Can hadronic resummation explain the large elliptic flow and small HBT radii seen at RHIC?
particle density dependence is assumed to be that of a projected uniform sphere of radius equal to the projectile radius, $R = r_0 A^{1/3}$, where $r_0 = 1.12$ fm and $A$ is the atomic mass number of the projectile. For $b > 0$ (non-central collisions) the transverse particle density is that of overlapping projected spheres. The longitudinal particle hadronization position ($z_{had}$) and time ($t_{had}$) are determined by the relativistic equations [13],

$$z_{had} = \tau_{had} \sinh y; \ t_{had} = \tau_{had} \cosh y$$

(3)

where $y$ is the particle rapidity and $\tau_{had}$ is the hadronization proper time. Thus, apart from particle multiplicities, the hadronization model has three free parameters to extract from experiment: $\sigma_0$, $T$, and $\tau_{had}$. The hadrons included in the calculation are pions, kaons, nucleons and lambda’s ($\pi$, $K$, $N$, and $\Lambda$), and the $\rho$, $\omega$, $\eta$, $\eta'$, $\phi$, $\Delta$, and $K^*$ resonances. For simplicity, the calculation is isospin averaged (e.g., no distinction is made among a $\pi^+$, $\pi^0$, and $\pi^-$). Resonances are present at hadronization and also can be produced as a result of rescattering. Initial resonance multiplicity fractions are taken from Hermann and Bertisch [12], who extracted results from the HELIOS experiment [14]. The initial resonance fractions used in the present calculations are: $\eta/\pi = 0.05$, $\rho/\pi = 0.1$, $\rho/\omega = 3$, $\phi/(\rho + \omega) = 0.12$, $\eta’/\eta = K^*/\omega = 1$ and, for simplicity, $\Delta/N = 0$.

The second stage in the calculation is rescattering which finishes with the freeze out and decay of all particles. Starting from the initial stage ($t = 0$ fm/c), the positions of all particles are allowed to evolve in time in small time steps ($dt = 0.1$ fm/c) according to their initial momenta. At each time step each particle is checked to see a) if it decays, and b) if it is sufficiently close to another particle to scatter with it. Isospin-averaged s-wave and p-wave cross sections for meson scattering are obtained from Prakash et al. [15]. The calculation is carried out to 100 fm/c, although most of the rescattering finishes by about 30 fm/c. The rescattering calculation is described in more detail elsewhere [11].

Calculations are carried out assuming initial parameter values and particle multiplicities for each type of particle. In the last stage of the calculation, the freeze-out and decay momenta and space-times are used to produce observables such as pion, kaon, and nucleon multiplicities and transverse momentum and rapidity distributions. The values of the initial parameters of the calculation and multiplicities are constrained to give observables which agree with available measured hadronic observables. As a cross-check on this, the total kinetic energy from the calculation is determined and compared with the RHIC center of mass energy of $\sqrt{s} = 130$ GeV to see that they are in reasonable agreement. Particle multiplicities were estimated from the charged hadron multiplicity measurements of the RHIC PHOBOS experiment [16]. Calculations were carried out using isospin-summed events containing at freezeout about 5000 pions, 500 kaons, and 650 nucleons. The hadronization model parameters used were $T = 500$ MeV, $\sigma_0 = 2.4$, and $\tau_{had} = 1$ fm/c. It is interesting to note that the same value of $\tau_{had}$ was required in a previous rescattering calculation to successfully describe results from SPS Pb+Pb collisions [11]. Figure 1 shows $m_T$ distributions for pions, kaons, and nucleons from the rescattering calculation for $b = 0$ fm near midrapidity ($-1 < y < 1$) fitted to exponentials of the form $\exp(-m_T/B)$, where $B$ is the slope parameter. The extracted slope parameters shown in Figure 1 are close in value to preliminary measurements from the PHENIX experiment for the $\pi^-$, $K^-$, and anti-proton of $190 \pm 10$, $300 \pm 30$, and $565 \pm 50$ MeV, respectively [17]. Thus, we see that if all hadrons begin at a common temperature of 300 MeV, the hadronic rescattering alone is able to generate enough radial flow to account for the differences in slope among the pion, kaon, and nucleon $m_T$ distributions.

The elliptic flow and two-pion HBT observables are also calculated from the freeze-out momenta and space-time positions of the particles at the end of the rescattering stage. The elliptic flow variable, $v_2$, is defined as [18]

$$v_2 = \langle \cos(2\phi) \rangle; \phi = \arctan(p_y/p_x)$$

(4)

where $p_x$ and $p_y$ are the $x$ and $y$ components of the particle momentum, and $x$ is in the impact parameter direction and $y$ is in the “out of plane” direction (i.e. $x - z$ is the reaction plane and $z$ is the beam direction). The HBT pion source parameters are extracted from the rescattering calculation using the same method as was applied for previous SPS-energy rescattering calculations [11]. The Pratt-Bertisch “out-side-long” radius parameterization is used [19, 20] yielding the four parameters $R_{T_{rad}}$, $R_{T_{out}}$, $R_{L_{out}}$, and $\lambda$, which represent two mutually perpendicular transverse (to the beam direction) radius parama-

FIG. 1: Transverse mass distributions from the rescattering model. The lines are exponential fits to the distributions and the slope parameters are shown.
Rescatt. (b=8 fm) vs. STAR (11-45\%)

FIG. 2: Comparison of $v_2$ calculated from the rescattering model for $b = 8$ fm with STAR measurements for pions and nucleons. The plotted points with error bars are the rescattering calculations and the lines show the trends of the STAR measurements. Average errors on the STAR measurements are $\approx 0.002$ for pions and $0.006$ for protons-antiprotons.

ters, a radius parameter along the beam direction, and a parameter related to the “strength” of the two-pion correlations, respectively.

