The Higgs Boson Production Cross Section
as a Precision Observable?

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Abstract

We investigate what can be learned at a linear collider about the sector of electroweak symmetry breaking from a precise measurement of the Higgs boson production cross section through the process $e^+e^- \rightarrow hZ$. We focus on deviations from the Standard Model arising in its minimal supersymmetric extension. The analysis is performed within two realistic future scenarios, taking into account all prospective experimental errors on supersymmetric particle masses as well as uncertainties from unknown higher order corrections. We find that information on $\tan \beta$ and $M_A$ could be obtained from a cross section measurement with a precision of $0.5 - 1\%$. Alternatively, information could be obtained on the gaugino mass parameters $M_2$ and $\mu$ if they are relatively small, $M_2, \mu \approx 200$ GeV.
1 Introduction

One of the fundamental problems facing particle physics is understanding the nature of electroweak symmetry breaking. If this symmetry breaking is due to a light Higgs boson, then the Higgs boson will certainly be discovered at the Tevatron [1] or the LHC [2–4]. The remaining challenge will then be to understand whether this new object is the Higgs boson of the Standard Model (SM) or some more exotic particle. In the SM, the couplings of the Higgs boson to all particles are completely fixed once the mass is known and so the validity of the SM can be confirmed by measuring Higgs production and decay rates and eventually the Higgs potential itself [5]. In alternative models, the Higgs couplings can be quite different from the SM values and so can potentially be used to distinguish between models.

A linear collider (LC) with an energy in the range $\sqrt{s} \sim 350 - 500$ GeV has the capability for performing precision measurements of both Higgs boson production and decay rates [6–8], provided that the Higgs boson mass, $M_h$, lies in the range predicted by electroweak precision observables, $M_h \lesssim 200$ GeV [9, 10]. The dominant production mechanism for such a light Higgs boson is $e^+e^- \rightarrow hZ$ [11], with the largest decay channel being $h \rightarrow b\bar{b}$ or $h \rightarrow WW^*$. The measurements must be interpreted in terms of SM expectations or some model of physics beyond the SM. The goal is then to use the experimental data to disentangle the underlying structure of the model. An important question is thus the required experimental precision for production rates and branching ratios in order to distinguish it from the SM and perhaps to measure the parameters of the new theory.

An integrated luminosity of $L \sim 500$ fb$^{-1}$ and $\sqrt{s} = 350$ GeV is expected to produce measurements of the various Higgs branching ratios with precisions in the $2 - 10\%$ range at an $e^+e^-$ collider [6–8]. The precision will be less at $\sqrt{s} = 500$ GeV, primarily due to the reduced rate [6,12]. The LHC can measure some, but not all, Higgs branching ratios, with a precision which is typically less than that obtainable at a LC [13].

Supersymmetric (SUSY) theories [14] are widely considered as the theoretically most appealing extension of the SM. They are consistent with the approximate unification of the three gauge coupling constants at the GUT scale and provide a way to cancel the quadratic divergences in the Higgs sector, hence stabilizing the huge hierarchy between the GUT and the Fermi scales. Furthermore, in SUSY theories the breaking of the electroweak symmetry is naturally induced at the Fermi scale, and the lightest supersymmetric particle can be neutral, weakly interacting and absolutely stable, providing a natural solution for the Dark Matter problem. Therefore the implications of the measurements of Higgs boson branching ratios have been extensively studied within the context of the minimal supersymmetric model (MSSM) [15]. This model is extremely predictive and so is useful for comparing the experimental reach achievable in various channels.

In this note we address the question of whether the total $e^+e^- \rightarrow hZ$ cross section, $\sigma_{hZ}$, can be used as a precision observable to help determine the structure of the electroweak sector of the MSSM. The measurement of the $e^+e^- \rightarrow hZ$ Higgsstrahlung production cross section is expected to achieve a $2 - 3\%$ accuracy at $\sqrt{s} = 350$ GeV [12].
This assumes $\mathcal{L} = 500\text{ fb}^{-1}$ and the analysis of the $Z \to l^+l^-$ events only. From this measurement, some restrictions can be inferred about the parameters of the MSSM, which we investigate here. For the Higgsstrahlung process, the complete next-to-leading order corrections (involving SM and SUSY particles), including all vertex and box corrections have been calculated [16]. More recently also the leading two-loop corrections have been included [17]. Since these corrections are significant, their inclusion is crucial for drawing conclusions about the underlying model.

