + p_K)}{m_K^2} \right] \right\} \\
\times (\vec{p}_+ \times \vec{p}_-) \cdot \vec{p}_{l^+}. \quad (11)
\end{aligned}$$

As can be noted, if the form factors S , T , and H were all relatively real, this term would vanish in the absence of CP violation. In the CP -conjugate process $K^- \rightarrow \pi^+ \pi^- l^- \nu$, the analogous interference term is given by

$$\begin{aligned}
\sum_{\text{pol}} \mathcal{M}^2(K^-) |_{T\text{-odd}} = +16 G_F^2 |V_{us}|^2 \frac{m_l}{m_K^2} \\
\times \text{Im} \left\{ (C_+^V)^* H C_-^S (S)^* \right. \\
\left. \times \left[1 - 2 \frac{C_-^T (T)^*}{C_-^S (S)^*} \frac{(p_l - p_\nu)(p_- + p_K)}{m_K^2} \right] \right\} \\
\times (\vec{p}_+ \times \vec{p}_-) \cdot \vec{p}_{l^-}. \quad (12)
\end{aligned}$$

Thus if we neglect the highly suppressed tensor term quadratic in $(\vec{p}_+ - \vec{p}_-)$, the sum of these two T -odd correlations becomes a clear CP -violating observable.

The unpolarized differential decay rate of K_{l4} decays can generally be written as

$$\begin{aligned}
d\Gamma = G_F^2 |V_{us}|^2 N(s_\pi, s_l) J_5(s_\pi, s_l, \theta_\pi, \theta_l, \phi) \\
\times ds_\pi ds_l d(\cos \theta_\pi) d(\cos \theta_l) d\phi, \quad (13)
\end{aligned}$$

where the five independent kinematical variables are defined as in [11] and all the dynamical information is encoded in J_5 . Furthermore, from the experimental point of view it is convenient to decompose J_5 as [17]

$$\begin{aligned}
J_5 = 2(1 - z_l) [I_1 + I_2 \cos 2\theta_l + I_3 \sin^2 \theta_l \cos 2\phi \\
+ I_4 \sin 2\theta_l \cos \phi + I_5 \sin \theta_l \cos \phi + I_6 \cos \theta_l \\
+ I_7 \sin \theta_l \sin \phi + I_8 \sin 2\theta_l \sin \phi \\
+ I_9 \sin^2 \theta_l \sin 2\phi], \quad (14)
\end{aligned}$$

showing the explicit dependence on the leptonic variables θ_l and ϕ . The T -odd correlations (11) and (12) contribute to I_7 and, neglecting the suppressed contribution proportional to C_-^T , the CP -violating combination $[I_7(K_{l4}^+) + I_7(K_{l4}^-)]$ is given by

$$\begin{aligned}
I_7(K_{l4}^+) + I_7(K_{l4}^-) = \frac{m_l}{m_K} \frac{\lambda^{1/2}(m_K^2, s_\pi, s_l)}{(1 - z_l)} \gamma \sqrt{s_l} \sqrt{s_\pi - 4m_\pi^2} \\
\times \sin \theta_\pi \times \text{Re}(HS^*) |C_+^V|^2 \text{Im} \left(\frac{C_-^S}{C_+^V} \right) \quad (15)
\end{aligned}$$

$[\gamma = \{1 - \lambda(m_K^2, s_\pi, s_l)/(m_K^2 - s_l + s_\pi)^2\}^{-1/2}]$. As expected, this observable is suppressed by the lepton mass and in practice can be studied only in the muon case. In these channels, I_7 would reach 20% of the dominant form factor (I_1) for $\text{Im}(C_-^S/C_+^V) \sim 1$ and taking into account $\mathcal{O}(10^{-1})$ FSI phases [11]. Thus in high-statistics experiments the possible bounds on $|\text{Im} C_-^S|$ could become competitive with the $\mathcal{O}(10^{-3})$ bounds on $|\text{Im} C_+^S|$, derived from $P_\mu^T(K_{\mu 3}^+)$.

In particular, for an integrated flux of 10^{10} tagged $K^\pm$ decays (yielding $\approx 10^5$ reconstructed $K_{\mu 4}$ events), as expected both at KLOE [2] and NA48 [3], we estimate that $|\text{Im} C_-^S|$ could be bounded at the 10^{-2} level. Although these constraints on $|\text{Im} C_-^S|$ are less strict than the ones on $|\text{Im} C_+^S|$ derived from $K_{\mu 3}^+$ decays, we recall that the two bounds are in principle independent. Thus, constraining $|\text{Im} C_-^S|$ by means of an analysis of T -odd correlations in K_{l4} decays is very useful since we cannot exclude *a priori* a scenario where $|\text{Im} C_-^S| \gg |\text{Im} C_+^S|$.

In this Brief Report we have presented a general analysis of T -odd correlations accessible in K_{l4} decays, without the

measurement of lepton polarization. As we have shown, in general these are not clean CP -violating observables, due to the large FSI phases affecting K_{l4} amplitudes. However, combining information from K^+ and K^- modes it is possible to construct a clean CP -violating observable, sensitive to nonstandard $\Delta S=1$ charged-current interactions. Future

high-statistics experiments on K_{l4} decays could use this observable to put new constraints on exotic nonstandard scenarios, such as R -parity-violating supersymmetry, complementary to those obtained from K_{l3} decays [4].

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