Figure 2 shows the $p_T$ dependence of $v_2$ for pions and nucleons extracted from the $b = 8$ fm rescattering calculation compared with the trends of the STAR measurements for $\pi^+ + \pi^-$ and $p + p$--bar at 11--45\% centrality [1], which roughly corresponds to this impact parameter. As seen, the rescattering calculation values are in reasonable agreement with the STAR measurements. Thus, the same rescattering mechanism that can account for the radial flow seen in Figure 1 also can account for the magnitude and $p_T$ dependence of the elliptic flow for pions and nucleons.

The pion source parameters extracted from HBT analyses of rescattering calculations for three different impact parameters, $b = 0$, 5, and 8 fm, are compared with STAR $\pi^-$ measurements at three centrality bins [2] in Figure 3. Note that the PHENIX HBT results [3] are in basic agreement with the STAR results. The STAR centrality bins labeled “3”, “2”, and “1” in the figure correspond to 12\% of central, the next 20\%, and the next 40\%, respectively. These bins are roughly approximated by the impact parameters used in the rescattering calculations, i.e. the average impact parameters of the STAR centrality bins are estimated to be within $\pm 2$ fm of the rescattering calculation impact parameters used to compare with them. In the left panel, the centrality dependence of the HBT parameters is plotted for a $p_T$ bin of 0.125 -- 0.225 GeV/c. In the right panel, the $m_T$ dependence of the HBT parameters is plotted for centrality bin 3, for the STAR measurements, or $b = 0$ fm, for the rescattering calculations. Although there are differences in some of the details, the trends of the STAR HBT measurements are seen to be described rather well by the rescattering calculation.

As shown above, the radial and elliptic flow as well as the features of the HBT measurements at RHIC can be adequately described by the rescattering model with the hadronization model parameters given earlier. The results of the calculations are found to be sensitive to the value of $\tau_{\text{had}}$ used, as was studied in detail for SPS rescattering calculations [11]. For calculations with $\tau_{\text{had}} > 1$ fm/c the initial hadron density is smaller, fewer collisions occur, and the rescattering-generated flow is reduced, reducing in magnitude the radial and elliptic flow and most of the HBT observables. Only the HBT parameter $R_{\text{long}}$ increases for larger $\tau_{\text{had}}$ reflecting the increased longitudinal size of the initial hadron source, as seen in Equation 3. One can compensate for this reduced flow in the other observables by introducing an ad hoc initial “flow velocity parameter”, but the increased $R_{\text{long}}$ cannot be compensated by this new parameter. In this sense, the initial hadron model used in the present calculations with $\tau_{\text{had}} \sim 1$ fm/c and no initial flow is uniquely determined with the help of $R_{\text{long}}$.

At this point, it is appropriate to discuss how physical the initial conditions of the present rescattering calculation are. In order to use this picture, one must assume: 1) hadronization occurs very rapidly after the nuclei have passed through each other, i.e. $\tau_{\text{had}} = 1$ fm/c, 2) the hadrons thermalize rapidly, and 3) hadrons or at least hadron-like objects can exist in the early stage of the collision where there are maximum values of $T$ and $\rho$ of 300 MeV and 8 GeV/fm$^3$, respectively.

Addressing assumption 3) first, in the calculation the maximum number density of hadrons at mid-rapidity at \( t = 0 \) fm/c is \( 6.8 \text{ fm}^{-3} \), rapidly dropping to about \( 1 \text{ fm}^{-3} \) at \( t = 4 \) fm/c. Since most of these hadrons are pions, it is useful as a comparison to estimate the effective volume of a pion in the context of the $\pi^{-} \pi^{-}$ scattering cross section, which is about \( 0.8 \text{ fm}^{2} \) for s-waves [15]. The “radius” of a pion is found to be 0.25 fm and the effective pion volume is 0.065 fm$^{3}$, the reciprocal of which is about \( 15 \text{ fm}^{-3} \). From this it is seen that at the maximum hadron number density in the calculation, the particle occupancy of space is estimated to be less than 50\%, falling rapidly with time. One could speculate that this may be enough spacial separation to allow individual hadrons or hadron-like objects to keep their identities and not melt into quark matter, resulting in a “super-heated” semi-classical gas of hadrons at very early times, as assumed in the present calculation.

Assumptions 1) and 2) can both be motivated by the Color Glass Condensate model [21, 22]. Before the collision of two relativistic nuclei, the nuclei can be described as two thin (lorentz-contracted) sheets of Color Glass (a dense glass of gluons). The use of the term “glass” is not merely an analogy but is mathematically rigorous due to the time-dilated nature of the gluon field which
behave like a liquid on long timescales but a solid on short ones [21]. In the usual picture, after the collision takes place the Color Glass melts into quarks and gluons in a timescale of about 0.3 fm/c at RHIC energy, and then the matter expands and thermalizes into quark matter by about 1 fm/c. The timescale in the Color Glass Condensate model for thermalized matter matches the timescale needed in the rescattering model for an initial thermalized hadron gas. Thus one is tempted to modify the collision scenario such that instead of the Color Glass melting into quarks and gluons just after the collision, the sudden impact of the collision “shatters” it directly into hadronic fragments which then thermalize on the same timescale as in the parton scenario due to the hadronic strong interactions.

In summary, a thermal model + hadronic rescattering picture is able to adequately describe the large elliptic flow and small HBT radii recently measured at RHIC. A feature of this picture is a very early hadronization time of about 1 fm/c after the initial collision of the nuclei.

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