Our study differs considerably from previous studies of the Higgs branching ratios in that we investigate plausible future scenarios and estimate uncertainties from all relevant sources. We assume that some MSSM particle masses and mixing angles have been determined at the LHC and/or the LC, and vary all inputs accordingly within realistic errors, instead of fixing all parameters and then varying just one or two. Furthermore, the anticipated theory errors from unknown higher order corrections in the MSSM Higgs sector are taken into account in a consistent manner. We then ask what can be learned about the remaining unknown parameters of the model. These assumptions try to represent possible future scenarios and thus give an idea of what might be inferred from a precise $\sigma_{hZ}$ measurement.

The rest of this paper is organized as follows: In section 2 we review the necessary MSSM input parameters and existing higher order corrections in the Higgs sector. Our approach to the investigation, with emphasis on the attempt to look into realistic future scenarios, is explained in detail in section 3. Section 4 contains our analysis and the corresponding results, while conclusions can be found in section 5.

2 The MSSM: Basics

The Higgs sector of the MSSM consists of two Higgs doublets, $\mathcal{H}_1$ and $\mathcal{H}_2$ [18]. After electroweak symmetry breaking, there remain 5 physical Higgs bosons: $h, H, A,$ and $H^\pm$. In this note, we will be concerned only with the production of the lightest Higgs boson, $h$. The Higgs sector is described at tree level by two additional parameters (besides the SM parameters), which are usually chosen to be $\tan \beta$, the ratio of the Higgs VEVs, and $M_A$, the mass of the pseudoscalar Higgs boson. The mass eigenstates of the neutral scalar Higgs bosons are obtained from the interaction eigenstates $\phi_1$ and $\phi_2$ by the rotation,

$$
\begin{pmatrix}
H \\
h
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix},
$$

where at tree level

$$
\tan 2\alpha = \tan \beta \left(\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}\right).
$$

At tree level, the mass of the lightest Higgs boson is completely fixed in terms of $M_Z, M_A$ and $\tan \beta$.

The process $e^+e^- \to hZ$ proceeds (at the tree-level) via the Feynman diagram shown in Fig. 1 and is hence sensitive to the $ZZh$ coupling. At tree level, the $ZZh$ coupling in
the MSSM is altered from the SM value,
\[ g_{Zzh}^{\text{SUSY}} = g_{Zzh}^{\text{SM}} \sin(\beta - \alpha). \] (3)

For \( M_A \gg M_Z \), \( \sin(\beta - \alpha) \to 1 \) and the coupling of the lightest MSSM Higgs boson to the \( Z \) boson approaches that of the SM. We therefore expect that \( \sigma_{hZ}^{\text{SUSY}} \) will be sensitive to small \( M_A \).

Figure 1: Feynman diagram for lowest order contribution to \( e^+ e^- \to hZ \).

There are two important effects which arise when going beyond the tree level. The first is that the Higgs boson mass prediction is significantly increased by radiative corrections, leading to an upper bound at the two-loop level \([18–22]\) of \( M_h \lesssim 135 \text{ GeV} \) \([20]\). The most important corrections are those in the \( t/\tilde{t} \) sector \([22]\) and for large \( \tan \beta \) also those in the \( b/\tilde{b} \) sector. The mass matrices in the basis of the current eigenstates \( \tilde{t}_L, \tilde{t}_R \) and \( \tilde{b}_L, \tilde{b}_R \) are given by

\[ M_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_{\tilde{t}}^2 + \cos 2\beta \left( \frac{1}{2} - \frac{2}{3}s_W^2 \right) M_Z^2 & m_{\tilde{t}} X_{\tilde{t}} \\ m_{\tilde{t}} X_{\tilde{t}} & M_{\tilde{t}_R}^2 + m_{\tilde{t}}^2 + \frac{2}{3} \cos 2\beta s_W^2 M_Z^2 \end{pmatrix}, \] (4)

\[ M_{\tilde{b}}^2 = \begin{pmatrix} M_{\tilde{b}_L}^2 + m_{\tilde{b}}^2 + \cos 2\beta \left( \frac{1}{2} + \frac{1}{3}s_W^2 \right) M_Z^2 & m_{\tilde{b}} X_{\tilde{b}} \\ m_{\tilde{b}} X_{\tilde{b}} & M_{\tilde{b}_R}^2 + m_{\tilde{b}}^2 - \frac{1}{3} \cos 2\beta s_W^2 M_Z^2 \end{pmatrix}, \] (5)

where \( s_W^2 = 1 - c_W^2 = 1 - M_W^2/M_Z^2 \) and

\[ m_{\tilde{t}} X_{\tilde{t}} = m_t (A_t - \mu \cot \beta), \quad m_{\tilde{b}} X_{\tilde{b}} = m_b (A_b - \mu \tan \beta). \] (6)

Here \( A_t \) denotes the trilinear Higgs–stop coupling, \( A_b \) is the Higgs–sbottom coupling, and \( \mu \) is the Higgs mixing parameter. SU(2) gauge invariance leads to the relation

\[ M_{\tilde{t}_L} = M_{\tilde{b}_L}. \] (7)

The two mass matrices (4), (5) are diagonalized by the angles \( \theta_{\tilde{t}} \) and \( \theta_{\tilde{b}} \), respectively. The physical squark masses are \( m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1} \) and \( m_{\tilde{b}_2} \). Specifying \( m_{\tilde{t}_1}, m_{\tilde{t}_2}, \) and \( \theta_{\tilde{t}} \), along with \( \mu \) and \( \tan \beta \) therefore implicitly fixes the tri-linear mixing parameter \( A_t \), and similarly in the \( b \)-squark sector. The radiatively corrected value for the lightest MSSM Higgs boson mass, \( M_h \), depends sensitively on the parameters of the stop mass matrix (4).
The second important effect of going beyond tree level is that the SUSY particles enter into loop corrections. The complete set of one-loop corrections to the process $e^+e^- \rightarrow hZ$ has been computed in Ref. [16]. In addition, the leading two loop corrections have been included [17]. The effects of including these corrections are shown in Fig. 2 [17] and are seen to be large. This applies in particular for the two-loop corrections. Our analysis includes all one-loop SM and SUSY corrections, along with the leading two-loop corrections. From Fig. 2, we see that for $M_A \sim 100$ GeV, the inclusion of the two-loop corrections increases the rate over the one-loop result by up to $\sim 30\%$. From the analysis in Ref. [17] one can infer a theoretical uncertainty due to unknown higher order corrections for the prediction of $\sigma_{hZ}$ of $\sim 5\%$.

Figure 2: $\sigma_{hZ}$ as a function of $\sqrt{s}$ for $\tan \beta = 5$, $M_{tL} = M_{tR} = M_{bR} = 1$ TeV (the corresponding parameter for scalar leptons is set to $M_{l} = 300$ GeV), $X_t = 0$, $A_b = A_t$, $\mu = 200$ GeV, $M_2 = 200$ GeV and $m_{\tilde{g}} = 500$ GeV. The solid (dot-dashed) line represents the two-loop Feynman diagram result including (excluding) box contributions, the dotted line shows the RG $\alpha_{\text{eff}}$ approximation and the dashed line shows the one-loop Feynman diagramatic result. [17]

3 Concept of the analysis

The focus here is to determine in the context of SUSY what new information can be obtained from a precision measurement of $\sigma_{hZ}$, beyond the direct measurement of the lightest Higgs boson mass. At the time of a $\sigma_{hZ}$ measurement at the LC, SUSY (if it exists at a low mass scale) will have been discovered at the LHC and possibly confirmed by the LC. Therefore some SUSY parameters will be known with high precision from the LC measurements, while others (e.g. masses beyond the kinematic reach of the LC) will be
known with lesser precision from the LHC data. In the Higgs sector it is possible that only the lightest MSSM Higgs boson will have been measured (e.g. for $M_A \gtrsim 300$ and moderate tan $\beta$ values, tan $\beta \sim 10$) [2,3,6]. Only for relatively small masses, $M_H, M_A \lesssim \sqrt{s}/2$, will the heavy Higgs bosons be visible at the LC.

In a realistic analysis at the time of the LC the following has to be taken into account:

- uncertainties of the measured SM parameters
- uncertainties of the measured MSSM parameters
- intrinsic uncertainties on the theoretical prediction of the MSSM Higgs sector parameters ($M_h, \sigma_{hZ}, \ldots$) from unknown higher order corrections
- bremsstrahlung
- beamstrahlung
- other machine related uncertainties, e.g. due to the luminosity measurement, detector smearing etc.

A full simulation clearly goes beyond the scope of this exploratory analysis. However, we try to give a realistic impression about the information which can be obtained from a $\sigma_{hZ}$ measurement. To this end we include the following:

- the relevant SM uncertainties arising from the $m_t$ measurement
- we take into account all uncertainties on the MSSM parameters from their measurement at the LHC and/or the LC. To study the dependence of the cross section on the parameters, we vary all parameters within their expected precisions and include effects of SUSY particles beyond the leading order as described in the previous section.
- we assume a future theoretical uncertainty in the prediction of $M_h$ from the other SUSY input parameters of 0.5 GeV. For the theoretical prediction of $\sigma_{hZ}$ an uncertainty of 1% is assumed from unknown higher order corrections. (Numerically this is taken into account by allowing a variation of the Higgs boson mass as an input parameter in the $\sigma_{hZ}$ evaluation by ±0.5 GeV, although its experimental value that enters the calculation will be determined to a much higher accuracy, see below.)
- we do not include beamstrahlung, bremsstrahlung, or detector effects, which are beyond the scope of this note. While the latter can only be realized in a full simulation, the former mostly induce a shift in the numerical results, but have a much smaller effect on the errors.
- we neglect luminosity errors. Concerning these, it might be helpful not to investigate $\sigma_{hZ}$ directly, but to consider e.g. $\sigma_{hZ}/\sigma_{ZZ}$, since in this ratio many uncertainties cancel out. However, the idea of this analysis is to show the possible potential of a precise cross section measurement, which can already be obtained from an analysis of $\sigma_{hZ}$ alone.
Taking into account the relevant uncertainties in the above manner necessarily weakens the potential of a precise $\sigma_{hZ}$ measurement, see Sect. 4. However, contrary to existing analyses, which give more optimistic results, here more plausible scenarios for the LC are analyzed.

For this analysis we assume that $\sigma_{hZ}$ is measured at $\sqrt{s} = 350$ GeV with $L = 500$ fb$^{-1}$. In all the investigated scenarios, we assume that the Higgs boson mass will have been measured to an experimental accuracy of [6–8]

$$M_{h}^{\text{exp}} = 115 \pm 0.05 \text{ GeV}. \quad (8)$$

However, as mentioned above, within the MSSM this experimental error will always be dominated by the theoretical uncertainty on the prediction of $M_h$ due to unknown higher order corrections. While the current uncertainty in the $M_h$ prediction is estimated to be $\sim 3$ GeV [23], we assume for the future uncertainty

$$\delta M_h^{\text{theo}} (\text{future}) = \pm 0.5 \text{ GeV}. \quad (9)$$

Also the dependence of $M_h$ on the top quark mass is very strong, $\delta m_t / \delta M_h \approx 1$. However, $m_t$ will be determined to an accuracy better than $\sim 130$ MeV at a LC [6,24], so that the parametric uncertainty is smaller than the theoretical uncertainty. It is, however, taken into account.

Since the value of $M_h$ in the MSSM is not a free parameter, but depends on the other SUSY parameters, they have to be chosen such that the value of $M_h = 115 \pm 0.5$ GeV emerges. The numerical evaluation of the MSSM Higgs sector (including $M_h$ and $\sigma_{hZ}$) is based on the code $\text{FeynHiggsXS}$ [25,17].

4 Analysis and results

The approach explained above has been applied to two possible future scenarios. In both scenarios we make assumptions what parameters will be measured and what parameters are left free. This choice, since it involves the unknown MSSM parameters and their detectability, is of course subject to personal opinions. However, the scenarios certainly reflect the possible strength of the $\sigma_{hZ}$ measurement as explained in the previous section.

4.1 The Higgs sector scenario

In the first scenario we assume that the following masses and mixing angles have been measured at the LHC and/or the LC: In the $\tilde{t}$ sector, we chose

$$m_{\tilde{t}_1} = 500 \pm 2 \text{ GeV}$$
$$m_{\tilde{t}_2} = 700 \pm 10 \text{ GeV}$$
$$\sin \theta_{\tilde{t}} = -0.69 \pm 0.014. \quad (10)$$

$A_t$ is given implicitly in terms of $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\sin \theta_{\tilde{t}}$, $\mu$ and $\tan \beta$. We furthermore fix,

$$\mu = 200 \pm 1 \text{ GeV}. \quad (11)$$
We assume approximate unification of the trilinear Higgs-sfermion couplings and take:

\[
A_b = A_t \pm 10\% \\
A_t = A_t \pm 1\% .
\] (12)

In addition, we assume the relationship between gaugino masses predicted in many unified models. The specific values we take are:

\[
M_2 = 400 \pm 2 \text{ GeV} \\
M_1 = \frac{5}{3} s_W^2 M_2 \pm 1 \text{ GeV} \\
m_{\tilde{\beta}} = M_3 = 500 \pm 10 \text{ GeV} .
\] (13)

Finally, for the remaining sfermion sector we choose

\[
M_{\tilde{b}_R} = M_{\tilde{t}_R} \pm 10\% \\
m_{\tilde{e}_1}, m_{\tilde{e}_2} = 200 \pm 2 \text{ GeV} ,
\] (14)

where the selectron masses enter in the vertex and box corrections. The uncertainties chosen above are consistent with those given in Ref. Ref. [6].

With the above choices, the only remaining free parameters are \(M_A\) and \(\tan \beta\), which we assume to be only poorly known in this scenario. Our procedure is to pick a value for \(M_A\) and \(\tan \beta\) and check that the chosen parameters generate \(M_h = 115 \pm 0.5 \text{ GeV}\), which cuts out a slice of the \(M_A - \tan \beta\)-plane. For the above set of parameters, we then calculate \(\sigma_{hZ}\) and compare with the value obtained for the SM, \(\sigma_{hZ}^{\text{SM}} = 0.1530 \text{ pb}\). The resulting variations of the cross section from the SM value are shown in Fig. 3. Since the measurement of \(\sigma_{hZ}\) is a missing mass experiment, our results are independent of the Higgs boson decay channel.

The different panels of Fig. 3 show the regions where the rate differs from the SM prediction by a specified amount. This includes a theoretical uncertainty in the SM rate which we approximate by varying \(M_h\) within the range, \(M_h = 115 \pm 0.5 \text{ GeV}\) (as described in Sect. 3). The cross section is quite sensitive to \(\tan \beta\). A measurement which differs from the SM prediction by 1.4\% or less will restrict \(\tan \beta < 10\). Concerning the indirect \(M_A\) determination, a 1.4\% measurement would only be sensitive to \(M_A \lesssim 200 \text{ GeV}\). However, a measurement at the 0.5\% level, finding a deviation larger than 0.6\%, can mostly be realized only for \(M_A \lesssim 500 \text{ GeV}\) with only a few exceptions found at larger \(M_A\) values. Thus a weak upper bound might be established. Interestingly, this happens for values where the LHC can see only the lightest MSSM Higgs boson (in the so-called “LHC wedge region”). The currently envisaged accuracy on \(\sigma_{hZ}\) of 2−3\% is unfortunately not sufficient for \(\sigma_{hZ}\) to be used as such a precision determination.

4.2 The gaugino scenario

To demonstrate the possible amount of information that \(\sigma_{hZ}\) might deliver on \(\mu\) and \(M_2\), in this scenario we make the assumption that \(M_A\) and \(\tan \beta\) will have been measured,
Figure 3: The deviation of $\sigma_{hZ}^{\text{SUSY}}$ from $\sigma_{hZ}^{\text{SM}} = 0.1530$ pb is shown in the $M_A - \tan \beta$-plane for $M_h = 115 \pm 0.5$ GeV at $\sqrt{s} = 350$ GeV with $\mathcal{L} = 500$ fb$^{-1}$.

$$M_A = 250 \pm 10 \text{ GeV}$$
$$\tan \beta = 4 \pm 0.5 \ ,$$

(15)

but we leave the gaugino mass parameters $M_2$ and $\mu$ as free parameters (the scan stops at an upper bound of 1 TeV). The other MSSM parameters are assumed to have the same values as in Sect. 4.1, together with their corresponding uncertainties. As in the previous section, all experimental and theoretical errors are fully taken into account.
Figure 4: The deviation of $\sigma_{hZ}^{\text{SUSY}}$ from $\sigma_{hZ}^{\text{SM}} = 0.1530$ pb is shown in the $M_2 - \mu$-plane for $M_h = 115 \pm 0.5$ GeV at $\sqrt{s} = 350$ GeV with $L = 500$ fb$^{-1}$.

Fig. 4 shows the dependence of $\sigma_{hZ}^{\text{SUSY}}$ on $\mu$ and $M_2$. It is obvious that a reasonable sensitivity only appears for $M_2 \approx 200$ GeV or $\mu \approx 200$ GeV, where $\sigma_{hZ}^{\text{SUSY}}$ has a minimum. It is very unlikely that these two parameters, if they possess such a low value, will not have been measured directly, see e.g. [6] and references therein. Thus, in this scenario $\sigma_{hZ}$ can only offer complementary information which can verify the internal consistency of the MSSM (see Ref. [26] for detailed discussion on this subject).
5 Conclusions and Outlook

We have investigated whether a precise measurement of the Higgs production cross section, \( \sigma(e^+e^- \rightarrow hZ) \), offers additional information to pin down the unknown parameters of the MSSM. We have chosen two possible future scenarios. We have explained in detail what uncertainties will be present at the time of a \( \sigma_{hZ} \) measurement and how we take them into account. This includes realistic assumptions for all mass parameters together with the expected uncertainties obtainable at the LHC and/or LC. We also took into account realistic assumptions on the theoretical uncertainties for the predictions in the MSSM Higgs sector.

We find that the total rate needs to be measured to a 0.5 – 1% accuracy in order to be useful as a precision observable. Then additional information on \( \tan \beta \) or \( M_A \) (if it is not too high, \( M_A \lesssim 500 \) GeV) may be obtainable. The dependence of \( \sigma_{hZ} \) on the gaugino parameters \( \mu \) and \( M_2 \) shows a strong enough dependence to be useful only for very low values, \( \mu, M_2 \approx 200 \) GeV. Hence, in this case \( \sigma_{hZ} \) could only test the internal consistency of the MSSM.

The required precision for \( \sigma_{hZ} \) at the 1% level, as compared to the envisaged 2 – 3%, could possibly achieved by either accumulating a higher integrated luminosity (also at different center of mass energies) and/or by taking other than the leptonic Z decay modes into account.

Acknowledgement

This work supported by the Department of Energy under contract DE-AC02-98CH10886.

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